

Design of Switching Control Systems Using Control Performance Assessment Index

Arefeh Moridi, Shabnam Armaghan, Ali Khaki Sedigh and Saleheh Choobkar

Abstract—Switching control is employed in many adaptive control strategies to overcome difficulties encountered in the control design problems that cannot be routinely solved by conventional robust and adaptive control architectures. A key stage in switching control design is the switching logic. This paper proposes a new switching scheme based on the control performance index (CPI) concepts. The performance assessment index is primarily calculated using the Markov parameters of the closed loop transfer function to assess the closed loop performance of the regulatory and tracking control systems. It is shown that employing CPI can lead to proper switching between different controllers. Finally, simulation results are provided show the main points of the paper.

Index Terms— Performance Assessment, Switched System, Harris Index, Minimum Variance controller

I. INTRODUCTION

SWITCHING control systems are widely studied and used by control and systems engineers [1]. The main problems in linear switched systems are stability and poor transient responses, caused by switching between different controllers. Hence, improving the switched system's responses is of prime concern. A switched system consists of linear time invariant (LTI) subsystems and a regulated switching law. In general, a switched system is defined by the following equation:

$$\dot{x} = f_{\sigma}(x), x \in R^n \quad (1)$$

Where σ is a piecewise constant signal that is called the switching signal [2]. Various switching methods have already been introduced such as state-dependent versus time-dependent switching, autonomous (uncontrolled) versus controlled switching, chattering and slow switching, etc. [1]. In this paper, the principles of a new switching method based on the CPI are introduced. The CPI used is originated from the MV control.

The application of minimum variance (MV) as a performance index has been introduced in [3], [4]. Using the

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minimum variance controller (MVC), the least possible closed loop output variance is achieved, and it gives sufficient information concerning the performance of the closed loop system [5].

A popular MV-based performance index has been suggested by Harris in [6], and is referred to as the Harris index. The proposed method will use the Harris index for its switching logic.

The paper is organized as follow: In section 2, design of MVC and Incremental MV is considered. The performance assessment index is introduced in section 3. Sections 4 and 5 illustrate the main result of the paper by combining switching control and performance assessment index in a new switching logic strategy. Section 6 concludes the paper.

II. DESIGN OF MINIMUM VARIANCE CONTROLLER

Consider the plant described by the following equation:

$$y(t) = \frac{B}{A}u(t) + \frac{C}{A}e(t) \quad (2)$$

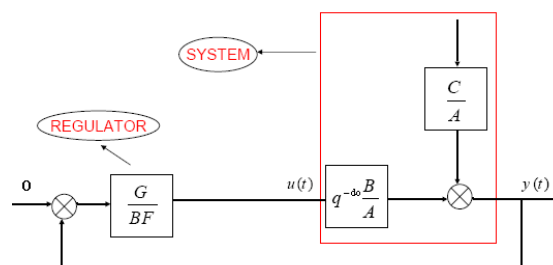


Fig. 1. Process model with MVC

The MVC can be described by the following set of equations:

$$d_0 = \text{deg}A - \text{deg}B$$

$$\frac{q^{d_0-1}C(q)}{A(q)} = F(q) + \frac{G(q)}{A(q)} \quad (3)$$

The Diophantine equation is as follow [5]:

$$q^{d_0-1}C(q) = A(q)F(q) + G(q) \quad (4)$$

Which gives

$$y(t + d_0) = \frac{B}{A}u(t + d_0) + Fe(t + 1) + \frac{qG}{A}e(t) \quad (5)$$

$e(t)$ is a random zero-mean sequence with finite variance σ^2 that is;

$$E[e(t)] = 0 \text{ and } E[e(t)^2] = \sigma^2$$

we have

$$e(t) = \frac{A}{C}y(t) - \frac{B}{C}u(t) \quad (6)$$

Then,

$$\begin{aligned} y(t + d_0) &= Fe(t + 1) + \frac{qG}{A} \left[\frac{A}{C}y(t) - \frac{B}{C}u(t) \right] \\ &\quad + \frac{B}{A}u(t + d_0) \\ &= Fe(t + 1) + \frac{qBF}{C}u(t) + \frac{qG}{C}y(t) \end{aligned} \quad (7)$$

