

Coevolution of Technical Trading Rules for High Frequency Trading

Kamal Adamu, Steve Phelps *

Abstract—Traders make trade decisions specifying *entry*, *exit*, and *stop loss* prices. Technicians often decide on *entry*, *exit*, and *stop loss* prices based on a predefined set of technical rules. In this paper, we employ a method based on grammatical evolution to coevolve technical rules for *entry*, *exit*, and *stop loss* for trading a London Stock Exchange (LSE) based stock in high frequency. We consider the case of two class of investors with risk averse, and loss preferences and build a partial trading frontier given the preferences considered. The performance of the rules evolved is compared to a publicly available trading system called the turtle trading system (TTS) and the best rules produced by our method outperforms TTS.

Keywords: *Coevolution, Grammatical Evolution, Trading Systems, Turtle Trading System, High Frequency Trading.*

1 Introduction

Traders make trade decisions specifying *entry*, *exit*, and *stop loss* prices. The *entry* rule dictates when to enter the market, the *exit* rule dictates when to *exit* the market, and the *stop loss* rule dictates when to *exit* a losing trade. There exists an interdependency between these prices [3, 6]. A trader that consistently fails to *exit* a losing trade when they have incurred a tolerable amount of loss will almost certainly be wiped out after a couple of losing trades. Moreover, a trader that takes profit too early or too late before making a required amount of profit will have very little to cover their costs and loss or loose part of the profit she has made [3, 6]. Technicians decide on *entry*, *exit*, and *stop loss* prices based on technical trading rules [3]. However, as the amount of candidate indicators increases, the search space of trading rules grows larger and more complex.

In a previous paper we employ a method based on grammatical evolution to coevolve technical trading rules for *entry*, *exit*, and *stop loss* for low frequency trading. The performance of the rules in [4] is assessed using the Sharpe ratio [4] and the objective is to find a collaborating set of *entry*, *exit*, and *stop loss* rules that maximise the Sharpe

ratio. In [9] they employ a four tree genetic programming approach in developing trading rules for *entering* and *exiting* a long or short GBP/EUR trade. They assess the performance of the rules evolved using a power utility function under three major assumptions about the preference of agents namely, loss aversion, risk aversion, and risk neutrality. The performance of the rules evolved in [9] is evaluated as the average performance under the three preferences. In this paper, we employ the method we used in [4] to coevolve *entry*, *exit*, and *stop loss* rules for trading Amvesco (Amvesco is listed on the London Stock Exchange) in high frequency. The performance of the rules evolved is compared to a publicly available trading system called the turtle trading system (TTS) [5, 6]. In addition, we compare the performance of our coevolutionary approach, which we will refer to henceforth as coevolutionary grammatical evolution (CGE), to a set of randomly distributed strategies. Similar to [9], we employ a power utility function as our fitness function however, we assess the performance of the rules evolved under three independent scenarios, risk aversion, and loss aversion and we build a partial trading frontier for these preferences (assuming a power utility, of course). Moreover, similar to [4, 9], we consider the case of a unit investor trading only one unit at any instant. The rest of the paper is organised as follows. Section 2 gives an overview of the TTS, coevolution, and Grammatical Evolution (GE). We present our data in Section 4, and a description of our framework is given in Section 3. We discuss our results in Section 5, and the paper ends with a conclusion in Section 6

2 Background

2.1 Turtle Trading System

The *entry*, and *exit* rules, for the turtle trading system are specified in Algorithm 1, and Algorithm 2 respectively. $H_t, t \in \{1, 2, 3, 4, \dots, T\}$ is the current highest price, and

Algorithm 1 Entry rules for TTS

```
if  $H_t > H_{t-55}$  or  $H_t > H_{t-20}$  then  
  GO LONG  
else if  $L_t < L_{t-55}$  or  $L_t < L_{t-20}$  then  
  GO SHORT  
end if
```

*Center for Computational Finance and Economic Agents University of Essex, CO4 3SQ, UK. Email: ksadam@essex.ac.uk, sphelps@essex.ac.uk

$L_t, t \in \{1, 2, 3, 4, \dots, T\}$ is the current lowest price. The *entry*, and *exit* rules of TTS are breakout rules. In other words, it is expected that if a global high (low) is made within a certain window (in this case a window of 55 bars or 20 bars) then there is a likelihood that prices will start to move in the direction of the breakout.

