High Frequency Trading using Fuzzy Momentum Analysis

A. Kablan *Member, IAENG*, and W. L. Ng.

Abstract — **High frequency trading holds a rapidly growing interest both for researchers and financial investment entities. Finding better order execution rates is an intriguing problem. For brokers trading large orders, the effect of order size and the market's trend and volatility are crucial for order scheduling. The cumulated order quantity of these institutional traders usually represents a big proportion of the daily trading volume, requiring sophisticated order splitting mechanisms to reduce market impact. This paper proposes a new framework for high frequency order execution using a novel way of momentum analysis which makes use of fuzzy logic reasoning mechanisms. The suggested order placement algorithm also considers the market's intraday volatility to minimize trading costs.**

Index Terms—**High Frequency Trading, Order Execution, Momentum Analysis, Fuzzy Logic.**

I. Introduction

The design of modern financial trading systems continues to gain popularity not only in the academic sector, but also for investors, financial institutions, and policymakers. However, due to their complexity and nonlinearity, many characteristics of financial signals could not be captured by traditional approaches in financial modeling. Moreover, recent research [2] suggests that adopting artificial intelligence techniques for financial modeling will yield positive results; hence, technical analysis is likely to continue to hold the interest of researchers and investors alike.

Originally, time series models were first combined with fuzzy theory by [8,9], resulting in fuzzy time-series, which is the fundamental framework of all of the investment systems. In general, these authors detail five steps for such a system:

1. Defining and partitioning the universe of discourse.
2. Defining the fuzzy sets.
3. Fuzzifying the observations.
4. Establishing the fuzzy relationships.
5. Forecasting and defuzzifying the results.

Researchers creating stock trading systems have implemented many variations of this model, of which the key adaptation primarily concerns the selection of appropriate observations, the definition of the fuzzy relationships, and the particular inference system used for forecasting.

Most systems use well-documented technical indicators from financial theory for their observations. For example, [2] use three technical indicators in their stock trading system:

- the rate of change,
- the stochastic momentum indicator and
- a support/resistance indicator that is based on the thirty-day price average.

A convergence module then maps these indices as well as the closing price on to a set of inputs for the fuzzy system, thus providing a total of seven inputs. In some cases, such as the rate of change, an indicator maps to a single input. However, it is also possible to map one indicator to multiple inputs. Four levels of quantification for each input value are used: small, medium, big and large. [8,9] used Mamdani's form of fuzzy rules [7] to combine these inputs and produce a single output variable with a value between 0 and 100. Low values indicate a strong sell, and high values a strong buy. The system is evaluated using three years of historical stock price data from four companies with variable performance during one period and employing two different strategies (risk-based and performance-based). In each strategy, the system begins with an initial investment of $10,000 and assumes a constant transaction cost of $10. Tax implications are not taken into consideration. System output is shown to compare favorably with stock price movement, outperforming the S&P 500 in the same period.

The problem of optimal order execution has been a main concern for financial trading and brokerage firms for decades [3]. The idea of executing a client's order to buy or sell a pre-specified number of shares at a price better than all other competitors seems intriguing. However, this involves the implementation of a system that considers the whole price formation process from a different point of view.

For financial brokers, profit is made by executing client's orders to buy or to sell a certain amount of shares of a specific stock at the best possible price. Many mathematical and algorithmic approaches have been implemented to perform this task. However, none of these models seem to overcome the standard volume based system widely applied in the industry. This paper suggests a different approach for tackling this problem and introduces a framework of using a fuzzy logic based momentum analysis system for high frequency trading. The objective is to analyse the current 'momentum' in the time series and to identify the current market condition which will then be used to decide the participation rate at that instance using the current traded volume. The system was applied to trading of two financial stocks, and tested against the standard volume based trading system. The results show how the proposed Fuzzy Logic Momentum Analysis System can outperform the traditional and standard systems that are used in the industry. The

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system uses fuzzy reasoning to analyse the current market conditions according to which a certain equity price is currently moving. The membership functions were later modified using ANFIS. This new approach in the optimisation of membership functions led to an improvement of the results.

