A Study of Numerical Modeling of Heat Transfer in Wall-Adjacent Turbulent Flows

A K Borah and A N Dev

Abstract --- ***I*n this paper we present some of the most practical problems of convective heat transport to or from a rigid surface, the flow in the vicinity of the body is in turbulent motion. On the hand, at the solid-fluid interface itself, the no-slip boundary condition ensures that turbulent velocity fluctuations vanish. But, at the wall, the diffusive transport of heat and momentum in the fluid is precisely expressible by the laws of applicable to laminar flow. Because, the turbulent shear stress, and often the turbulent heat flux, can, by continuity, increase only as the cube of the distance from the wall, there is a thin but very important sublayer immediately adjacent to the solid surface where the transport of heat and momentum is predominantly by molecular diffusion. Further from the wall, again by virtue of the cubic variation, there is a very rapid changeover to the state where turbulent transport dominates, a condition that normally prevails over the remainder of the flow. This thin *sublayer* and the *adjacent* transition region extending to the fully turbulent regime – collectively we shall term the viscosity affected-sublayer (VSL); is the subject of the present paper. Furthermore, we are concerned with how one can accurately model the flow in this region in a form suitable for use in CFD software. However, the accuracy is not only criterion.**

Index Terms---**Viscosity affected-sublayer, Nonlinear eddy viscosity models, Eddy-viscosity model, Unified methodology for integrated sublayer transport-analytical UMIST-A, Turbulent Elliptic Axisymmteric Manchester (TEAM) Power law differencing scheme (PLDS).**

I. INTRODUCTION

Wall-function strategies are certainly the approach of preferred by CFD code and their clients. However, the accuracy returned by many schemes when applied to new types of problems can be quite poor. Fig.(1) shows the computed heat transfer coefficients produced by a range of different computers for the problem of convective heat transfer downstream from an abrupt pipe enlargement (*X* represents the distance downstream of the enlargement). Evidently, there are vastly different predicted variations of Nusselt number among the entries. Despite the inevitably high computational cost, there has been a large effort in academic circles over the past forty years to develop models of turbulence that are applicable in both the fully turbulent regime and the viscous sublayer–so called low-Reynolds-number models. Models of this type range from the simple mixing-length schemes from the 1960s and two-equation eddy–viscosity models (EVMs) from the 1970s to more intricate connections between the turbulent fluxes and the mean-field gradients, exemplified by nonlinear eddy-viscosity models (NLEVMs) and second-moment closure. While such low-Reynolds number models have enabled accurate CFD computations to be made of a range of difficult flows, they are not the subject of this review (although results obtained with some are included in later comparisons). Instead attention is directed at much simpler approaches to handling the sublayer region known as wall functions [Patankar S. V and Spalding D. B Heat].

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II. ESSENTIAL FEATURES OF THE VSL AND SIMPLE APPROACHES TO ITS MODELING

We have considered that a *wall* whose surface lies in the *x-z* plane with the mean velocity, *U* (*y*), in the *x* direction. At the wall itself, the no-slip condition requires that the fluctuating velocity components should vanish. Moreover, if the density may locally be assumed uniform, from continuity the fluctuating velocity gradient in the direction normal to the wall, *y* must also vanish. On the other hand, if the velocity components are expanded in a Taylor series in terms of the wall-normal distance, we deduce that while the normal stresses $\overline{u^2}$ and $\overline{w^2}$ initially increase as y^2, $\overline{v^2}$ increases as y^4 (kinematics stresses are employed with typical dimensions $(m/s)^2$. Now, equally important, the turbulent shear stress $\overline{uv}$ increases only as y^3. These different exponents of dependency on *y* have been well confirmed both by experiment and direct numerical simulation in Fig.(2). The thinness of the sublayer across which the changeover from molecular to turbulent transport occurs, in simple flows the shear stress parallel to the wall within the fluid is often essentially uniform and equal to the wall kinematics shear stress, τ_w / ρ. As one moves away from the wall there is a progressive switchover from molecular to turbulent stress as exemplified by the y^3 variation. As Reynold's [Reynolds O] pioneering work first investigated, the rate of conversion of mean kinetic energy into turbulent kinetic energy by mean shear is equal to $-uv\partial U / \partial y$. In a constant stress layer this leads directly to the conclusion [Rotta J. C,] that the maximum rate of turbulence energy generation occurs where the turbulent and viscous stresses are equal; where $v\partial U / \partial y = -\overline{uv} = 0.5\tau_w / \rho$. In this case in simple wall shear flows the most intense turbulent velocity fluctuations normally appear within the VSL. If the region adjacent to the wall is at constant shear stress then dimensional analysis readily suggests that within that region