Algorithm 2 Exit rules for TTS

if $L_t < H_{t-20}$ or $L_t < L_{t-10}$ **then**
 Exit long position
else if $H_t > L_{t-20}$ or $H_t > H_{t-10}$ **then**
 Exit short position
end if

The TTS places the initial stop loss at *entry* using the following equation:

$$Stop_t = \begin{cases} Stop_{t-1} - 2N & \text{if Long} \\ Stop_{t-1} + 2N & \text{if short} \end{cases} \quad (1)$$

where N is the average true range and it is calculated as follows:

$$N = 19N_{t-1}TR_t/20 \quad (2)$$

$TR_t, t \in \{1, 2, 3, 4, \dots, T\}$ is the *true range* and its calculated as follows:

$$TR_t = \max(H_t - L_t, H_t - C_{t-1}, C_{t-1} - L_t) \quad (3)$$

$C_t, t \in \{1, 2, 3, 4, \dots, T\}$ is the price at the end of the time interval $t, t \in \{1, 2, 3, \dots, T\}$.

2.2 Grammatical Evolution

In GE, initially a population of random integer strings is initialized. The integer strings are a numeric representation of the solutions [1]. Solutions are mapped from integer strings to a human readable (executable) solutions using a set of production rules (grammar) [1].

The fitness of the mapped solutions is assessed and parents are selected for producing offspring solutions based on a roulette wheel principle [2]. Offspring solutions are mutated based on prespecified probability of mutation. Solutions with high fitness survive and pass down their genetic material to their offspring, and solutions with low fitness are replaced using tournament by solutions that surpass them in fitness. The process is repeated over several generations until a halting criterion is met.

2.3 Coevolution

Coevolution in the literature usually refers to a situation where a trait in one species evolves in response to a change in the trait in another species [10]. In other words, it is a situation where one species exerts evolutionary pressure on the other causing it to evolve and

vice versa [10]. Coevolution in nature can either be cooperative, or competitive. Coevolutionary computation borrows from the idea of coevolution in nature. In coevolutionary computing a problem is decomposed into subcomponents and the subcomponents are evolved simultaneously. This way, interdependencies between the different subcomponents is taken into account [10]. For instance, in this paper the trading problem is divided into different but interdependent subcomponents.

3 Framework

In our framework, we coevolve *entry* rules for long positions, *exit* rules for long positions, *stop loss* rules for long positions, *entry* rules for short positions, *exit* rules for short positions, and *stop loss* rules for short positions. Each set of rule is a species on its own. We denote the species of *entry* rules for long positions as $E_L^i, i \in \{1, 2, 3, \dots, P\}$, the species of *exit* rules for long positions as $C_L^i, i \in \{1, 2, 3, \dots, P\}$, and the *stop loss* rule for long positions as $S_L^i, i \in \{1, 2, 3, \dots, P\}$. E_S is the notation for *entry* rules for short positions, C_S is the notation for *exit* rules for short positions, and $S_S^i, i \in \{1, 2, 3, \dots, P\}$ is the notation for *entry* rules for short positions. Sexual reproduction is inter-species and solutions are rewarded based on how well they contribute to the overall problem. Collaborators are chosen at random from other species. For instance, when assessing a solution from the set $E_L^i, i \in \{1, 2, 3, \dots, P\}$, collaborators are chosen at random from $C_L^i, i \in \{1, 2, 3, \dots, P\}$, $S_L^i, i \in \{1, 2, 3, \dots, P\}$, $E_S^i, i \in \{1, 2, 3, \dots, P\}$, $C_S^i, i \in \{1, 2, 3, \dots, P\}$, and $S_S^i, i \in \{1, 2, 3, \dots, P\}$. Each species asserts evolutionary pressure on the other. Solutions that contribute to solving the problem attain high fitness and survive to pass down their genetic material to their offspring. On the other hand, solutions that do not contribute are awarded low fitness and are eventually replaced by solutions with higher fitness. Algorithm 3 illustrates the algorithm for our coevolutionary framework.