II. Fuzzy Inference

There are many types of fuzzy inference systems that have been proposed in literature. However, the most common is the Sugeno model [10], which makes use of a list of rules in the form of if-then rules to produce an output for each rule. Rule outputs consist of the linear combination of the input variables as well as a constant term; the final output is the weighted average of each rule's output. The rule base in the Sugeno model has rules of the form:

$$IF\ x\ is\ A_1\ and\ y\ is\ B_1\ ,\ THEN\ f_1 = p_1x + q_1y + r_1\ . \quad (1)$$

$$IF\ x\ is\ A_2\ and\ y\ is\ B_2\ ,\ THEN\ f_2 = p_2x + q_2y + r_2\ . \quad (2)$$

Where x and y are predefined membership functions, A_i and B_i are membership values, and p_i , q_i and r_i are the consequent parameters. When we calculate the equation of first-order Sugeno [10], the degree of membership variable of X_1 in membership function of A_i are multiplied by the degree of membership variable of X_2 and in membership function B_i, and the product is deemed a linear regression weight (W_i). Finally, the weighted average f_1 and f_2 is deemed the final output Z, which is calculated as follows:

$$Z = \frac{W_1\ \cdot\ f_1 +\ W_2\ \cdot\ f_2}{W_1 +\ W_2} \quad (3)$$

In the case of designing a fuzzy system for financial modelling, as is the case in this paper, one should opt to use the Mamdani model [7], which is based on linguistic variables and linguistic output. A fuzzy inference system is a rule-based fuzzy system that can be seen as an associative memory and is comprised of five stages:

1. An input stage, this consists of measuring what the input would be
2. Database stage which defines membership functions of the fuzzy sets used in the fuzzy rules.
3. A Processing stage that determines what action needs to be carried out based on fuzzy 'IF-THEN' rules.
4. An averaging stage that determines the centre of mass for all system conditions.
5. Finally an output stage that is a crisp control decision or an output.

An extension to Fuzzy Inference Systems is the Adaptive Neuro Fuzzy Inference System (ANFIS) which can be used for optimising the membership functions by combining fuzzy reasoning combined with the pattern recognition capability of neural networks [6]. The ANFIS approach learns the rules and membership functions from data [6]. It represents an adaptive network of nodes and directional links with associated learning rules. It is called adaptive because some, or all, of the nodes have parameters which affect the output of the node. These networks identify and learn relationships between inputs and outputs.

III. Fuzzy Logic Momentum Analysis System (FULMAS)

Creating a fuzzy inference system to detect momentum is a complex task. The identification of various market conditions has been a topic subject to various theories [5] and suggestions. This paper proposes a fuzzy inference system that categorises the market conditions into seven categories based on price movement, using the current volume to determine the participation rates (PR) of the trading system each time. The participation rate is the percentage of volume available in the market that the system will buy or sell at each trade.

Let P_i denote the current price, P_{i-1} the previous price and k_i is an up/down fluctuating counter that goes positive or negative according to the movement of the price. Whenever the price goes up, it adds 1, and when the price goes down, it subtracts 1. This indicator is used to identify market conditions for x amount of ticks (where one tick is one price observation), where if the market is moving strongly upwards, it will be detected by having more ones than -1 or zeros. This can be explained in the equation

$$Momentum\ = \sum_{i=1}^{x} k_i\ \ , \quad (3)$$

where x is the amount of elapsed time that we want to detect the momentum for. The first step in designing the FUzzy Logic Momentum Analysis System, FULMAS, involves defining the market conditions that the fuzzy system has to identify. In this study, the following seven market conditions are used to cover all possible movements of the price series:

1. "Rallying"
2. "Strong up"
3. "Slightly up"
4. "Average"
5. "Slightly down"
6. "Strong down"
7. "Crashing"

These conditions are considered linguistic values that the fuzzy logic system will use, and they will be responsible for determining the current state in the price formation process and its momentum. As momentum builds, the system considers the previous x amount of ticks and performs an inference procedure by adding all the up or down directional movements of the current price to the previous price in order to determine whether the general trend has been up or down after x points. The momentum is detected as displayed in the pseudo-code in Listing 1.

```
BEGIN
P = Price;
K = 0;
Start data feed (i);
REPEAT
  if P(i) > P(i-1)
    then k(i) = k + 1;
  else if P(i) < P(i-1)
    then k(i) = k - 1;
  else
    then k(i) = 0;
END
```

Listing 1: Calculating momentum

For example, if we want to detect the momentum of the last x=100 price observations (ticks), we add all the up, down fluctuations and then input the resulting crisp value to the fuzzy system, which would lie somewhere in the membership functions, as shown in the upper panel in Figure 1. The choice of triangular membership functions was made after using the expert based method due to their mathematical simplicity. Triangular shapes require three parameters and are made of a closed interval and a kernel comprised of a singleton. This simplifies the choice of placing the membership functions. The expert merely has to choose the central value and the curve slope on either side.