The first term in the right hand side of Equation (7) affects the system from $t+1 \dots t+d_0$ the other term is a mean square prediction of $y(t + d_0)$ up to t . The prediction error is calculated as:

$$\begin{aligned} \tilde{y}(t + d_0|t) &= y(t + d_0) - \hat{y}(t + d_0|t) \\ &= F(q)e(t + 1) \end{aligned} \quad (8)$$

$$\rightarrow \text{var } \tilde{y}(t + d_0|t) = \sigma^2(1 + f_1^2 + \dots + f_{d_0-1}^2)$$

So we have:

$$u(t) = -\frac{G}{BF}y(t) \quad (9)$$

Here the closed loop plant takes the form:

$$Ay = -\frac{G}{BF}By(t) + Ce \quad (10)$$

and the closed loop plant is shown in fig 1.

A. Incremental MV Controller

Incremental MV controller is applied to the tracking system to minimize the output variance. Fig. 2 shows the incremental MV control of the plant introduced in section II.

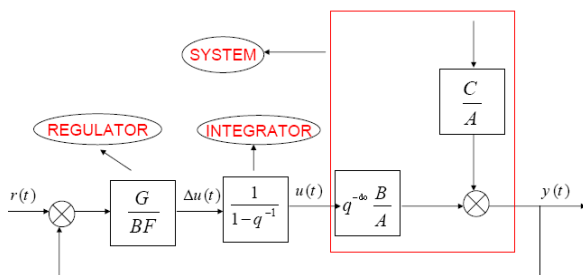


Fig. 2. Process model with incremental MVC

As we can see, the only difference between MVC and Incremental MVC is in an integrator block. The supplementary term changes the equations as follow:

$$(1 - q^{-1})u(t) = \frac{G}{BF}(r(t) - y(t)) \quad (11)$$

$$u(t) = (1 - q^{-1})u(t) + u(t - 1) \quad (12)$$

The Diophantine equation changes as:

$$C = FA(1 - q^{-1}) + q^{-d_0}G \quad (13)$$

The closed loop plant can thus be written as:

$$y(t) = \frac{G}{C}r(t - k) + (1 - q^{-1})Fe(t) \quad (14)$$

III. CONTROL PERFORMANCE ASSESSMENT

In the past decades, control performance monitoring has become an active field of research with many potential applications in the industry. CPA techniques are used to indicate whether the controller performance meets the closed loop requirements. A successful and widely used CPA is the Harris index. It is used to detect any changes in the closed loop performance [7] and is defined as follows [6]:

$$\eta_{Harris} = \frac{\sigma_{mv}^2}{\sigma_y^2} = \frac{\sum_{i=0}^{d_0-1} f_i^2}{\sum_{i=0}^{\infty} f_i^2} \quad (15)$$

The coefficient η_{Harris} is derived from the impulse response of noise to output transfer function. The mentioned coefficients are the Markov parameters. In (15) the numerator is the square summation of Markov parameters till d_0-1 , where d_0 shows the system delay and the denominator is the square summation of all Markov parameters which indicates the output variance. This benchmark (η_{Harris}) varies within [0, 1]. It is obvious that the closer the value is to 1, the better the performance will be [6],[8].

If the set point is a square wave input, then the calculation of η also requires the impulse response of the reference to output transfer function [9].

IV. SWITCHING CONTROL STRATEGY BASED ON THE PERFORMANCE ASSESSMENT TECHNIQUES

To develop an effective switching control strategy, the closed loop plant performance is monitored by on line calculation of the Harris index and based on this observation the controller with the best performance index is chosen for control. In fig. 3 a block diagram of the switching control strategy is shown briefly.

