Developing Slip-Flow and Heat Transfer in Annular Microchannels with Constant Heat Flux

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Abstract— Rarefied gas flow and heat transfer in the entrance region between two concentric cylinders are investigated. The continuum momentum and energy equations, with the velocity slip and temperature jump condition at the solid walls, are solved numerically. The slip velocity and temperature jump at the walls are taken into account. Two different cases of the thermal boundary conditions are considered: uniform heat flux at the outer wall and adiabatic inner wall (Case A) and uniform heat flux at the inner wall and adiabatic outer wall (Case B). The effects of the channel radius ratio \( \frac{r^*}{r_0} \), and Knudsen number \( Kn \) on the simultaneously developing velocity and temperature field, the friction coefficient, and Nusselt number are studied. Results show that the micro-scale effects that being associated with rarefaction are caused enhancement in the rate of heat transfer. The results are compared with those available in the literature and excellent agreement is observed.

Keywords— Slip-flow, Rarefaction, Annular microchannel, Entrance region, Knudsen number

I. INTRODUCTION

Microscale single-phase heat transfer has attracted an important research interest in the last decades due to using of the microchannels is a way to solve the current and future problems of the dissipation of high heat fluxes. These researches has been motivated by recent advances and recent fields of applications of microchannels in microelectromechanical systems (MEMS), inject print heads, infrared detectors, diode lasers, miniature gas chromatographs [1], DNA sequencing, automotive, chemical and food industries, environmental technologies, and the aviation and space industries.

For continuity assumption to be fulfilled the mean free path, \( \lambda \), of the fluid molecules, has to be much lower than the hydraulic characteristic length scale of microchannel. The Knudsen number, \( Kn = \frac{\lambda}{D_h} \), is used to check whether the flow can be considered continuum or not [2]. Under standard condition, the molecular mean free path \( \lambda \) is typically 70 nm. Considering the typical hydraulic diameters of microchannels (1 to 100 \( \mu \)m), the associated Knudsen numbers indicate some level of rarefaction even at normal pressures [2]. So as the channel size shrink down, the standard expressions of continuity should be reviewed.

For \( Kn < 10^{-2} \), the flow is a continuum flow and it is modeled by the classic Navier-Stokes and energy equations. But the initial deviations from classic continuum models appear at the walls because at the walls there are fewer interactions between the gas molecules than in the core of the flow [3]. So for \( 10^{-3} < Kn < 10^{-1} \), the flow is a slip flow [4] and the Navier-Stokes and energy equations remain applicable, provide a velocity slip and a temperature jump are taken into account at the walls.

It is experimentally shown that the rarefaction effects encountered in the microfluidic devices are important. Experiments conducted by Arkilic, Breuer and Schmidt [5,6, Pfalher, Harley, Bau and Zemel [7,8], Hareley, Huang, Bau and Zemel [9], and Maurer Tabeling, Joseph and Willaime [10] on the transport phenomena of gases flow in microchannels confirm that fluid flow and heat transfer at microscale differ greatly from those at macroscale.

Duan and Muzyczka [1] and Avci and Aydin [11] performed an analytical analysis of fully developed laminar slip flow and heat transfer in annular microchannels with constant heat flux boundary conditions, with viscous dissipation. Rarefaction and aspect effects on the Nusselt number had been studied in their papers, in addition to [11] has been studied the viscous dissipation effects.

Tune and Bayazitoglu [12] studied temperature developing and viscous dissipation effects in microtubes by the integral transform technique, for steady state and laminar gas flows with uniform heat flux and uniform temperature boundary conditions. They observed that the Nusselt number decreasing is greater when considering the viscous dissipation, also M. Renksizbulut, Niaznand and Tercan [13] and Niaznand, Renksizbulut and Saeed [14] solved the problem of laminar
simultaneously developing gas flow in a rectangular and trapezoidal microchannel respectively with a constant wall temperature boundary condition at the walls and aspect ratio and rarefaction effects are presented.

In the present work, incompressible gas flow and heat transfer in a concentric annular microreactor with constant heat flux boundary condition are investigated. Two dimensional Navier-Stokes and energy equations with velocity-slip boundary and temperature jump conditions at the walls have been solved numerically using a control-volume method. Axial molecular diffusion of momentum and heat are also included in analysis. The effect of radius ratio, Knudsen number related to the slip flow regime and the Brinkman number on the Nusselt number have been investigated particularly in the entrance region.

