A Location Within Distribution Network Design Problem With Flexible Demand

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Abstract — Nowadays, flexibility is one of the strategic targets of many supply chain manufacturing systems. One of the approaches to state flexibility in supply chain is considering demand uncertainty in transportation issues. In this paper, we develop a simulated annealing algorithm for location distribution problem. We use statistical upper bound for objective function as a risk measure to state demand uncertainty and compare it with the deterministic approach.

Index Terms — Location, Multi commodity, Multi objective, Uncertain Demand, Simulated Annealing.

I. INTRODUCTION

Supply chain management (SCM) is the process of planning, implementing and controlling the operations of the supply chain in an efficient way. SCM spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption (Council of Supply Chain Management Professionals 2007, Simchi-Levi et al. 2004).

There are more works in literature considering concepts of SCM in variant areas that we state some of them in distribution network and location problem as follows. Altiparmak et al. (2006) developed a multi-objective genetic algorithm (MOGA) to find a set of optimal pareto solution for Supply chain network (SCN) design. Thanh et al. (2008) proposed a mixed integer programming (MIP) formulation to design and plan a production-distribution system along the supply chain. Pujari et al. (2008) presented an integrated approach for incorporation of location, production, inventory and transportation issues within a supply chain. Shu and Karimi (2009) developed two heuristic algorithms for considering concept of safety stock in supply chain networks. Kaminsky and Kaya (2008) proposed effective heuristics for inventory positioning in supply chain networks involving several centrally managed production facilities and external suppliers. Monthatipkul and Yenradee (2008) introduced an MIP model to find an optimal inventory/distribution plan (IDP) control system for a one-warehouse/multi-retailer supply chain system. Chauhan et al. (2009) designed a heuristic for Multi-commodity supply network planning and a branch and price for large-sized problems. For more detailed study, Gunasekaran and Ngai (2009) and Minner (2003) can be useful.

Nowadays, flexibility is one of the strategic goals of many supply chain manufacturing systems. Here we explain some previous studies in flexible SCM. Manzini et al. (2008) present a decision support platform towards the development of an expert system capable of supporting the integration of planning, design, management, control, and optimization of the activities in a flexible production-distribution system. The flexibility and applicability of the proposed modeling framework is illustrated through two different case studies, which highlight the benefits of coordinating both activities in such a complex supply chain (SC) environment (Bonfill et al. 2003). Jain et al. (2008) develop a new approach based on Fuzzy Association Rule Mining to support the decision makers by enhancing the flexibility in making decisions for evaluating agility with both tangibles and intangibles attributes/criteria such as

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Flexibility, Profitability, Quality, Innovativeness, Proactiveness, Speed of response, Cost and Robustness.

One of the approaches which states flexibility in supply chain is considering demand uncertainty in transportation issues. In this paper we use problem definition that is presented by Afshari et al. (2010) and develop a simulated annealing (SA) algorithm to consider uncertain demand. We use statistical upper bound for objective function as a risk measure to state demand uncertainty and compare it with the deterministic approach.

The rests of paper are as follows. Model description is stated in section II. In Section III, mathematical model is formulated, SA algorithm is developed in Section IV, computational results are indicated in section V and conclusions are discussed in section VI.

II. MODEL DESCRIPTION

We define problem as follows:

Components of supply chain such as are illustrated in Figure 1 are introduced.

Central warehouses: The main demanded stocks of supply chain are supplied here. There are two potential location for central warehouses, capital of country and south port.

Regional warehouses: Demanded stocks between central warehouses and customers are distributed here. There are eight potential locations for regional warehouses that they are in the capital of provinces.

Customers: There are twenty eight customers that are located in the cities of the provinces.

Goods: Five types of commodities can be supplied for the customers demanding five families of cars Afshari et al. (2010).

Assumptions of problem are as follows:

We consider two potential central warehouses which at least one of them should be located; there are restricted capacities for both central and regional warehouses. Transportation cost per unit is put as a coefficient of distance between central and regional warehouses and also between regional warehouses and customers. There is a minimum level of customer satisfaction. We have two objectives for problem, the former is minimizing total cost including establishment and transportation cost and the latter is maximizing customer satisfaction.