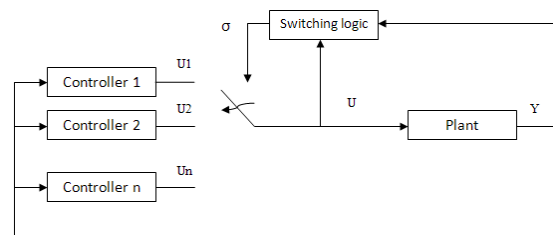


Fig. 3. A block diagram of the switching system

This will provide a practical answer to the following two fundamental questions:

- Is the active controller, at any time, appropriate for controlling the system?
- Is it possible to switch to a better controller among the designed controllers at any time?

In the present proposed scheme, the Harris index is used to examine the control loop performance for switching. It is assumed that there are designed individual MVC for a set of plants. This set consists of possible plant models at different operating points and operating conditions.

During closed loop operation and while the plant is under the control of a specific controller, the Harris index is calculated from the closed loop transfer function, or it can be calculated from the closed loop input-output data. If the Harris index indicates a deteriorating performance, the controller is switched to the best available controller. It is shown that using this method, switching occurs at exactly the expected times, i.e. the times which the plants have changed.

V. SIMULATION RESULTS

In this section, the above strategy is illustrated via simulation results for a regulatory and tracking control system.

(a) *Switching Control Strategy Based on CPA in Regulatory Control*

Consider the following discrete system

$$G_p = \frac{K_p}{z - 0.7}, K_p = \pm 1 \quad (16)$$

The noise dynamic is described by the following transfer function:

$$G_L = \frac{z}{z - 0.4} \quad (17)$$

The minimum variance controllers for this process are

derived from section II as:

$$mv_1 = \frac{0.4z - 0.28}{z - 0.4} \quad (18)$$

$$mv_2 = \frac{0.4z - 0.28}{-z + 0.4} \quad (19)$$

Note that due to the sign change in the open loop plant, a single conventional controller cannot reach the minimum output variance and best performance of the system, hence implementing a new controller is inevitable for switching.

First, assume that there is no switching and the plants and controllers change arbitrary. Fig.4 shows the different intervals either for plants or controllers. This figure indicates that in some intervals the performance becomes deteriorated and the output variance is increased.

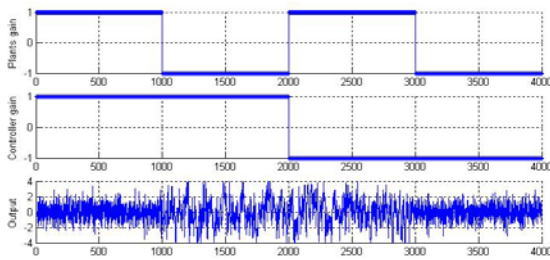


Fig. 4. Controllers gain, plants gain and output without switching

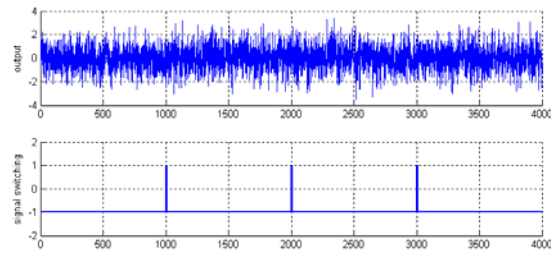


Fig. 5. Output and switching signal

Then, the switching law is employed to achieve better performance results. As it is shown in Fig. 4, the switching leads to minimum expected output variance. In the mean time, Fig. 5 shows the switching signal. As it can be seen, switching has taken place precisely where the variances have an unexpected value.

If we define the accumulated-loss function as $V(t) = \sum_{i=1}^t y^2(i)$ we can show the superiority of the switched system. Fig. 6 shows the loss function. The lower curve is the loss-function when switching is occurred and the upper curve is the loss-function without switching.