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$$U^{+} \equiv \frac{U}{U_{\tau}} = f\left(y^{+}\right) \equiv f\left(\frac{yU_{\tau}}{\nu}\right) \quad (1)$$

Where U_{τ} represents friction velocity $\sqrt{\tau_w / \rho}$. If the region of validity of Eq.(1) extends into the fully turbulent regime, then various arguments, ranging from the mixing-length hypothesis to Millikan's [Millikan, C. B] overlap concept, may be employed to infer that there Eq.(1) may be particularized to

$$U^{+} = \frac{1}{\kappa} \ln\left(Ey^{+}\right) \quad (2)$$

Where κ *and* E represents the von Karman constant, reflects the structure of turbulence in this *fully* turbulent region and E is the coefficient dependent on the flow structure over the VSL. Eq. (2) has been used directly for applying effective wall boundary conditions in CFD methods to avoid having to resolve the viscous sublayer [Bradshaw, P, Ferris D. Hand Attell, N. P]. Eq.(1) and Eq.(2) is applicable only if the shear stress remains very nearly constant across the region to which it is applied. Furthermore, a decrease in shear stress across the sublayer of just 5% causes a marked increase in the constant E in Eq.(2). On the other hand, physically this amounts to a thickening, in terms of y^{+}, of the VSL due, ultimately, to the decline of turbulence energy generation relative to viscous dissipation in the sublayer. Such a decrease in shear stress may arise interalia from flow acceleration [Jones, W. P and Launder B. E ; Perkins, K. R and M. Eligot; Kays, W. M and Moffat, R. J], suction through the wall [Kays, W. M and Moffat, R. J], net buoyant force on fluid moving along vertical walls [Jackson, J. D and Hall W. B], or, even in fully developed pipe flow at bulk Reynolds numbers below 10^4 [Kudva, A. A and Sesonske A ; Patel, V. C and Head M. R]. Likewise, a shear stress that increases strongly with distance from the wall can lead to thinning-of the sublayer [Simpson, R. L Kays W. M and Moffat R. J ; Spalart P and Leonard A; Launder, B. E]. On the other hand, the picture is complicated by flow impingement where turbulence energy is generated by the interaction of normal stresses and normal strains rather than by shear. The thermal equivalent to Eq. (2) is

$$\Theta^{+} = \frac{1}{\tilde{\kappa}} \ln\left(\tilde{E}\, y^{+}\right) \quad (3)$$

where Θ^{+} represents the dimensionless temperature difference $\left(\Theta_w - \Theta\right)\rho U_{\tau} C_p / \dot{q}''_w$ and $\tilde{k}$ and $\tilde{E}$ are the thermal counterparts of k and E. By introducing the Eq. (2), Eq. (3) may be rewritten as

$$\Theta^{+} = \frac{k}{\tilde{k}} U^{+} + \frac{1}{k} \ln\left(\tilde{E}\, y^{+}\right) \quad (4)$$

The ratio $\left(\frac{k}{\tilde{k}}\right)$ represents the *turbulent Prandtl number,* σ_t and the result may be recast as

$$\Theta^{+} = \sigma_t \left[U^{+} + P\left(\frac{\sigma}{\sigma_t}\right)\right] \quad (5)$$