3.1 Utility Function

The framework for assessing the fitness of the rules produced using CGE is as follows:

In this paper, we employ a power utility function as our fitness function [9]. The power utility function is defined by the following equation [9]:

$$U(W_i) = \frac{W_i^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma}, \gamma > 1 \quad (4)$$

$$W_i = \begin{cases} W_0(1 + v_i) & v_i > 0 \\ w_0(1 + v_i)^\lambda & v_i < 0, \lambda > 1 \end{cases} \quad (5)$$

$$v_i = (P_t - P_{t-k})/P_{t-k} \times I_i, k \in \{1, 2, 3, \dots, T\} \quad (6)$$

$$I_i = \begin{cases} +1 & \text{Long position} \\ -1 & \text{Short position} \end{cases} \quad (7)$$

Algorithm 3 Framework for Coevolutionary Grammatical Evolution (CGE)

```

for each population do
  Initialise random population of integer strings
  Map integer strings
  Evaluate fitness of population
  Check for best solution (elitist)
end for
while halting criterion is not met do
  for each population do
    Generate offspring solutions
    Map offspring solutions
    Evaluate fitness of offspring solutions
    Select new population of solutions
  end for
  Check for new elitist
  Evaluate fitness of ecosystem
end while

```

v_i is the return for trade interval $i, i \in \{1, 2, \dots, N\}$, W_i is a modified level of wealth for the given trade interval $i, i \in \{1, 2, \dots, N\}$, and $I_i, i \in \{1, 2, 3 \dots N\}$ is the trade indicator for a given trade interval. For this study we consider the case of a unit investor and set the initial level of wealth $W_0 = 1$. λ , and γ define the risk, and loss preference of the agents respectively. The fitness, f , of a trading strategy is then taken to be the expected utility, which is calculated as follows:

$$f = E(U(\hat{W})) = \frac{1}{N} \sum_i^N U(W) \quad (8)$$

The following assumptions are implicit in the fitness evaluation:

1. Only one position can be traded at any instant.
2. Only one unit can be traded at any instant.
3. There is no market friction (zero transaction cost, zero slippage, zero market impact). Arguably, since only one unit is traded at any instant, the effect of market impact can be considered to be negligible.

Our objective is to find rules for *entry*, *exit*, and *stop loss* that maximise the expected utility, $E(U(\hat{W}))$.

Algorithm 4 Framework for fitness assessment

```

if Entry rule for long position is met then
  Go Long
else if Entry rule for short position is met then
  Go short
end if
if Long and (Exit rule for long position is met) then
  Exit long position
  calculate utility of wealth
else if Stop rule for long position then
  Exit long position
  calculate utility of wealth
end if
if Short and (Exit rule for short position is met) then
  Exit short position
  calculate utility of wealth
else if stop rule for short position is met then
  Exit short position
  calculate utility of wealth
end if

```

3.2 Parameter Settings

In this paper, we set the population size of $E_L^i, E_S^i, C_L^i, C_S^i, S_L^i, S_S^i$ to 50 (i.e. $N=50$). Collaborators are chosen at random for cooperation and this is done every generation (the epoch length for cooperation is $epoch=1$). The maximum number of generations, G_{max} , is set to 200 and if after $G_{max}/2$ there is no improvement in the mean fitness of E_L^i , the search is terminated and a new search is initialised. We run 10 searches in this fashion. For one set of 10 searches we consider the case of a loss averse agent with $\gamma=35$ and $\lambda=1.15$. For a second set of 10 searches we consider the case of risk averse agents with $\gamma=35$, and $\lambda=1$.

