Hybrid Phase Unwrapping for Three-Dimensional Surface Reconstruction Based on Pixel Quality Classification using a Laplacian Filter

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Abstract—In this research, a three-dimensional surface reconstruction system based on structured light is built by projecting three sinusoidal phase-shifted light patterns encoded as RGB intensity values and calculating the phase angle values proportional to the surface shape. These values are calculated using an inverse tangent function and, therefore, wrapped into the interval \([-\pi,\pi]\). Due to this, an additional step known as phase unwrapping is needed to obtain the absolute phase distribution.

Phase unwrapping is a trivial task in absence of image noise or discontinuities, reflections and shadows in the target but, in practice, calculating the correct unwrapped phase distribution is an ill-posed problem. We propose a very simple, fast and reliable phase unwrapping solution by adequately combining the speed of the traditional scanline algorithm with the robustness of a quality map based approach. The integration of the two methods is based on the estimation of noise and phase jumps from the second spatial derivative changes calculated with a Laplacian filter applied to the wrapped phase.

Index Terms—3D surface reconstruction, phase unwrapping, structured light

I. INTRODUCTION

The application of structured light based reconstruction techniques have been found useful in diverse fields such as human body shape measurement [6], manufacturing process quality inspection [1] and 3D media content generation [10] among other areas. One of the most noticeable advantages of these reconstruction techniques is their ability to generate a contactless, fairly accurate reconstruction of a real object at video frame rates. This characteristic has made possible to use this technology in applications that need real-time data analysis such as virtual reality [3].

The basic layout for the pattern projection-based surface acquisition system used on this research consists of a DLP projector, a CCD camera and a computer where the data analysis takes place. The reconstruction process involves: i) projecting three phase-shifted sinusoidal patterns encoded in each one of the RGB intensity channels onto the target surface, ii) acquiring an image of this arrangement where the projected light is phase modulated proportionally to the changes in the surface, iii) calculating the phase modulation in the image, since we use a phase shift of \(2\pi/3\) the projected phase shifted patterns for each color band are given by (1).

\[
\begin{align*}
B(x, y) &= \sin\left(\theta(x, y) - \frac{2}{3}\pi\right) \\
G(x, y) &= \sin(\theta(x, y)) \\
R(x, y) &= \sin\left(\theta(x, y) + \frac{2}{3}\pi\right)
\end{align*}
\]

\(\theta(x, y) = \arctan\left(\sqrt{3} \frac{B(x, y) - R(x, y)}{2G(x, y) - B(x, y) - R(x, y)}\right)\) (2)

These phase values are wrapped in the interval \(-\pi\) to \(\pi\) due to the arctangent function applied so we need to iv) use a phase unwrapping algorithm (Fig. 1) to detect the discontinuity jumps inserted by the inverse tangent and, finally, v) map the unwrapped phase distribution to real world 3D coordinates using the camera parameters obtained by traditional calibration methods such as [12].

II. PHASE UNWRAPPING

In this reconstruction system, what is referred to as phase unwrapping is the process of determining the unknown integral multiple of \(2\pi\) to be added at each pixel of the wrapped phase map to make it continuous by removing the artificial \(2\pi\) discontinuities.
Thus, phase unwrapping is a trivial task if the wrapped phase map is ideal. However, in practice, it is an ill-posed problem where the presence of shadows, reflections, object and fringe discontinuities, noise etc. makes correct phase unwrapping difficult and path dependent.

Simple scanline phase unwrapping is carried out by comparing the phase at neighboring pixels and adding or subtracting $2\pi$ to bring the relative phase between the two pixels into the range of $-\pi$ to $\pi$ but this method can easily generate errors in the unwrapping direction, called stretch-lines (Fig. 2) [7].

![RGB phase shifted source](image1)

(a) RGB phase shifted source

![Surface with stretch-lines](image2)

(b) Surface with stretch-lines

Fig. 2. Typical propagation of errors in form of stretch-lines when scanline phase unwrapping is applied.

Several unwrapping algorithms have been developed to try to improve the robustness of this stage such as branch-cut based algorithms [5], and methods based on a quality map [2], among others. However, these robust algorithms are usually too slow for high resolution, real-time 3D surface reconstruction applications.