The quantity P, Jayatilleke pee-function can be determined [Jayatilleke, C. L. V] and from analysis a distribution of turbulent viscosity and turbulent Prandtl number over the viscous region [Patankar S. V and Spalding D. B; Spalding, D. B]. And simple form [Spalding, D. B] is

$$P \equiv 9.24 \left\{\left(\frac{\sigma}{\sigma_t}\right)^{3/4} - \left(\frac{\sigma}{\sigma_t}\right)^{1/4}\right\} \quad (6)$$

In Eq. (6) P represents a measure of the different *resistances* of the sublayer to heat and momentum transport when $\sigma < \sigma_t$, P is negative. The presumption that the viscous sublayer is of universal thickness renders and limited applicability even in simple shear flow, but more serious weaknesses appear in situations where the near-wall flow ceases to be shear dominated; at separation or stagnation points. The friction velocity U_{τ} as the normalizing velocity scale leads to absurd results such as a zero heat transfer coefficient at a stagnation point the weakness was removed [Spalding, D. B; Gosman, A. D Pun, W. M, Runchal A. K, Spalding D. B and Wolfshtein, M] by replacing U_{τ} in Eq. (2) by $c_{\mu}^{1/4} k_r^{1/2}$ where k_r represents the turbulent kinetic energy at some reference near-wall point in the fully turbulent region, and c_{μ} is a constant (as 0.09). Eq.(2) and Eq.(3) can be generalized to

$$U^{*} = \left(\frac{1}{k^{*}}\right) \ln\left(Ey^{*}\right) \qquad \Theta = \sigma_t\left(U^{*} + P^{*}\right) \quad (7)$$

where

$$U^{*} \equiv \left(\frac{\rho U k_r^{1/2}}{\tau_w}\right) \qquad \Theta \equiv \frac{\left(\Theta_w - \Theta\right)\rho C_{\rho} k_r^{1/2}}{\dot{q}''_w} \quad (8)$$

and $k^{*} \equiv c_{\mu}^{1/4} \kappa$; $E^{*} \equiv c_{\mu}^{1/4} E$; $P^{*} \equiv c_{\mu}^{-1/4} P$

Wall functions also need to be provided for any turbulence variables computed during the course of the computations, for the turbulence energy, k its dissipation rate, ε. For the simple shear

$$\varepsilon = -\overline{uv} \frac{\partial U}{\partial y} \quad (9)$$

On the other hand, Eq. (9) is used to fix the value of ε at the near-wall node in boundary –layer (marching) solvers where the flow next to the wall is, indeed, often close to local equilibrium. The turbulent kinetic energy is likewise demonstrated in terms of the wall shear surface as

$$k = \left(\frac{c_{\mu}^{-1/2} \tau_w}{\rho}\right) \quad (10)$$

The most complete statement of this approach Chieng and Launder [Cheing, C. C and Launder B. E]. A crucial element in the procedure lies in deciding the average generation and dissipation rates of k over the near-wall cell,

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the average generation rate of the turbulence energy, presuming the generation arises simply from shearing, is

$$\overline{P} = -\frac{1}{y_n}\int_0^{y_n} \overline{uv}\frac{dU}{dy}dy \qquad (11)$$

$$\overline{P} = \frac{\tau_w U_n}{\rho\, y_n} \qquad (12)$$

The problem with the above is that – within the truly viscous sublayer, the shear stress is transmitted by molecular interactions not by turbulence and there is no creation of turbulence linked with the intense velocity gradient there as