The complimentary characteristics of neural networks and fuzzy inference systems have been recognised and the methodologies have been combined to create neuro-fuzzy techniques. Indeed, earlier work by [11] described an artificial neural network with processing elements that could handle fuzzy logic and probabilistic information, although the preliminary results were less than satisfactory.

In this study, we use ANFIS from another perspective, i.e. to optimise the membership functions already created in the previous section. This is performed by feeding the ANFIS system both the training data and the desired output, and tuning the ANFIS in order to reach the target result by modifying the membership functions.

In other words, at each instance, ANFIS is fed the results currently obtained from the fuzzy system in the previous section, and then a set of 'target' prices or data is inserted into ANFIS. This target price will be an optimal price that is far better than the current one (a cheaper price if on buy mode or a higher price if in sell mode). The system runs and modifies the membership functions in each epoch in order to get as close to the optimal price as possible.

We also test the approach on another type of membership functions, which is the bell-shaped membership function (see Figure 2, upper panel). Figure 1 and 2 show both applied membership functions before (upper panel) and after (lower panel) ANFIS was initialised to reach optimal 'target' results.

Comparing the results of both optimised membership functions, an improvement in the original system was discovered. The optimised triangular membership functions have also outperformed the optimised bell-shaped membership functions; this confirms the experts' opinion mentioned above concerning the choice of the triangular membership functions.

The same procedure is applied for calculating the linguistic variable "volatility", where the linguistic values

1. "Very High"
2. "High"
3. "Medium"
4. "Low"
5. "Very Low"

The fuzzy logic system considers both market momentum and volatility. It generates the rules and then takes a decision based on the amount of market participation. The combined approach is summarised in Figure 3.