III. MODEL FORMULATION

A. Sets and indices

- $L$: Sets of central warehouses ($|L| = l, k \in L$),
- $M$: Sets of regional warehouses ($|M| = m, j \in M$),
- $N$: Sets of customers ($|n| = n, i \in N$),
- $O$: Sets of good types ($|O| = o, t \in O$).

B. Variables

- $v_k = \begin{cases} 1, & \text{if the potential point of } k \text{ for central warehouses is located} \\ 0, & \text{Otherwise} \end{cases}$
- $u_j = \begin{cases} 1, & \text{if the potential point of } j \text{ for regional warehouses is located} \\ 0, & \text{Otherwise} \end{cases}$
- $x_{ijt}$ Percentage of demand customer $i$ for commodity $t$ that is supplied by regional warehouse $j$,
- $y_{kt}$ Percentage of demand regional warehouse $j$ for commodity $t$ that is supplied by central warehouse $k$.

C. Parameters

- $a_{it}$ Demand of customer $i$ for commodity $t$,
- $b_{jt}$ Capacity of regional warehouse $j$ for commodity $t$,
- $c$ Cost of transportation per unit,
- $d_{ij}$ Distance between regional warehouse $j$ and customer $i$,
- $d_{jk}$ Distance between regional warehouse $j$ and central warehouse $k$,
- $e_{kt}$ Capacity of central warehouse $k$ for commodity $t$,
- $P$ Coefficient of total cost in objective function,
- $q_k$ Cost of installation central warehouse $k$,
- $s_{it}$ Minimum level of customer satisfaction $i$ for commodity $t$.
- $w_j$ Cost of installation regional warehouse $j$.
D. Mathematical Model

\[
\text{Min } Z_1 = \sum_{t=1}^{a} \sum_{j=1}^{m} \sum_{l=1}^{n} c_{d_{ij}} a_{it} x_{ijt} \\
+ \sum_{t=1}^{a} \sum_{j=1}^{m} \sum_{l=1}^{n} c_{d_{ij}} a_{it} y_{jkt} + \sum_{j=1}^{m} w_{j} u_{j} + \sum_{k=1}^{l} q_{k} v_{k}
\]

\[
\text{Max } Z_2 = (1 - P) \sum_{t=1}^{a} \sum_{j=1}^{m} \sum_{l=1}^{n} x_{ijt}
\]

\[
\sum_{t=1}^{a} \sum_{j=1}^{m} x_{ijt} \leq n \cdot o_{u_{j}} \quad \forall j \quad (1)
\]

\[
\sum_{t=1}^{a} \sum_{j=1}^{m} y_{jkt} \leq m \cdot o \cdot v_{k} \quad \forall k \quad (2)
\]

\[
\sum_{j=1}^{m} a_{it} x_{ijt} \leq b_{jt} \quad \forall j, t \quad (3)
\]

\[
\sum_{j=1}^{m} b_{jt} y_{jkt} \leq e_{kt} \quad \forall k, t \quad (4)
\]

\[
\sum_{j=1}^{m} x_{ijt} \geq s_{it} \quad \forall i, t \quad (5)
\]

\[
\sum_{k=1}^{l} b_{jt} y_{jkt} \geq \sum_{i=1}^{n} a_{it} x_{ijt} \quad \forall j, t \quad (6)
\]

\[
\sum_{j=1}^{m} x_{ijt} \leq 1 \quad \forall i, t \quad (7)
\]

\[
\sum_{k=1}^{l} y_{jkt} \leq 1 \quad \forall j, t \quad (8)
\]

First objective, \(Z_1\), is summation of:
- Transportation cost between central and regional warehouses, \(\sum_{t=1}^{a} \sum_{j=1}^{m} \sum_{l=1}^{n} c_{d_{ij}} a_{it} x_{ijt}\).
- Transportation cost between regional warehouses and customer, \(\sum_{t=1}^{a} \sum_{j=1}^{m} \sum_{l=1}^{n} c_{d_{ij}} a_{it} x_{ijt}\).
- Installation cost for central warehouses, \(\sum_{i=1}^{n} w_{j} u_{j}\).
- Installation cost for regional warehouses, \(\sum_{k=1}^{l} q_{k} v_{k}\), that is multiplied by weighted coefficient \(P\).

