A Multistage Approach to the Design of Prototype Filters for Modulated Filter Banks

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Abstract: When filter banks are used in multicarrier modulation the transition bandwidth of the filter is usually very narrow. This is because the bandwidth of the filters is determined by the number of subchannels and the greater the number of channels lesser will be the transition bandwidth. The order of the filter being inversely proportional to the transition bandwidth is generally very high. Hence computational complexity is also very high. We propose a method that will help reduce this complexity by designing the decimators/interpolators in the subchannels using a multistage approach. We compare the saving obtained in comparison to direct design approaches. We will also compare the performance of a cosine modulated filter bank designed in this way with direct design methods.

Keywords: decimator, interpolator, multicarrier modulation, multistage, transition bandwidth,

1. Introduction

Digital Subscriber Line (xDSL) applications use a multicarrier modulation technique called as Discrete Multitone (DMT) for the purpose of high bit rate transmission over the commonly used twisted pair copper wires. This eliminates bottlenecks in the data access network between the central office and the end user. This has been made possible by the use of, amongst other things , advanced signal processing technology. One such area of signal processing, filter banks , has found vast applications is several areas of digital communication , such as high speed DSL services for internet [1][2].

In Fig.1 is shown the block diagram of a multicarrier modulation system that uses multirate filter banks . The modulation and demodulation is performed using filters and fast transforms such as FFT and DCT. The multirate filter bank is composed of the synthesis and the analysis filter banks that are used for performing modulation and demodulation.

Each channel of the synthesis filter bank consists of an upsampler cascaded with a filter, together called as the interpolator, and the analysis filter bank consists of a filter followed by a downsampler together termed as a decimator.

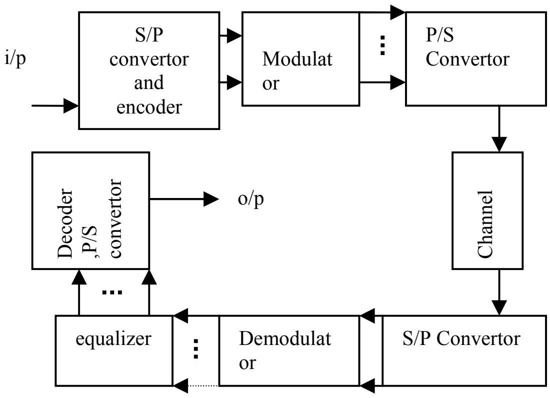

Fig.1 Multicarrier modulation System using multirate filter banks

The up sampling and the down sampling factors will be determined by the number of subchannels of the system and greater the number of channels greater will be upsampling/downsampling factor and smaller will be the transition bandwidth. This will obviously lead to longer length filters being used. This will in turn increase the computational complexity involved in the implementation of the decimators and the interpolators.

In order to minimize computation a multistage approach to the decimator and interpolator shown in Fig2. and Fig 3. is used. This is based on the IFIR approach suggested by Neuvo et al [3]. We will briefly explain this method and see how it may used to obtain multistage implementations.

Manuscript submitted on February 15, 2010.

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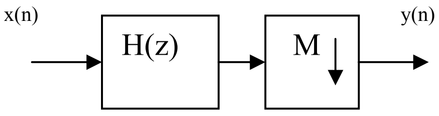


Fig. 2. Decimation Filter Decimator

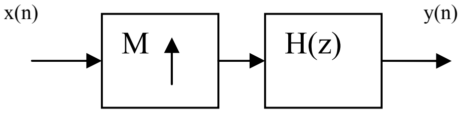
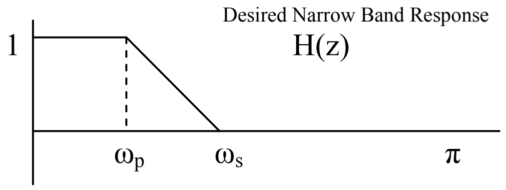
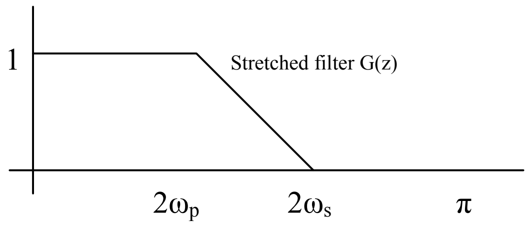