$$\overline{P} = \frac{\tau_w (U_n - U_v)}{\rho\, y_n} \qquad (13)$$

Which is based on the simple idealized notion that there is an abrupt changeover from molecular to turbulent transport at a distance y_v from the wall. In addition, a corresponding strategy is applied to obtain the mean energy dissipation rate, $\overline{\varepsilon}$. The first attempt to incorporate dissipation in the viscous sublayer into a wall-function treatment appeared in [Cheing, C. C and Launder B. E]. However, it was found that the level of *Nu* in separated flows underestimated by 20-30%. Reasonable accord with experiment was achieved, however, by allowing the sublayer to become thinner when there was substantial diffusion of turbulent kinetic energy toward the wall, which is broadly in line with [Johnson, R. W and Launder, B. E]. Amano [Amano R] demonstrated a more elaborate the *wall-function* treatment by decomposing the viscosity-affected zone into a laminar sublayer and a buffer region where turbulent transport was increasingly important as one proceeded away from the wall. Furthermore, another significant difference was his practice of determining the near- wall value of $\overline{\varepsilon}$ from its transport equation rather than by prescribing the length scale. It was investigated also similar-pipe expansion test flows to those of [Johnson and Launder] but concluded that that this two-layer viscous/b model gave satisfactory agreement with experiment, whereas [Chieng Launder] single-layer version produced too *high* value of *Nu* even though, in representing the velocity field, he adhered to a constant dimensionless sublayer thickness. The reason for his strikingly different behaviour from that reported in [Johnson and Launder] was probably linked with the necessarily crude, coarse-grid approximation of the source terms in the ε equation over the near wall cell. [Ciofalo and Collins] confirmed the conclusion of [Johnson and Launder] that the variation of the sublayer thickness was, indeed, a vital element of any wall treatment for impinging or separated flows. However, they related the sublayer thickness not to be diffusive inflow (or outflow) of turbulence energy but to the local turbulence intensity, $k^{1/2}/U$ at the near wall node, a practice that from a numerical point of view was certainly more stable.

III. TWO CURRENT WALL-FUNCTION APPROACHES

All the computations have been performed with suitably adapted versions of the TEAM (Turbulence Elliptic Axisymmetric Manchester) computer code [Huang, P. G and Leschzinger], which is a finite-volume based solver, employing a Cartesian grid with fully staggered storage arrangement and the SIMPLE pressure correction scheme of [Patankar, S. V]. Furthermore, the most of the calculation the QUICK scheme of [Leonard, B. P] has been used to convection of the mean variables, with the power law differencing scheme (PLDS) of [Patankar. S] applied for the turbulence quantities. In all cases, grid-refinement studies have shown that the results presented are free from numerical discretization errors.

The Unified methodology for integrated sublayer transport – ***analytical*** it provides a clear, albeit simple, physical model based on an analytical solution of the streamwise momentum and energy equations in the near-wall region. Henceforth the approach has been designed to be able to cope with

(a) forced, mixed, or natural convection flow on near vertical surfaces
(b) strong variations of molecular transport properties across the VSL
(c) laminarization, i.e. a marked thickening of the VSL in buoyancy-aided mixed convection and comprehensive description.

In this paper we have investigated the main elements that especially relates to the above capabilities. On the other hand, the starting point is a prescribed ramp distribution of turbulent viscosity in Fig. 3(a).

$$\frac{\mu_t}{\mu_v} = c_\mu c_l\,(y^* - y_v^*) \qquad for\;\; y^* \geq y_v^* \qquad (14)$$

The coefficients c_μ *and* c_l represents the conventional ones adopted in one equation turbulence models (0.09, 2.55) where now $y^* = \rho_v\, y k_P^{1/2}/\mu_v$ and the subscript denotes where the quantity is evaluated: *v*, at the edge of the viscous layer; *P* at the near wall *node*. The simple viscosity profile is essential to retain a form of the near-wall differential equations that can be analytically integrated to give velocity and temperature profiles. Furthermore, one important aspect of this integration is that source terms in the stream wise momentum equation representing pressure gradients or buoyancy can be retained. One important aspects of this integration is that source terms in the stream wise momentum equation representing pressure gradients or buoyancy can be retained. Now, the subsequent profiles are then used to obtain quantities such as *wall shear stress* and *cell-averaged* source terms, which are required for wall function treatment.