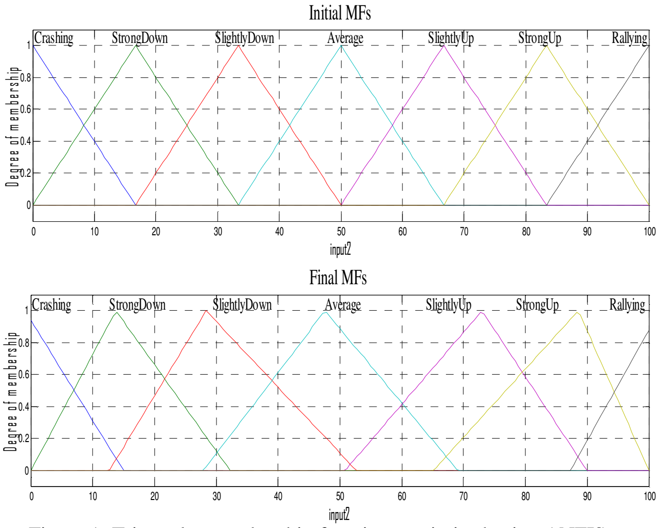

Figure 1: Triangular membership functions optimised using ANFIS

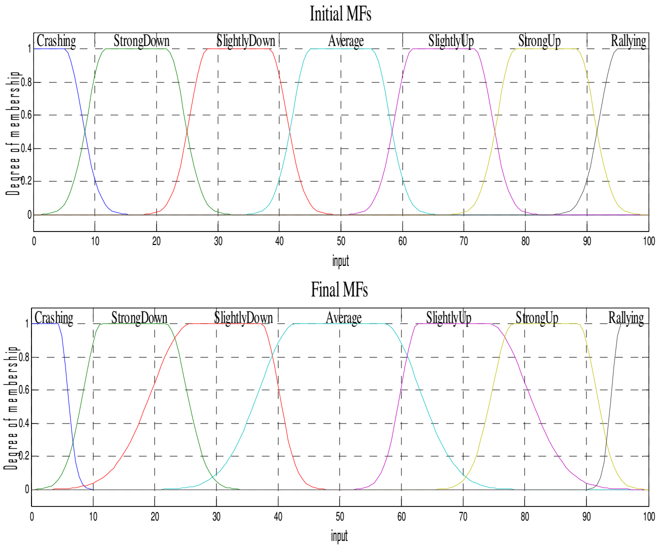

Figure 2: Bell-shaped membership functions optimised using ANFIS

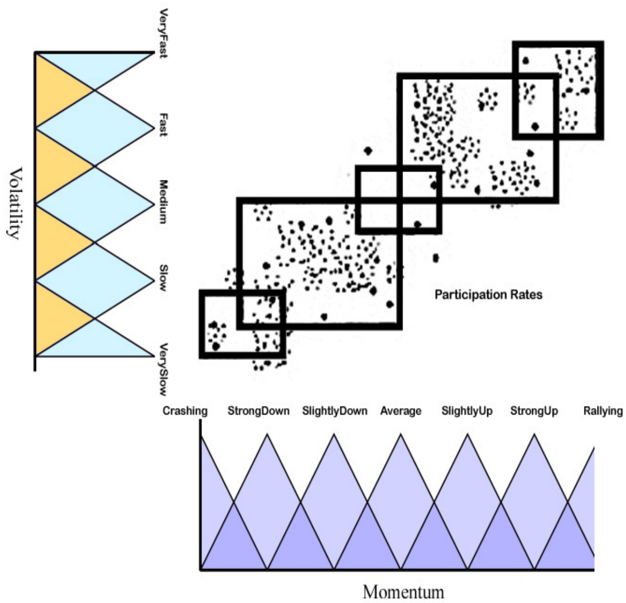

Figure 3: Extracting fuzzy rules from both volatility and momentum

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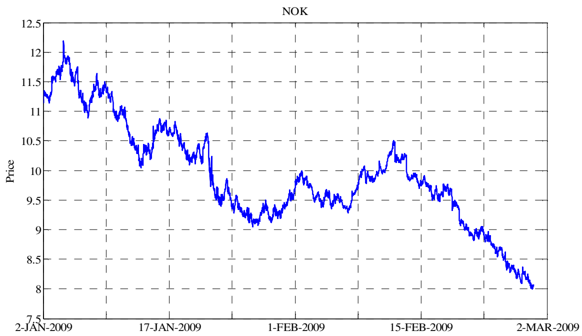

Figure 4: Price for NOK from 2 January 2009 and 27 February 2009

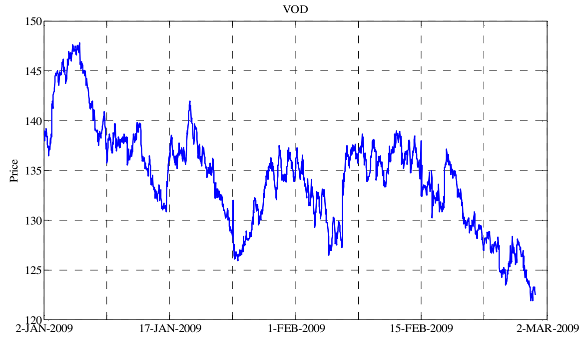

Figure 5: Price for VOD from 2 January 2009 and 27 February 2009

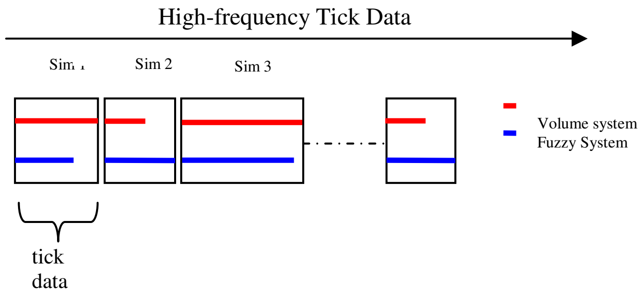

Figure 6: Tick data-splitting mechanism, the new simulation starts where the last simulation has ended

IV. Empirical Application

A. The Data

Experiments in this paper have been carried out on high-frequency tick data obtained from ICAP plc. For the stocks Vodafone Group plc (VOD) and Nokia Corporation (NOK). Figure 4 and Figure 5 show the time series of their share prices.