Second objective, \(Z_2\), is the summation of the level of the customer satisfaction that is multiplied by \((1 - P)\).

Constraints (1) and (2) states if regional warehouse \(j\) or central warehouse \(k\) satisfy the demand, it has been installed. Constraints (3) and (4) show capacity restriction for each regional warehouse. Constraint (5) implies that there is a minimum level of customer \(i\) satisfaction for commodity \(t\). Constraint (6) considers that amount of supply should be greater than amount of demand. Constraint (7) shows that the maximum level of customer \(i\) satisfaction for commodity \(t\) should be less than or equal to 1. Constraint (8) shows that the total percentage of demand of regional warehouse \(j\) for commodity \(t\) should be less than or equal to 1.

IV. SIMULATED ANNEALING

Our contribution is to consider uncertainty in the problem. We use a risk measure that Norman and Smith (1997) previously used in facility layout problem.

To solve combinatorial optimization problems, simulated annealing algorithm is first proposed by Kirkpatrick et al. (1983). The name of SA algorithm is attained from the simulation of the annealing of solids. Annealing refers to a process of cooling material gradually to reach a steady state. SA algorithm starts with solution and moves to a neighborhood solution \(\text{Max}_{\text{iteration}}\) times in each temperature. The move space consists of changing location of warehouses. Enhancing moves are always accepted while not enhancing moves are only accepted with the below possibility function

\[
\exp \left\{ -K \cdot (\Delta OF) / t \right\}
\]

Where \(K\) is a constant number that it is set before running algorithm, \(\Delta OF\) is the increase in objective function value and \(t\) is the temperature.

\[
T_{\text{new}} = y T_{\text{old}} \quad y = 0.9
\]

Parameters of SA algorithm are as follows:

\(T_0\): Initial temperature
\(T_{\text{end}}\): Final temperature
\(\text{Max}_{\text{iteration}}\): Maximum move per each temperature
\(y\): Cooling coefficient
\(K\): Convertor coefficient of objective function

First objective, according to Norman and smith [11], is update as follows:
Min \( Z_1 = P \sum_{t=1}^{n} \sum_{j=1}^{m} c_i d_{ij} \bar{a}_{it} x_{ijt} + Z_{1-a} \sum_{t=1}^{n} \sum_{j=1}^{m} c_i d_{ij} \sigma_{it}^2 x_{ijt} \)

\( \bar{a}_{it} \): Variance of demand of good type \( t \) for customer \( i \),
\( \bar{a}_{it} \): Expected value of demand of good type \( t \) for customer \( i \),

V. COMPUTATIONAL RESULT

We consider demand as Normal distribution \( a_{it} \sim N(\bar{a}_{it}, \sigma_{it}^2) \), \( \alpha = 0.01, 0.05, 0.1 \) and minimum customer satisfaction level as \( \beta_{it} = 0.1 \). Table 1 shows the increase of satisfaction level between deterministic approach and uncertainty approach using satisfaction level as a measure.

<table>
<thead>
<tr>
<th>( \sigma_{it}^2 )</th>
<th>( \alpha = 0.01 )</th>
<th>( \alpha = 0.05 )</th>
<th>( \alpha = 0.1 )</th>
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</thead>
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<td>0.100</td>
<td>0.100</td>
<td>0.100</td>
</tr>
<tr>
<td>( \sigma_{it}^2 = 5 )</td>
<td>0.121</td>
<td>0.132</td>
<td>0.143</td>
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<tr>
<td>( \sigma_{it}^2 = 10 )</td>
<td>0.111</td>
<td>0.116</td>
<td>0.120</td>
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<tr>
<td>( \sigma_{it}^2 = 15 )</td>
<td>0.104</td>
<td>0.105</td>
<td>0.105</td>
</tr>
</tbody>
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VI. CONCLUSION

In this paper, we develop a simulated annealing algorithm for location distribution problem. We use statistical upper bound for objective function as a risk measure to state demand uncertainty and compare it with the deterministic approach. Computational results show flexible demand increase customer satisfaction level respect to deterministic demand. As expected, increase of \( \sigma_{it}^2 \) causes reduction in satisfaction level and increase of \( \alpha \), enhance satisfaction level.

References