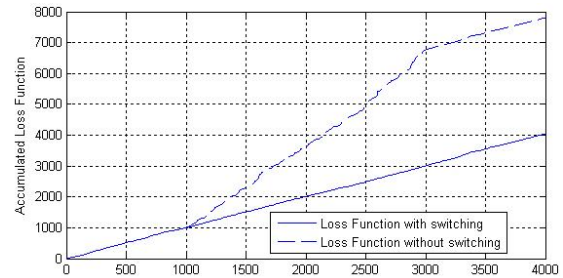


Fig. 6: Accumulated-loss function comparison

Then, two PID controllers are designed for the mentioned system to compare the result with a practical controller in industry. The parameters of the PID controllers were specified as:

$$\begin{aligned} K_{p1} &= -K_{p2} = 0.43 \\ K_{I1} &= -K_{I2} = 0.5 \\ K_{D1} &= -K_{D2} = 0.0019 \end{aligned}$$

It can be seen from fig. 7 that without switching, the closed loop system with a single PID controller becomes unstable. The figure shows the accumulated loss function of the system with a PID controller and a MVC.

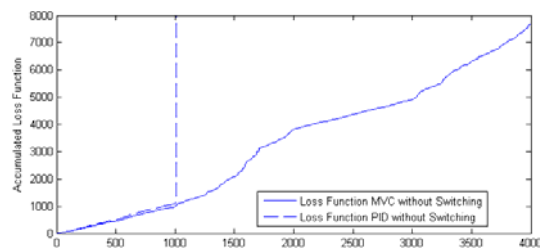


Fig. 7: Comparison between MV and PID accumulated-loss function without switching

From Fig. 8 we see that while the switching is taken place the loss function of MV is located under the loss function of PID.

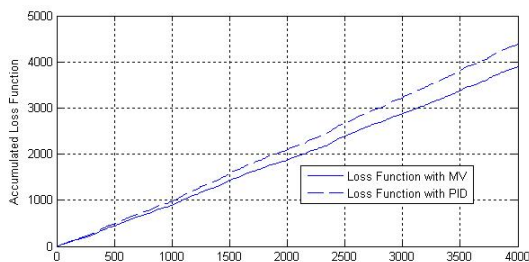


Fig. 8: comparison between MV and PID accumulated-loss function with switching

(b) *Switching control strategy based on CPA in Set Point Tracking*

Consider the previous system; the incremental MV controller is given by:

$$Inc_{MV1} = \frac{1.4z^2 - 1.38z + 0.28}{z - 0.4} \quad (20)$$

$$Inc_{MV2} = \frac{1.4z^2 - 1.38z + 0.28}{-z + 0.4} \quad (21)$$

Due to the plant change during the simulation, the system with a single incremental controller becomes unstable. So the significance of switching is prominent. By using the introduced strategy in section IV the closed system becomes stable. Fig. 9 depicts the output signal which exactly tracks the reference input with minimal variance. Besides, the switching signal is illustrated in this figure.

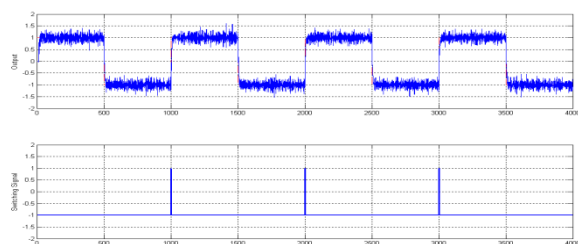


Fig. 9. Output and switching signal

We can find out that the switching is taken place exactly where the system is to become unstable.

VI. CONCLUSION

In this paper, a new switching control scheme based on the performance assessment monitoring for both regulatory and set point tracking control is presented. The Harris index as a switching logic and minimum variance and Incremental MV controllers is used to minimize the output variance. The simulation results with conventional PID and MV controllers show that by using this method, switching is taken place exactly at the expected times. Also, the minimum variance performance is achieved by switching between controllers based on the proposed switching scheme.

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