

Determining the Inventory Policy for Slow-Moving Items: A Case Study

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Abstract—Inventory control of slow-moving items is essential for many establishments since these items have a low lead time demand but a high price. Besides, as the demand pattern for slow-moving items is irregular, the estimation of the lead time demand is challenging. This study gives a comparison of the different methods of modelling the lead time demand, motivated by a case study at a retailing establishment. After modelling the lead time demand with different methods for the selected slow-moving items, optimum reorder points are obtained.

Index Terms— bootstrapping, continuous review policy, reorder point, slow-moving inventory

I. INTRODUCTION

Variation in demand increases the challenge of maintaining inventory to avoid stockouts or to satisfy the customer fill rate. Since it is hard to obtain an accurate estimate of the lead time demand, the inventory control problem is getting complicated by the fact that demand is uncertain or the variation of demand is highly volatile. A random demand with a large proportion of zero values is described as an intermittent demand [1]. Such items are also referred to as slow-moving items. A demand that is intermittent is often also 'lumpy', meaning that there is great variability among the nonzero values [2].

Inventory control of slow-moving items is essential to many establishments, since excess inventory leads to high holding costs and stockouts can have a great impact on the performance of operations. As the demands for slow-moving items are extremely stochastic and as the demand might sometimes be zero or as a lumpy demand, it is difficult to develop efficient strategies for the inventory management of items with such a demand owing to their nature. This complicates the estimation of the lead time demand distribution that is essential to obtain the control parameters of most inventory policies [3-4].

This paper deals with a case study on both forecasting lead time demand and developing an inventory policy for Class A inventories for a company that produces handmade items. In the following section the related literature is briefly reviewed. Section 3 gives the description and assumptions of the problem. Computational results of five

different techniques for modelling the lead time demand and

II. REVIEW OF RELATED LITERATURE

An early paper about the inventory control policy of low demand items with Poisson demand belongs to [5]. Ever since, the theoretical studies in the literature on the inventory control of slow-moving items have been abundant, whereas the case studies have been few. In addition, the selected product in the case studies performed is generally spare parts.

The base stock policy in the continuous review inventory models when the demand distribution is Poisson was examined by [6-8]. A forecasting method superior to the exponential smoothing was developed by [9], assuming the demand is Bernoulli process and demand size is assumed to have a Normal distribution. According to the Croston's method, separate exponential smoothing estimates of the average size of the demand and the average demand interval are made after demand occurs. If no demand occurs, the estimates do not change. Certain limitations of the Croston's method are identified in [10]. The authors quantify the bias associated with the Croston's method and they present a modification to the Croston's method that gives approximately unbiased demand estimates. [11] gives a discussion about the comparison of forecasting methods and accuracy of resulting estimates. A Markovian bootstrap approach was used by [2] to forecast lead time demand. The bootstrap method allows of creating the demand pattern and then estimating the demand size if it occurs. Different inventory policies are discussed for slow-moving items [12-15].

Many of the studies in the existing literature generally concentrate on the theoretical aspects of the demand forecasting problem or inventory management problem or else both problems together. However, the studies working with empirical data are not encountered much although there are some examples, such as [3] and [16]. An empirical comparison of different reorder point methods is studied in [3]. The authors construct the lead time demand with respect to different approaches and give an optimization by the decomposition approach. [16] propose a new method of determining the order-up-to levels for intermittent demand items in a periodic review system. They model the lead time demand as a compound binomial process and show that the proposed method is better than the existing ones using empirical data. Unlike the existing literature, our work analyzes an empirical data set to forecast the lead time demand and to optimize the customer service level. Spare parts are considered as products in almost all studies in the

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literature on the inventory control of slow-moving items. In a fashion that will not be encountered much in the literature, our study uses real demand data concerning a product other than spare parts and covers the development of an inventory control policy to optimize the customer service level for slow-moving items.

III. PROBLEM DESCRIPTION

The data used in this study are obtained from a firm that has been active in the area of production and sale of touristic carpets in Turkey since 2000. Monthly demand data are obtained from the company for the period from 01.01.2004 to 30.06.2009. About 95% of the customers are tourists who generally come as a group whose size is usually unknown in advance. Therefore, on some days, sales are high according to the size of the group, whereas there is no sale on some days. This nature of the data makes it highly volatile and complicates the development of an inventory policy. The slow-moving item in our study is carpet, whereas almost all studies in the existing literature evaluate spare parts as slow-moving items.