The grammar used in mapping E_L^i, E_S^i, C_L^i , and C_S^i is shown in Table 1 and the grammar used in mapping S_L^i, S_S^i is shown in Table 2. In our notation, $O(t-n:t-1)$ represents a set of open prices, $C(t-n:t-1)$ represents a set of closing prices, $H(t-n,t-1)$ represents a set of highest prices, and $L(t-n:t-1)$ represents a set of lowest prices between $t-n$ and $t-1$. $O(t-n)$ represents the open price at $t-n$, $C(t-n)$ represents the closing price at $t-n$, $H(t-n)$ represents the highest price at $t-n$, and $L(t-n)$ represents the lowest price at $t-n$. Where $n \in \{10, 11, 12, \dots, 99\}$ and $t \in \{1, 2, \dots\}$.

Table 1: Grammar for mapping E_L^i , E_S^i , C_L^i , and C_S^i . ϕ is the set of non terminals, r are the rules for mapping the non-terminal ϕ , and n is the number of rules for mapping the non-terminal ϕ

ϕ	r	n
$\langle \text{expr} \rangle ::$	$\langle \text{binop} \rangle (\langle \text{expr} \rangle, \langle \text{expr} \rangle)$	(2)
$\langle \text{rule} \rangle ::$	$\langle \text{var} \rangle \langle \text{op} \rangle \langle \text{var} \rangle$ $\langle \text{var} \rangle \langle \text{op} \rangle \langle \text{fun} \rangle$ $\langle \text{fun} \rangle \langle \text{op} \rangle \langle \text{fun} \rangle$	(3)
$\langle \text{binop} \rangle ::$	and, or, xor	(3)
$\langle \text{var} \rangle ::$	$H(t - \langle \text{window} \rangle)$ $L(t - \langle \text{window} \rangle)$ $O(t - \langle \text{window} \rangle)$ $C(t - \langle \text{window} \rangle)$	(4)
$\langle \text{op} \rangle ::$	$>, <, =, \leq, \geq,$	(1)
$\langle \text{window} \rangle ::$	$\langle \text{integer} \rangle \langle \text{integer} \rangle$	(9)
$\langle \text{integer} \rangle ::$	1, 2, 3, 4, 5, 6, 7, 8, 9	(9)
$\langle \text{fun} \rangle ::$	$\text{sma}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{sma}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{sma}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{sma}(C(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(C(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(C(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(C(t - \langle \text{window} \rangle : t - 1))$	(15)

Table 2: Grammar for mapping S_L^i , and S_S^i . ϕ is the set of non terminals, r are the rules for mapping the non-terminal ϕ , and n is the number of rules for mapping the non-terminal ϕ

ϕ	r	n
$\langle \text{expr} \rangle ::$	$\langle \text{preop} \rangle (\langle \text{expr} \rangle, \langle \text{expr} \rangle)$	(2)
$\langle \text{rule} \rangle ::$	$\langle \text{rule} \rangle \langle \text{op} \rangle \langle \text{rule} \rangle$ $\langle \text{var} \rangle \langle \text{op} \rangle \langle \text{var} \rangle$ $\langle \text{var} \rangle \langle \text{op} \rangle \langle \text{fun} \rangle$ $\langle \text{fun} \rangle \langle \text{op} \rangle \langle \text{fun} \rangle$ $\langle \text{fun} \rangle$ $\langle \text{var} \rangle$	(6)
$\langle \text{preop} \rangle ::$	min, max	(2)
$\langle \text{var} \rangle ::$	$H(t - \langle \text{window} \rangle)$ $L(t - \langle \text{window} \rangle)$ $O(t - \langle \text{window} \rangle)$ $C(t - \langle \text{window} \rangle)$	(4)
$\langle \text{fun} \rangle ::$	$\text{sma}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(H(t - \langle \text{window} \rangle : t - 1))$ $\text{sma}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(L(t - \langle \text{window} \rangle : t - 1))$ $\text{sma}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(O(t - \langle \text{window} \rangle : t - 1))$ $\text{sma}(C(t - \langle \text{window} \rangle : t - 1))$ $\text{ema}(C(t - \langle \text{window} \rangle : t - 1))$ $\text{max}(C(t - \langle \text{window} \rangle : t - 1))$ $\text{min}(C(t - \langle \text{window} \rangle : t - 1))$	(15)
$\langle \text{window} \rangle ::$	$\langle \text{integer} \rangle \langle \text{integer} \rangle$	(1)
$\langle \text{ineteger} \rangle ::$	1, 2, 3, 4, 5, 6, 7, 8, 9	(9)