A very important characteristic of this type of data is its irregular spacing in time, which means that the price observations (ticks) are taken at different intervals (as they arrive). As it will be described later in the system design, the system does not sample data at fixed time intervals; instead it reads real-time ticks. Both price and volume information are available as data. The application is designed for an interdealer broker[1], which means that they have the ability to create orders with any amount of volume. For both stocks, two months of high-frequency tick data sampled from 2 January 2009 to 27 February 2009; simulations are terminated whenever 1 million shares have been bought/sold. Since we are comparing two approaches, each simulation starts at the point where the last simulation of either system has been terminated. It must be mentioned that two months of high-frequency tick data is a significantly large amount of data, considering every iteration, the system analyses the momentum of the past 100 ticks. Figure 6 shows how the data is split after each simulation in order to avoid any possible similarities or autocorrelation in the price in subsequent subsamples.

[1] An interdealer broker is a member of a major stock exchange who is permitted to deal with market makers, rather than the public, and can sometimes act as a market maker.

B. Approach

a) Standard Volume System (SVS)

A standard brokerage/trading mechanism for executing large orders is a simple volume-based system that splits the target transaction volume whenever a pre-specified threshold number of shares is executed; the system will then submit bid or ask orders (depending on the order) at a certain percentage.

Assume a threshold of 10,000 shares. If this threshold value is executed and *PR=25%*, the system will buy or sell 25% of the average volume currently available in the market. This is done as follows:

$$IF\ sum(volume) > Threshold, \quad THEN\ amount\ of\ shares = (\% * Volume_i)\,. \tag{4}$$

Therefore, the

$$Total\ SVS\ Cost = \sum_{i=1}^{n} Price_i * (\,amount\ of\ shares_i\,)\,, \tag{5}$$

where n is the number of operations (trades) required to reach the target order of buying or selling a certain amount of shares. For example, If there is an order to buy 1 million shares of a stock, The participation rate PR will be a fixed %. In most cases this rate is 25%. Thus, whenever a threshold is exceeded, (e.g. 10000 shares have been traded on the market), the SVS will constantly buy or sell 25% of that amount. This system was perceived to be efficient and is being adopted by many brokerage firms around the world. The aim of this paper is to show that FULMAS outperforms this type of system in the long run. This is assessed using order execution costs for buy and sell orders.

b) FULMAS for Trading

The aim of the Fuzzy Logic Momentum Analysis System (FULMAS) implemented in this paper is to outperform the industry's standard volume system that has been used by brokerage firms to execute large orders of buying or selling a certain stock. Although many systems have used various techniques such as quantum modelling and analysis to determine the various participation rates (PR), they usually fail to surpass the standard volume system in the long term [1]. This paper uses FULMAS to determine the PR in the market according to the current momentum. E.g., for a buy order, it is preferable to increase the PR (number of shares bought at that time) when the price is low and to decrease the participation when the price is high.

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TABLE I. PARTICIPATION RATES FOR BUY SIDE AND THE SELL SIDE OF FULMAS

Market Condition	Buying PR	Selling PR
Rallying	10%	40%
Strong up	15%	35%
Slightly up	20%	30%
Average	25%	25%
Slightly down	30%	20%
Strong down	35%	15%
Crashing	40%	10%

The idea here is to use the momentum analysis system described above to identify the current market condition that the price is residing in. This will enable us to vary the PR (%), providing an advantage, since the system can trade aggressively when the condition is at an extreme. It would also adjust its trading frequency when the condition is at another extreme. In other words, if we are selling one million shares, the system will make a trade whenever the threshold of volume has been exceeded. However, if the current market condition indicates that the price is very high or *rallying,* then we know that this is a suitable time to sell more shares, for example, 40% of the current volume. The same concept applies when the momentum indicates that the price is *strong down,* which means that the system should sell a lower volume at this low price, for example, 15%. The reverse mechanism applies for buying shares. When the market is *crashing*, this is a good indicator that we should buy a large volume (40%), and when the price is at an *average* point, it would behave like the SVS system, i.e. buying 25%of the volume. This is shown in Table 1. The same procedure is applied to volatility and then combined with volume to produce the fuzzy rules.

c) Performance Measures

After the implementation of both trading systems, the assessment criteria at which both systems will be compared against each other will be the outperformance of FULMAS on the SVS measured in basis points (bp)[2]. To calculate the outperformance (*OP*) for the buy side in basis points, the following formula is used:

$$OP_{Buy} = \left(1 - \frac{FULMAS\ price}{SVS\ price}\right) * 10^4 \ , \qquad (6)$$

where $FULMAS\ price$ is the total cost of buying *x* amount of shares using FULMAS, and $SVS\ price$ is the total cost of buying the same number of shares using the traditional SVS. Similarly, for the sell side the improvement in basis points is

$$OP_{Sell} = \left(\frac{FULMAS\ price}{SVS\ price} - 1\right) * 10^4 \ . \qquad (7)$$

[2] One basis point is equal to 1/100th of a percent or 0.01 percent. It is also equivalent to 0.0001 in decimal form. The basis point is a unit of measure often used to describe the percentage at which a change in the value or rate of a financial instrument has occurred.