Fig. 3. Up Sampler Interpolation Filter

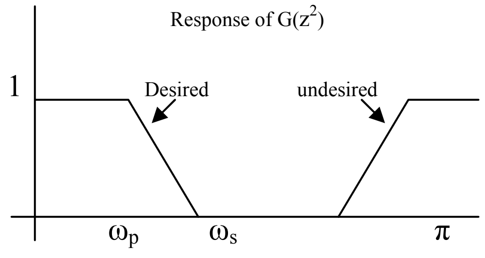
2. **IFIR filter**



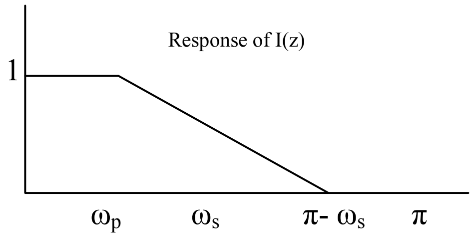
(a)



(b)



(c)



(d)

Fig 4. Stages of the IFIR filter.

Say the desired filter designed according to certain set specifications has order N and a transition bandwidth of Δf. If this filter is stretched 2 fold then the transition band will be $2\ \Delta f$ and the order of the stretched filter G(z) as shown in Fig.4(b) will be N/2. The frequency response of $G(z^2)$ is as shown in (c). It has two pass bands, one the desired passband and the other is its image centered on $2\pi/L$ where L is the stretch factor, in this case 2. This image is to be suppressed by cascading the filter $G(z^2)$ with an image suppressor filter I(z) as shown in (d), to obtain the desired response. The order of I(z) is very small as it has a very large transition band. The order of G(z) (model filter) will be a little more than N/2. Therefore since the order of the two filters is low the amount of computational complexity involved in implementing the filter is greatly reduced.

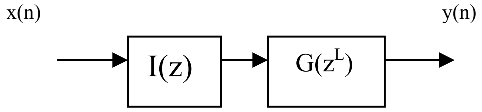


Fig. 5 IFIR filter for stretch factor L

3. **Multistage Design**

Consider the analysis filter bank in the figure shown below. For number of subchannels M that is large the analysis filter H(z) will be narrowband and may designed using the IFIR approach.

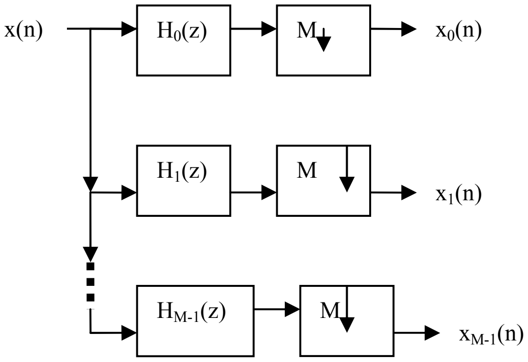


Fig. 6 Analysis Filter Bank

Let us take the stretch factor M_1 to be a factor of the decimation factor M. Then we may represent the decimator structure in each channel as a cascade of the image suppressor filter then down sampling by M_1, the model filter G(z) and then down sampling by M_2 as shown in Fig. 5 below.