II. GOVERNING EQUATIONS

The flow between two concentric cylinders is analyzed, and the basic geometry shown in Fig. 1. In this study it is assumed that the flow is symmetric in \( \theta \) direction, so the flow is two dimensional. The inflow has a flat velocity profile such that \( u_0 = 1 \) and \( v_0 = 0 \) with uniform temperature \( T_i = 0 \). The flow is hydrodynamically and thermally developing, steady, laminar viscous flow with constant properties (i.e. the thermal conductivity and the thermal diffusivity of the fluid are considered to be independent of temperature). The dimensionless variables are defined as follows:

\[
\begin{align*}
z &= \frac{z}{D_h}, & r &= \frac{r}{D_h}, & u &= \frac{u}{U_i}, & v &= \frac{v}{U_i}, \\
T &= \frac{T - T_i}{q'D_h/k}, & u^* &= \frac{u U_i}{\alpha \sqrt{\text{Ra}}}, & P &= \frac{P_l}{\rho U_i^2}, \\
\text{Re} &= \frac{\rho U_i D_h}{\mu}, & \text{Kn} &= \frac{\lambda}{D_h}, & \text{Pr} &= \frac{\mu C_P}{k}.
\end{align*}
\]

(1)

Therefore the non-dimensional governing equations in a cylindrical coordinates can be expressed as (the stars were omitted for simplicity):

Case A

\[
\frac{\partial T}{\partial r} \bigg|_{r=R_i} = \text{Re Pr}, \quad \frac{\partial T}{\partial r} \bigg|_{r=R_o} = 0
\]

(7)

And in the case B, thermal boundary conditions are:

\[
\frac{\partial T}{\partial r} \bigg|_{r=R_i} = \text{Re Pr}, \quad \frac{\partial T}{\partial r} \bigg|_{r=R_o} = 0
\]

(8)

It is suggested that in analogy with the velocity slip phenomenon, there might be a temperature jump at the walls.
Smoluchowski [17] was derived the below equation to describe the temperature jump phenomena.

\[
T - T_w = \frac{2 - \sigma_T}{\sigma_T} \left( \frac{2\gamma}{\gamma + 1} \frac{Kn}{Re Pr} \frac{\partial T}{\partial n} \right)_{w} \tag{9}
\]

As for the slip, it can be assumed that a fraction \(\sigma_T\) of the molecules have a long contact with the wall and that the wall adjusts their mean thermal energy. The other fraction, \(1 - \sigma_T\), is reflected keeping its incident thermal energy [3]. \(\sigma_T\) and \(\sigma_T\) are functions of the composition and temperature of gas, the gas velocity over the surface, and the solid surface temperature, chemical state and roughness [18]. We can assume that the accommodation is constant and if the gas is air and the surface is iron, \(\sigma_T = 0.9\) [19].

Note that the body forces are ignored in this work. For outflow, zero-gradients along the axial flow direction are applied to \(u\), \(P\), and \(T\), so \(\frac{\partial u}{\partial x} = \frac{\partial P}{\partial x} = \frac{\partial T}{\partial x} = 0\) and the radial velocity is zero, i.e. \(v_r = 0\).

In the all above equations \(D_h\) is hydraulic diameter. The Reynolds number (Re), is based on the uniform inlet velocity and hydraulic diameter. Since the density is constant, mean velocity \((U_m)\) is equal to inlet velocity \((U_i)\) everywhere in the channel; also the Knudsen number is based on hydraulic diameter.

As it is said, the flow is incompressible, i.e. \(Ma = 0.3\), moreover Mach number (Ma) can be written as a function of Reynolds number (Re) and Knudsen number (Kn). Hence the selected range for Re and Kn should satisfy the incompressibility of flow.

\[
Ma = \frac{Kn Re}{k^2 \sqrt{\gamma}} \tag{10}
\]

III. NUMERICAL METHOD

The governing equations are integrated over the control volumes of the solution domain and are discrete. So the governing equations are converted into a system of algebraic equations. The numerical method goal is solving this system of algebraic equations. At the first a velocity, pressure and temperature fields are guessed and the velocity field from the momentum equations are calculated using the approximate pressure and temperature field without the velocity slip condition. Then pressure correction and velocity correction are solved. Finally temperature field is calculated from energy equation without temperature jump condition by the updated velocities. The obtained velocity field does not necessarily satisfy the mass conservation equation, and thus a velocity correction is introduced. It is an iterative solution, after substitution of the new velocity, pressure and temperature field with the guessed fields the above steps should be repeated sequentially. After some iteration the velocity slip and temperature jump conditions with under relaxation coefficients is exerted in the algebraic system in return the classic boundary condition. These under relaxation coefficients should be smaller for the higher Knudsen numbers. The iterations are continued until both of the maximum temperature difference between two sequence iteration and maximum continuity imbalance was smaller than a criterion.