4 Data

In this paper, we compress historical high frequency tic-data into a sequence of high, low, open, and close proxy prices for five minutely trading intervals [7]. The data we use is Amvesco tic-data for the period between 1 March 2007 to 1 April 2007. The descriptive statistics of the data is shown in Table 4

Table 3: Descriptive statistics of data

Mean	Standard deviation	Skewness	Kurtosis
-0.0193	1.6072	-1.4265	24.6282

Table 4:

The data was divided into four blocks for K-fold cross validation [8].

5 Results & Discussion

Table 5 shows the descriptive statistics for the set of loss averse agents ($\lambda=1.15, \gamma=35$) produced using CGE, the set of risk averse agents ($\lambda=1, \gamma=35$) produced using CGE, and random strategies (MC) with loss averse ($\lambda=1.15, \gamma=35$), and risk averse ($\lambda=1, \gamma=35$) preferences. Also included in Table 5 is the expected utility obtained by the TTS for a risk averse ($\lambda=1, \gamma=35$), and loss averse ($\lambda=1.15, \gamma=35$) preference.

The results in Table 6 are results for a sign test for the null hypothesis that the median of the set of loss averse strategies produced using CGE is different from the expected utility of the TTS with the same preference, and for the null hypothesis that the median of the set of risk averse strategies produced using CGE is different from the expected utility of the TTS with the same preference. The results in Table 6 indicates a failure to reject the hypothesis at 95% confidence interval. This means given the preferences considered we have not produced sets of strategies with medians that are statistically different from the expected utility of TTS. However, the best strategies produced using CGE outperform TTS for both scenarios considered.

The results in Table 7 is for the null hypothesis that the median of the set of loss averse agents is different from the median of random strategies with the same preference. It also includes results for the null hypothesis that, the median of the set of risk averse agents is different from the median of a set of random strategies with the same preference, and the median of a set of random strategies is different from the expected utility of the TTS. The results in Table 7 show that we should reject the null at 95% confidence interval for all of the strategies.

Figure 1 shows the trading frontier from the two sets of strategies produced using CGE. $\mu_r, \sigma_r, \alpha_r,$ and γ_r are

Table 5: Descriptive statistics of the distribution of utility for different strategies. μ is the mean, σ is the standard deviation, α is the skewness, and ω is the kurtosis. MC is a set of random strategies

	μ	σ	α	ω
CGE($\lambda=1.15, \gamma=35$)	+0.0001	0.008	-1.49	5.31
CGE($\lambda=1, \gamma=35$)	-0.0055	0.007	-0.75	1.94
MC($\lambda=1.15, \gamma=35$)	-0.0018	0.008	-3.52	20.80
MC($\lambda=1, \gamma=35$)	-0.0003	0.003	-0.99	21.65
TTS ($\lambda=1.15, \gamma=35$)	-0.001	0.0001	-1.10	2.29
TTS ($\lambda=1, \gamma=35$)	-0.0003	0.0001	-1.05	2.24

Table 6: Sign test for the null hypothesis that the medians of the sets strategies produced using CGE is the same as the expected utility of the TTS with the same preference.

	z-value	p-value	sign
($\lambda=1.15, \gamma=35$)	1.5811	0.10940	2
($\lambda=1, \gamma=35$)	-0.9487	0.34380	2

the mean, standard deviation, skewness, and kurtosis of the strategies respectively. Figure 1 shows that quite a number of loss averse CGE strategies dominate their risk averse strategies in the μ_r and σ_r frontier, and produce considerably positively skewed returns compared to their loss averse counterparts.