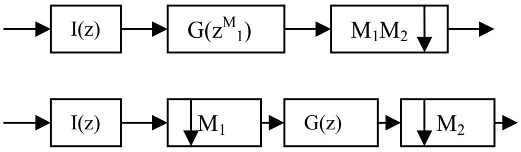


Fig. 7 2-Stage decimator

The computational complexity involved in the implementation of the filters is dependent on the order of the filter and the order of the Model filter G(z), N_g and the Image suppressor filter I(z), N_i can be obtained from the equations below.

$$N_g = \frac{20\log\sqrt{0.5\delta_1\delta_2} - 13}{14.6(\omega_s - \omega_p)/2\pi * M_1} \qquad (1)$$

$$N_i = \frac{20\log\sqrt{0.5\delta_1\delta_2} - 13}{14.6[2\pi - (\omega_s + \omega_p)M_1]/2\pi} \qquad (2)$$

The number of multiplications per unit time(MPU) is

$$N_g/M \ + N_i/M_1 \qquad (3)$$

In the tables shown below the direct and multistage designs are compared on the basis of the filter order and the number of additions and multiplications involved in the implementation of the filter for different values of M and M_1. Note that M will remain fixed and M_1 can take different values for given M

Table I: Variation of filter order with M.

		Filter order			
M		Direct Design	IFIR Method		
			G(z)	I(z)	Total
8	M_1=4	75	21	17	101
16	M_1=8	150	21	32	200
	M_1=4		41	13	177
32	M_1=16	298	21	64	400
	M_1=8		41	25	353
64	M_1=32	596	21	129	801
	M_1=16		41	50	706
	M_1=8		81	23	671

From Table 1. the following inference can be drawn. As the value of M is increased the order of the conventional filter approximately doubles and the order of the model filter G(z) and of I(z) are significantly less. Also the order of G(z) and I(z) is dependent on the value of the factor M_1. As M_1 reduces the order of G(z) increases and that of I(z) reduces. Table 2. will show the effect of the multistage approach on the computation involved. The specifications we have chosen are δ_1=.02, and δ_2=.001. The passband and stop band edges have been chosen to be π/2M and π/M respectively.

Table II. Comparison on the basis of MPU

M		Direct Design	IFIR Method		
			G(z)	I(z)	Total
8	M_1=4	4.69	1.31	2.3	2.61
16	M_1=8	4.69	.656	2	2.656
	M_1=4		2.56	1.63	4.19
32	M_1=16	4.66	.328	2	2.328
	M_1=8		.641	1.56	2.2
64	M_1=32	4.66	.164	2.02	2.184
	M_1=16		.32	1.56	1.88
	M_1=8		.633	1.44	2.073

Table III. Comparison on the basis of APU

M		Direct Design	IFIR Method		
			G(z)	I(z)	Total
8	M_1=4	9.38	2.63	4.25	6.88
16	M_1=8	9.38	2.63	4	2.63
	M_1=4		2.56	3.25	5.81
32	M_1=16	9.31	.656	4	4.656
	M_1=8		1.28	3.13	4.41
64	M_1=32	9.31	.328	4.03	4.358
	M_1=16		.641	3.13	3.771
	M_1=8		1.27	2.88	4.15

What can be inferred from the Tables 2 and 3 is the following. The cost of implementation of the filter using direct design methods is almost independent of the number of subchannels. But the cost using multistage designs varies for different M. Also for a given M the computational cost is different for different values of M_1. This means therefore that there must be some value of M_1, where the computational cost is minimum. We will now proceed to determine that value of M_1 where the cost of implementation in terms of MPU is minimum for a fixed M.

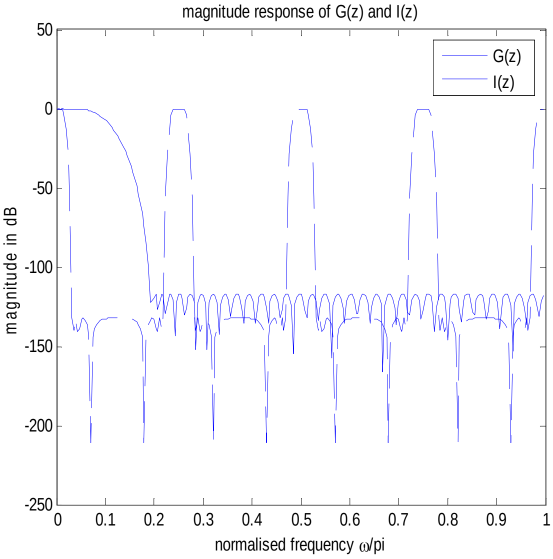
Substituting (1) and (2) in (3) , differentiating the result with respect to M_1 and equating to zero we will get the value of M_1 that will obtain minimum MPU for a given value of M. This is given by