In our research, a set of these algorithms were evaluated against artificially generated images containing different levels of additive white Gaussian noise, speckle noise and salt and pepper noise. It was found that the methods based on a quality map offered the most robust solution against all types of noise but also had one of the worst execution time performances, which is consistent with the results presented in [9].

In a quality map based approach, the phase is unwrapped by using a quality measure to guide the unwrapping path. The process starts from the highest quality point and continues to the lower quality ones until it finishes which results in avoiding errors to be propagated along the unwrapping path. The quality of a pixel in the phase map can be calculated by different methods such as pseudo-correlation, derivative variance, maximum phase gradient, and other measures that estimate the possibility of noise presence in a pixel neighborhood that affects the reliability of the phase encoded on its intensity [4].

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III. PROPOSED SOLUTION

In theory, the simple scanline algorithm should only fail in pixels where the quality map methods normally assign a low quality value. This low quality value makes the noisy pixel unwrapping to be postponed and prevents the error propagation along stretch-lines when the quality map algorithm is used.

Based on this observation, we established a criterion to separate the pixels of the image in two groups: those likely to be unwrapped correctly by the simple scanline process and those likely to cause errors according to their reliability. The pixels likely to cause errors could then be unwrapped correctly by a quality map algorithm.

Previously, in [11], a similar idea is proposed where different quality levels are calculated using thresholds involving the quality map mean and standard deviation values. A variation of the simple scanline method is then used in each quality level defined postponing the lower quality pixels that might cause errors. This approach has the following disadvantages:

First, the quality map mean based criterion is not always an accurate way of splitting the pixels in groups. In images with a generalized low quality some error inducing pixels might be left for early stages of the unwrapping causing them to propagate errors over subsequent levels.

Second, the use of the scanline algorithm for all the quality levels is probably the fastest option but does not necessarily improve the robustness in the unwrapping of regions belonging to a single quality level.

Lastly, due to the fact that the quality map mean value is used, we need to calculate the quality map for the full image. Since some equations used to compute the quality of a phase distribution are computationally expensive, this stage of the algorithm can take a representative amount of processing time.

In our approach, we propose to define the low quality pixels by finding noisy pixels and phase jumps in the wrapped phase map using a Laplacian filter. Concretely, the phase unwrapping process in our system is executed in the following stages (Fig. 3):

![Flowchart of the proposed phase unwrapping algorithm](image3)

Fig. 3. Flowchart of the proposed phase unwrapping algorithm.

A. Signal power mask

As an initial quality measurement of the pattern projected over the target, a function proportional to the signal power is estimated using (3). This allows us to separate the target from the background and decrease the size of the processing space by discarding image pixels not being reached properly by the projected light. A simple threshold is set accordingly to the reflective properties of the scene background and the quality mask is defined through binarization.

$$P(x, y) = (\sqrt{3}(B(x, y) - R(x, y)))^2 + (2G(x, y) - B(x, y) - R(x, y))^2$$

(3)
B. Quality mask

The unwrapping will start from a high quality pixel near to the centroid of the signal power mask. As this central region will become the reference for the rest of the process, a line in the direction perpendicular to the fringes is marked to be unwrapped using the quality map method to increase reliability.

Then we proceed to locate high gradient areas using a simple Laplacian filter defined by the 3x3 element L in (4). The use of this filter is justified by its innate high sensitivity to noise.

\[
L = \begin{bmatrix}
0 & 1 & 0 \\
1 & -4 & 1 \\
0 & 1 & 0
\end{bmatrix} \tag{4}
\]

The threshold used to decide the quality clustering is calculated automatically based on an estimate of noise using the fact that the mean of the magnitude squared image is known to be roughly proportional to the signal-to-noise ratio [8]. Using a threshold cutoff instead of a zero-crossing criterion as is usual when applying a Laplacian filter, allows us to select not the actual noisy pixel but the pixels adjacent to it where the unwrapping path should be reviewed to avoid error propagation.

To increase the connectivity of the low quality regions and add confidence to the pixel selection, we calculate the morphological gradient of the resulting mask with a 3x3 square kernel.

Finally, we add boundaries of regions surrounded by object discontinuities where the scanline unwrapping can fail due to the lack of a previously unwrapped reference pixel.

An example of the quality mask obtained with this process is shown in Fig. 4 where pixels in white correspond to the low quality group to be processed using a quality map algorithm and pixels in gray correspond to the high quality group to be processed with a scanline approach.