IV. GRID INDEPENDENCE AND VALIDATION

The grids are finer near the channel walls and in the entrance region, where the flow parameters variations are large. The grid resolution is studied by performing computations at Re=40 with Kn=0.01 for flow in an annular duct of \(r^*=0.5\). Supplementary runs were performed with coarse grid size (200x40), medium grid size (250x50) and fine grid size (300x65). The effect of this three different grid sizes on Nusselt number (Nu) and friction factor coefficient \(f/Re\) for case A are presented in Fig. 2. As can be seen, Nu decrease and \(f/Re\) increases with increasing the number of grid sizes. But the values obtained from fine and medium grid sizes are very close, so medium grid size (250x50) provides satisfactory resolution and it is a suitable compromise between accuracy and solution cost. Therefore, all the computations were performed with this grid size. Furthermore, channel lengths are set to a value greater than estimated entrance length of the flow to ensure that fully developed conditions are achieved at the exit.

![Fig. 2. Effect of grid size on Nu and \(f/Re\)](image)

The computations were performed for two cases A and B with Kn=0 for \(r^*=0.5\) in the entrance region. Fig. 3 shows the variations of the local Nusselt number in the entrance region in the present study and those available in the handbook of heat transfer [19], which shows an excellent agreement. In addition in Fig. 4 fully developed values of friction factor of different radius aspect ratio \((r^*)\) have been compared the handbook of heat transfer [19] for Kn=0 and Re=100. For more case validation it can be refer two the papers of Duan and Muzychka. [1] and Avci and Aydin [11].
V. RESULT AND DISCUSSION

V.1. FLOW FIELD:

The fluid is air with $Pr = 0.71$. If the microchannel is short, such that the hydrodynamic and thermal development length is no longer negligible compared to the microchannel length, entrance region have to be taken into account. In the entrance region the pressure in not uniform and the radial velocity is not zero over a given cross-section. However, in the fully developed region, as can be seen in the Fig. 5 the isobar lines are normal to the $z$ direction, the radial velocity is zero.

Fig. 6 illustrate the nondimensional velocity profiles of these channel at three nondimensional axial locations. It is clearly can be seen that by increasing the Kn from 0.001 to 0.1 the velocity slips on the walls, by developing the slip velocity decreases. The velocity profiles of the fully developed region are not symmetric, and the maximum in the velocity occurs closer to the inner wall. Furthermore the slip velocity at inner wall is greater than the slip at outer wall. As shown in this figure increasing the Kn results an increase the magnitude of slip velocities at both inner and outer walls while the magnitude of core velocity decreases. Also for a given Kn, the magnitude of slip velocity decreases and the core velocity increases as the flow develops.

The pressure drop along the channel can be expressed as [13]:

$$\frac{dP}{dx} = 4f \frac{1}{2} \frac{\rho u_m^2}{D_h}$$  \hspace{1cm} (11-a)

Where $f$ and $\frac{dP}{dx}$ is the Fanning friction factor coefficient and axial pressure gradients, respectively. By nondimensionalizing the equation (11-a) we have:

$$\frac{dP^*}{dx} = 2f \text{Re Pr}$$  \hspace{1cm} (11-b)

So:

$$f \text{Re} = \frac{1}{2Pr} \frac{dP^*}{dx}$$  \hspace{1cm} (11-c)

Slip at the walls reduces the axial pressure gradients and the fluid molecules adjacent to the walls can keep part of their inlet momentum depending on Kn, it can be seen from Fig. 6. Fig. 7 illustrates the $f$Re along the channel for three different Knudsen numbers. For a given Knudsen number, as the flow develops $f$Re decreases and approaches to a constant value at fully developed region. Also $f$Re decreases as Kn increases. But differences between $f$Re in the entrance region for a constant nondimensional axial location are very greater than differences at fully developed region. The pressure drop in the fully developed region the pressure drop is only due to wall
friction while in developing region the change in the momentum rate accounts for a major part of pressure drop.