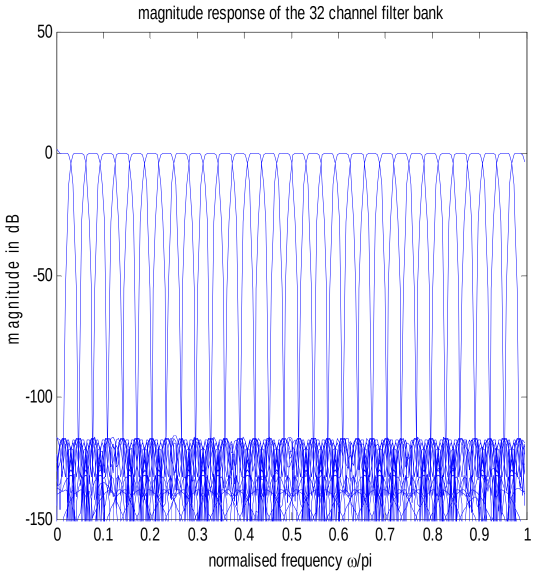
M_1(optimal) =

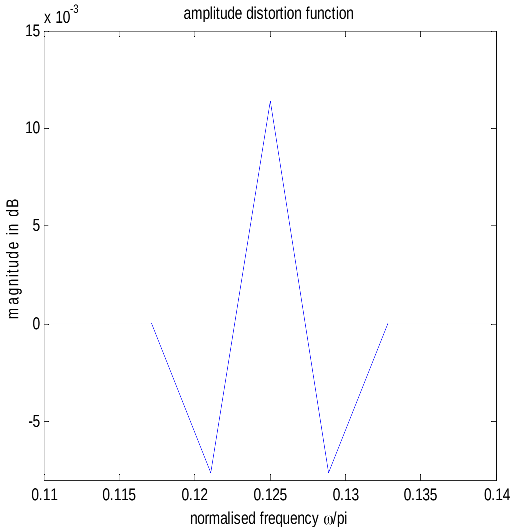
$$-\frac{2\pi}{M(\omega_s-\omega_p)-(\omega_s+\omega_p)}+\frac{2\pi\sqrt{M(\omega_s^2-\omega_p^2)}}{M(\omega_s^2-\omega_p^2)-(\omega_s+\omega_p)^2} \qquad (4)$$

Since the value of M_1 we obtain from (4) may not exactly be a factor of M we must choose an appropriate value that is close to it. On testing the values for different M_1 and comparing, the values that we have tabulated in Table (3) match very closely. So we may conclude that by factoring M suitably we can obtain maximum saving in computation cost. We have shown here, how the decimators of the analysis filter bank can be designed using the multistage approach. The interpolators in the case of the synthesis filter bank may also be designed using the multistage approach.

Next we will study that the characteristics of a prototype filter for a cosine modulated filter bank designed by this method and see how it compares with that of a filter obtained by direct design methods.







Comparisons are obtained in the case of a 32 channel filter bank, using the method proposed by creusere and Mitra in [5] where the order of the filter by direct design is 511 where as for the same method using the multistage implementation the order of G(z) ia 69 and I(z) is 73 all specifications of the filters being the same for both methods. We have chosen M_1 to be 8 is is the factor closest to the optimal value calculated using (4)
In the case of [5] the peak to peak amplitude distortion was marginally lower at .01

In conclusion we may say that the multistage implementation allows us to design the filter banks using filters of much less order and it would be highly desirable to explore this method for implementation of filter banks.

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