V. RESULTS

Initial simulations have been carried out on the described data, using triangular membership functions. These triangular membership functions were further optimised using ANFIS, and have produced better results. We now present both results from the initial simulations, and the simulations using the optimised system.

A. *FULMAS vs SVS*

This section displays the results of using both FULMAS and SVS to buy one million shares of VOD and NOK. For each stock, simulations have been carried on the tick dataset described. The data has been arranged as described in order to make use of us much data available as possible, and both systems have been run and tested on the same datasets. For each instrument, approximately 30 simulations have been carried on the tick dataset using a rolling windows approach as described above. The cost at each simulation for buying 1 million shares of NOK using both systems has been noted, and the outperformance of FULMAS over SVS is calculated. The average price of the entire set for each simulation is also computed; this price is calculated by taking the average price of the equity throughout the time of each simulation. Finally the improvement of FULMAS against SVS is displayed. This improvement rate can be either positive; when FULMAS has outperformed SVS, or negative, when FULMAS is outperformed by SVS. Figures 7-10 exhibit the outperformance of FULMAS over SVS in basis points for each simulation carried. Whenever that rate is positive it means that FULMAS has outperformed SVS, and vice versa. As discernible, in most cases the proposed FULMAS show an improvement.

Table 2 and Table 3 list the average outperformance rate of buying one million shares of NOK and VOD respectively using the optimised FULMAS. Here we see a much higher improvement than the previous system, which confirms that the use of ANFIS to optimise the membership functions has increased the performance of the system by a high rate on both the buy and sell sides. The results show that on the buying side, the system outperforms the standard system by more than six basis points on average. On an industrial scale, this means a large amount of savings for financial institutions that employ such systems to vary the participation rates. Table 3 shows the final results for the sell side.

TABLE 2: OUTPERFORMANCE OF FULMAS IN BUYING 1 MILLION SHARES

	Mean (OP)	Median (OP)
Buying NOK	6.94 bp	6.57 bp
Buying VOD	14.48 bp	4.33 bp

TABLE 3: OUTPERFORMANCE OF FULMAS IN SELLING 1 MILLION SHARES

	Mean (OP)	Median (OP)
Selling NOK	9.36 bp	5.79 bp
Selling VOD	7.71 bp	6.91 bp

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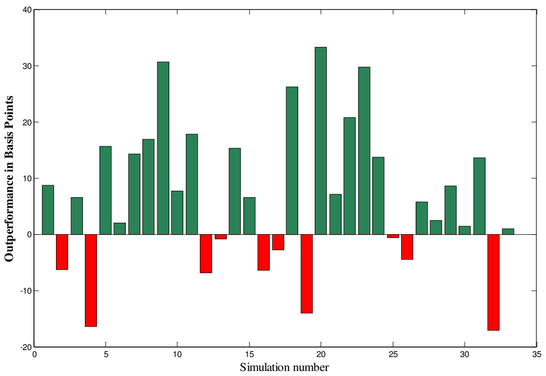