In Fig. 8 the $f/Re$ of the different aspect ratios ($r^*$) are compared in a given Knudsen number. For lower Knudsen numbers the friction factor at the inlet decreases by increasing $r^*$ and the difference is great, but in the fully developed region it becomes reverse and the $f/Re$ increases by increasing $r^*$, as shown in the Fig 8. For higher Kn the $f/Re$ has a constant behavior from inlet to outlet and it is always decreases by increasing the $r^*$.

V.II. TEMPERATURE FIELD:

In slip flow regime, the heat transfer of our problem can be treated, providing the energy equation (4) and temperature boundary conditions (7 for case A and 8 for case B) are solved with the momentum equations (3, 4) and the slip-flow boundary conditions. As can be observed from energy equation, the temperature distribution in the entrance region is affected by the geometry, Re, Pr, Kn and Br numbers.

The local Nusselt number is defined as:

$$Nu = \frac{h_x D_h}{k} = \frac{D_h}{T_w - T_m} \left( \frac{\partial T}{\partial n} \right)_w = \frac{q^* D_h}{k} = \frac{1}{T_w - T_m} \frac{T_w - T_m}{T_w - T_m}$$  \(\text{(12)}\)

Where $q^*$ is the heat flux on the outer wall (case A) or on the inner wall (case B), $T_w$ is the outer wall temperature for case A and inner wall temperature for case B and can be obtained from temperature jump condition (9):

And $T_m$ is the bulk mean temperature, defined as:

$$T_m = \frac{1}{A u} \int T \, \nu \, dA$$  \(\text{(13)}\)

The velocity-slip condition (6) and temperature-jump (9) have apposite effects on the heat transfer rate. The velocity-slip at the wall tends to decrease the temperature difference between the gas and the wall by increasing the advection in the wall, while temperature-jump would act to increase the temperature difference by creating an effect similar to contact resistance. Larrode, Housiadas and Drossinos [20], and Yu and Ameel [21] proposed that in the slip flow regime heat
transfer rate could be increased or decreased compared to continuum flow regime depending on gas and wall interaction. The variation of the Nusselt number along the main flow direction in the channel at three different Knudsen numbers for both cases A and B are shown in figures 9 and 10 for $r^*=0.5$. It can be seen from these figures that an increasing in Kn leads to decrease the Nusselt number. On the other hand the temperature-jump effect conquer the velocity-slip effect and the net effect result a reduction in the Nusselt number.

Figures 11 and 12 illustrate the variation of the Nusselt number with the radius ratio ($r^*$) in both cases A and B along the channel. It can be seen that the $Nu$ is independent of the geometry. For all radius ratios, the Nusselt numbers of the inlet approach a constant asymptotic value. Yet in the fully developed region, there is a distinction between $Nu$. For a given Knudsen number, in the inlet of the channels with smaller aspect ratios $r^*$ have a bigger $Nu$, but behavior of the fully developed Nusselt numbers versus $r^*$ in case A and B are different. In case A, fully developed Nusselt number decreases with the radius ratio while in case B it is increases with the aspect ratio, so Nusselt number in case B has a uniform behavior from inlet to outlet.

VI. CONCLUSIONS

In this work, the effects of rarefaction have been examined for simultaneously developing laminar, constant-property flows in concentric annular microchannels for $Kn \leq 10^{-1}$. Different radius aspect ratios ($0.1 \leq r^* \leq 0.9$) were considered with $Pr=0.71$. Two different orientations of the wall thermal boundary conditions have been considered.

The results indicate that the slip flow Nusselt numbers and friction factors are lower than those for continuum flow and decrease with increasing the Knudsen number. Especially in the entrance region the slip-flow $Nu$ and $f/Re$ have too lower values than the no-slip cases and as the flow develops the differences become smaller. In the other hand, friction factors which are obtained from the slip flow is limited despite of the continuum flow which has a singularity at $z=0$. Velocity-slip and temperature-jump are both very large in the entrance region due to the existence of large gradients. However, the slip velocities at both wall is reduced as the flow develops along the channel.

REFERENCES