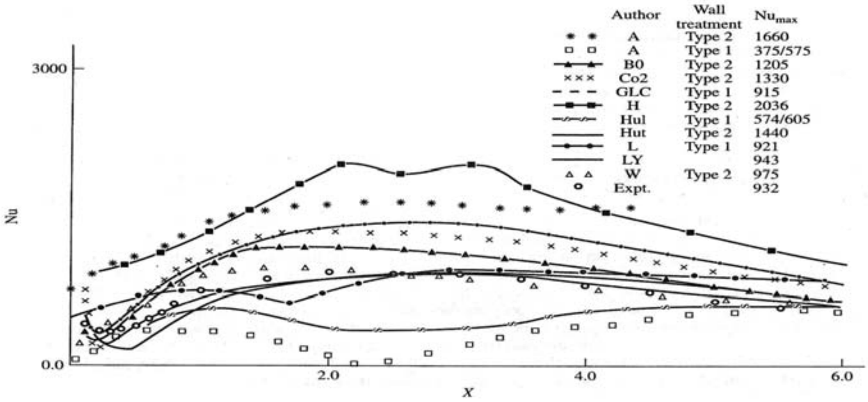

Fig. (1): Nusselt number distributins downstream of an abrupt pipe enlargement submitted for an IAHR Workshop,1987. Personal communicaton, A.G. Hutton and R. szczepura.

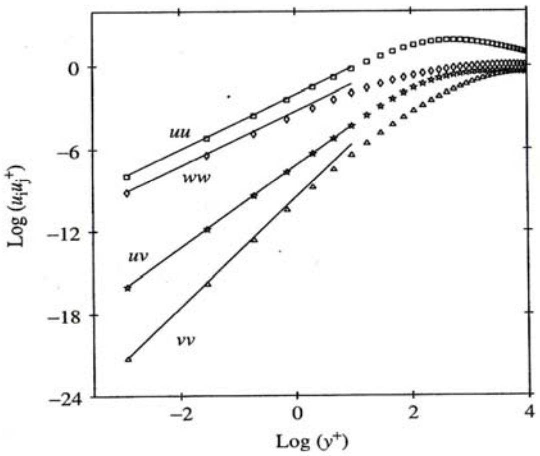

Fig. (2): Near-wall variation of the Reynolds stresses. Symbols: DNS data of Kim et al. [37]; solid lines are of slope 2 (for $\overline{u^2}$ and $\overline{w^2}$), 3 (for $\overline{uv}$), and 4 (for $\overline{v^2}$).

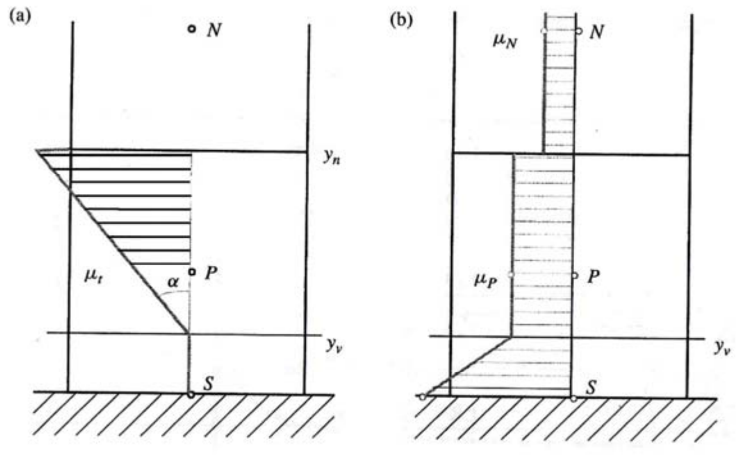

Fig. (3): Viscosity distributions assumed over near-wall cell:
(a) turbulent viscosity; (b) molecular viscosity.

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IV. CONCLUSION AND FURTHER RESEARCH

In this work two new wall-function approaches have been demonstrated. The first one is based on the analytical solution of simplified near-wall momentum and temperature equations accounting for pressure gradients and other force fields as buoyancy, while the second is based on a local one-dimensional solution of the governing equations. Furthermore, both approached have been applied to a range of flows in which standard log-law based wall functions are known to perform badly. In each case the present methods have been shown to mimic the result obtainable with full low Reynolds number solutions- but a fraction of the computational cost. As a final observation on both the wall-function approaches all the applications so far considered are relatively straightforward compared with their types of flows the industrial user *needs* to compute. However, we observe no evident impediment to their use in these more complex flows. Indeed, we have observed that the turbulent flow CFD community will contribute to this wider testing and where necessary the improvement of this prototype forms.

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