(a) Source image  
(b) Resulting quality mask.

Fig. 4. Quality mask pixel classification.

C. Quality map

The quality map used to guide the unwrapping process for the low quality pixels is calculated using the maximum phase gradient approach defined by (5) and used in [11].

\[
Q(x, y) = \nabla_{\text{max}} \theta(x, y) = \max \left[ \begin{array}{c}
\theta(x, y) - \theta(x - 1, y) \\
\theta(x + 1, y) - \theta(x, y) \\
\theta(x, y) - \theta(x, y - 1) \\
\theta(x, y + 1) - \theta(x, y)
\end{array} \right] \tag{5}
\]

D. Unwrapping path

We start from the centroid reference line using the quality map method, change to the scanline approach and switch back to quality map unwrapping when a low quality pixel is found. After all the connected low quality pixels in that region are unwrapped we go back to the scanline order.

E. Unwrapped phase correction

An optional scanning of the resulting phase map is performed to correct possible jumps left between patches of the unwrapped surface caused by the alternation of the two methods.

IV. RESULTS AND EVALUATION

A. Synthetic data

An undistorted pattern representing an ideal flat surface is modified by introducing additive white Gaussian (AWGN), speckle and salt and pepper noise artificially to evaluate the robustness of the different unwrapping approaches against these noise models.

The additive white Gaussian noise model can be described by the equation shown in (6) where the generated image \( f(i,j) \) is the sum of the original \( s(i,j) \) and the noise \( n(i,j) \). The white Gaussian noise follows a normal distribution over the frequency domain with a zero-mean as described by (7).

\[
f(i, j) = s(i, j) + n(i, j) \tag{6}
\]

\[
n(i, j) \sim N(0, \sigma^2) \tag{7}
\]

The speckle noise is a multiplicative noise model defined by (8) and follows a uniform distribution defined in terms of its mean and variance as shown in (9).

\[
f(i, j) = s(i, j) \cdot n(i, j) \tag{8}
\]

\[
n(i, j) \sim U(\mu, \sigma^2) \tag{9}
\]

The salt and pepper noise is described through its presence percentage in the image.

The scanline, quality map and proposed hybrid algorithms are executed over these noisy samples and the results are assessed using the root-mean-square error (\( E_{\text{rms}} \)) definition in (10) where \( I \) represents the original unwrapped surface and \( K \) its noisy sampled version.

\[
E_{\text{rms}} = \sqrt{\frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (I(x, y) - K(x, y))^2} \tag{10}
\]

The figures 5 to 7 show the \( E_{\text{rms}} \) obtained for different levels of noise in the unwrapped phase distribution for each algorithm. We can corroborate the reliability of the proposed hybrid algorithm is comparable to the reliability observed for the quality map method with the exception of some samples in the salt and pepper noise exercise.
A comparison of the three methods in terms of processing improvement by a factor of approximately 10.5 times.

The algorithm was tested and compared with the scanline and quality map algorithms using 206 samples in different views for 15 different target objects with most of them having discontinuities, holes and internal shadows. Images of size of 640x480 pixels were processed using a Core 2 Duo 1.2GHz processor with 3 GB of RAM. Due to the difficult surfaces selected we were able to correctly unwrap 23.3% of the samples using the scanline method. This percentage rose to 90.3% when we used the hybrid approach whereas the quality map algorithm was successful in all the samples. The total number of errors in form of stretch-lines in the samples decreased from 1277 to only 38 which indicate an improvement of 97%. An example of the results obtained is shown in Fig. 8 in which the difference with the scanline method in most of the target objects tested while decreasing the execution time by a factor of more than 10 for images of size 640x480.

Another advantage of the proposed method is that the trade-off between robustness and speed can be controlled through the threshold fed to the Laplacian noise detection mechanism and allows the optimization of these variables according to particular application requirements.

V. CONCLUSIONS

The solution to phase unwrapping here presented combines the speed of the traditional scan line algorithm with the robustness of a quality map based approach. Despite the rudimentary method to classify the pixels according to its second spatial derivative values, we obtained results comparable with the ones achieved by the quality map only method in most of the target objects tested while decreasing the execution time by a factor of more than 10 for images of size 640x480.

REFERENCES