6 Conclusions and Future Work

Technical traders make trade decisions specifying entry exit, and stop loss prices based on technical trading rules. In this paper, we have coevolved rules that tell a trader when to *enter* the market, and when to *exit* the market for long, and short positions respectively as well as rules that tell a trader when to *exit* a loss making position (*stop loss*). Two set of rules were produced for two different preferences. The performance of the rules produced is compared with a publicly available trading system called the turtle trading system (TTS) and the best rule produced by our method outperforms the TTS. Our rules were also compared to a set of random strategies and the strategies produced by our method are statistically better

Table 7: Sign test for the null hypothesis that the medians of random strategies is same as the median of strategies produced using CGE, and for the null hypothesis that the median of random strategies is the same as the expected utility of the TTS

	z - value	p-value	sign
($\lambda=1.15, \gamma=35$)	-8.5000	0.0000	7
($\lambda=1, \gamma=35$)	+8.5000	0.0000	7
TTS	+3.1000	0.0019	34
TTS	+2.7000	0.0069	36

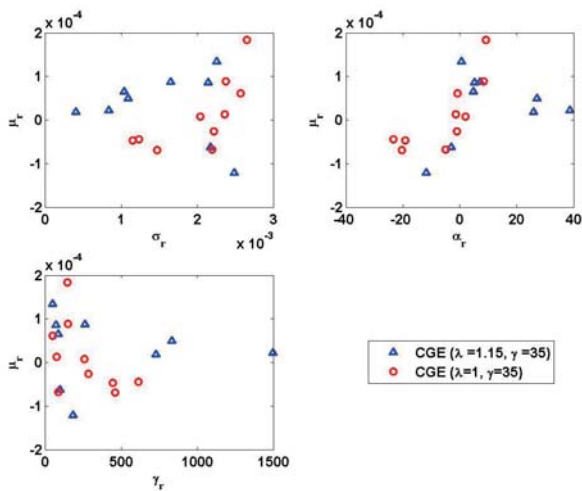


Figure 1: Trading frontier of CGE strategies for different preferences. μ_r is the mean return, σ_r is the standard deviation of return, α_r is the skewness of return, γ_r is the kurtosis of return

than random strategies in trading the stock of Amvesco for both of the preferences considered (i.e loss aversion, and risk aversion). The TTS is also shown to be statistically better than a set of random strategies in trading the stock of Amvesco. Moreover, we have generated a partial trading frontier and most of the loss averse strategies dominate the risk averse strategies in the mean, and standard deviation setting, and produced highly skewed positive returns.

In this paper, we make an assumption of frictionless markets. In future work we will include transaction cost. Moreover, we will allow for multiple positions to be traded.

References

- [1] O'Neill, M., Ryan, C., *Grammatical Evolution: Evolutionary Automatic Programming in an Arbitrary Language*, Kluwer Academic Publishers, 2003.
- [2] Fogel, B.F., *Evolutionary Computation: Toward a New Philosophy of Machine Intelligence*, IEEE Press, 1995.
- [3] Murphy, M.J., *Technical Analysis of the Futures Markets*, New York Institute of Finance, Prentice Hall, 1986.
- [4] Adamu, K., Phelps., S., "A Coevolutionary Grammatical Evolution Approach for Developing Technical Trading Rules," *Asia Pacific Industrial Engineering and Management Sys. Conf.*, Kitakyushu, Japan, pp. 187.

- [5] Anderson, J.A., "Taking A peek Inside the Turtle's Shell," *School of Economics and finance, Queensland University of Technology.*, Australia.
- [6] Faith, C., "The original turtle trading system," www.turtletrader.com
- [7] Ahmad, K., Ahmad, S., "Reuters Data Analysis, Brand Protection: Comparing the Provenance of two Time Series," *University of Surrey Department of Computing* 2003
- [8] Kohavi, R. "A study of Cross-Validation and Bootstrap for Accuracy Estimation and Model Selection," *International joint Conference on artificial intelligence*, 1995
- [9] Saks, P., Maringer, D., "Evolutionary Money Management," *Lecture Notes in Computer Science, Springer*, V5484/2009, pp. 162-171
- [10] DeJong, M. P. K., "Cooperative coevolution: an architecture for developing coadapted subcomponents," *Evolutionary Computation, The MIT Press*, V8, no.9, pp.1-29