Figure 7: Optimised FULMAS outperformance to SVS system for buying NOK

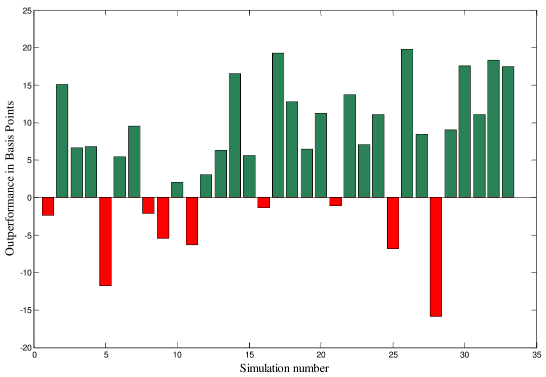

Figure 8: Optimised FULMAS outperformance to SVS system for selling NOK

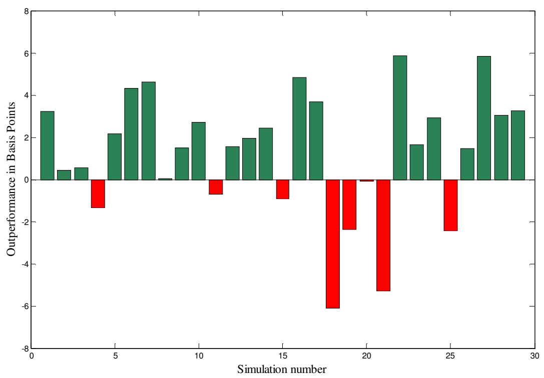

Figure 9: Optimised FULMAS outperformance to SVS system for buying VOD

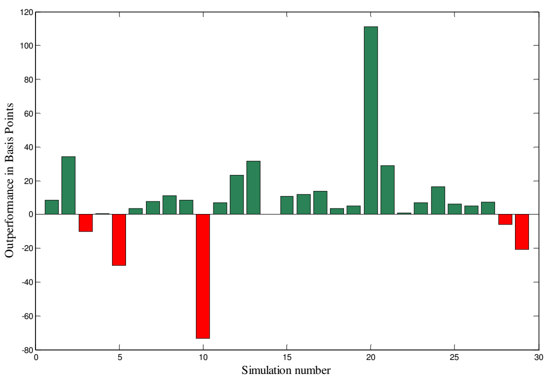

Figure 10: Optimised FULMAS outperformance to SVS system for selling VOD

VI. Conclusion

There are still relatively few studies comparing artificial intelligence approaches and classical models such as time series theory. However, recent research continues to suggest that adopting artificial intelligence techniques for the technical analysis of financial systems will yield positive results. With several researchers now questioning the Efficient Market Hypothesis [4], particularly in light of the recent global financial crisis, artificial intelligence approaches such as time series that can improve analysis will likely continue to hold the interest of researchers and investors alike.

The problem of order execution is a very complicated one. To be able to provide the best price, an execution system must dynamically change the participation rates at each instance in order to cater to price changes that are driven by momentum and volatility.

This study has introduced a system that utilises fuzzy logic in order to justify the current market condition that is produced by the accumulation of momentum. The proposed FULMAS is a fuzzy logic momentum analysis system that outperforms the traditional SVS used in the industry, which is often based on executing orders dependent on the weighted average of the current volume. Results of the implemented system have been displayed and compared against the traditional system. The system reveals that the AI enhanced approach increases the average profitability on orders on both the buy and sell sides.

References

[1] Boehmer, E (2005). Dimensions of execution quality: Recent evidence for U.S. equity markets. Journal of Financial Economics, 78, pp. 553–582.

[2] Dourra, H. and Siy, P. (2002). Investment using technical analysis and fuzzy logic. Fuzzy Sets and Systems, Vol. 127 (2002), pp. 221-240.

[3] Ellul, A., C. Holden, P. Jain, and R. Jennings, (2003), Determinants of order choice on the New York Stock Exchange. Working Paper.

[4] Fama, E. F. (1965). The behaviour of stock market prices. The Journal of Business, Vol. 38, pp. 34-105.

[5] Griffin, J. (2007). Stock market trading and market conditions, forthcoming, Review of Financial Studies.

[6] Jang, R. (1993). ANFIS: Adaptive network-based fuzzy inference system, IEEE Transactions on Systems, Man and Cybernetics, Vol. 23 (3) pp 665-685.

[7] Mamdani, E. and Assilian, S. An experiment in linguistic synthesis with a fuzzy logic controller. International Journal of Man-Machine Studies 7 (1), pp. 1-13.

[8] Song, Q. and Chissom, B.S. (1993). Forecasting enrollments with fuzzy time-series. Part I. Fuzzy Sets and Systems 54, pp. 1-10.

[9] Song, Q. and Chissom, B. S. (1994). Forecasting enrollments with fuzzy time-series. Part II. Fuzzy Sets and Systems 62, pp. 1-8.

[10] T. Takagi and M. Sugeno. Fuzzy identification of systems and its application to modeling and control, IEEE Transactions on Systems, Man and Cybernetics, Vol. 15, pp 116-132, 1985.

[11] Wong, F.S. and Wang, P. Z. (1990). A stock selection strategy using fuzzy neural networks. Neurocomputing 2 (1990/91), pp. 233-242.

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