Around forty (40) different types of carpet are being sold, but since there are different dimensions for each carpet, the number of the products approaches a hundred. Within the scope of the study, an ABC analysis is performed and the inventory policy is proposed only for Class A products. 65% of the total sales and 20% of the total items are classified as Class A items. Seven (7) types of carpet are included in this category. Detailed information can be found in [17] for the ABC classification.

The turnover ratio is a ratio that shows the speed of sale of the products in the stock throughout the year. The low rate in the turnover rate indicates that the product has a low sale. The stock of popular, fast-moving items should turn more often (up to 12 times per year), whereas slow-moving items may turn only once or not at all. The turnover ratio is evaluated for Class A items. The turnover ratios for two of them are found less than 1. The turnover ratios for the two products, which are coded as AZR02 and ML03, are calculated as 0.71 and 0.83, respectively. Therefore, it is concluded that since the carpets AZR02 and ML03 have a low turnover rate, they can be classified as slow-moving items. Each type of carpet has different dimensions.

One particular type of slow-moving demand forecasting, which is especially difficult to predict, involves that of the "intermittent demand" that is characterized by frequent zero values intermixed with nonzero values. Traditionally, the characteristics of intermittent demand are derived from two parameters: the average inter-demand interval (ADI) and the coefficient of variation (CV). ADI measures the average number of time periods between two successive demands and CV represents the standard deviation of requirements divided by the average requirement over a number of time periods. The data demand patterns are explicitly considered in relation to the pattern and the size of the demand when it occurs. These are classified into four categories [10] on the basis of modified criteria [18]. More detailed information about the definitions of the categories can be found in [19].

From the demand data, the CV² and ADI values are

calculated for each dimension of the selected carpets AZR2 and ML3 summarized in Table I. The dimensions, which are classified as intermittent, are used for the analysis, and inventory policies for these items are generated.

TABLE I
DEMAND STRUCTURE

Code	CV ²	ADI	DEMAND STRUCTURE
AZR2D06	0.846	1.19	ERRATIC
AZR2D07	0.522	1.94	LUMPY
AZR2D08	0.512	1.47	LUMPY
AZR2D09	0.213	3.82	INTERMITTENT
AZR2D10	0.367	1.94	INTERMITTENT
AZR2D11	0.507	1.94	LUMPY
ML3D05	0.377	1.00	SMOOTH
ML3D06	0.522	1.00	ERRATIC
ML3D07	0.488	0	SMOOTH
ML3D08	0.311	2.06	INTERMITTENT
ML3D09	0.484	1.32	SMOOTH
ML3D10	0.316	2.83	INTERMITTENT
ML3D11	0	10.17	INTERMITTENT
ML3D12	0.133	8.13	INTERMITTENT
ML3D13	0.109	10.00	INTERMITTENT

Usually the types of costs associated with inventory are holding cost, ordering cost and stockout cost. An annual fixed rate of 20% is used. This means unit holding cost of a carpet for a year is calculated as 20% of the purchasing cost of a carpet. By consulting the firm about the calculation of order cost, it is decided that 5% of the total amount approximately yields the unit order cost. As usual, unit order cost is independent of the order amount. Since it is aimed to establish the optimal balance between the service level and the inventory holding costs, the stockout cost is excluded from our study.

Continuous review (r,q) policy is implemented in our study, where r represents the reorder point and q represents the order amount. The order size q is obtained by the basic economic order quantity (EOQ) model, using the average annual demand. EOQ calculation is rounded, according to [20], as follows:

Evaluate $m=EOQ$

$$q = \begin{cases} 1 & \text{if } m = 0 \\ m & \text{if } m \neq 0 \text{ and } \frac{q}{m} \leq \frac{m+1}{q} \\ m+1 & \text{if otherwise} \end{cases} \quad (1)$$

Once the cumulative distribution of the lead time demand distribution is obtained, a reorder point that satisfies the desired customer service level or fill rate is determined. Using the cumulative distribution function of the lead time demand, for a possible reorder point r, the fill rate is calculated using (2).

$$fill\ rate = 1 - \frac{E(s)}{q} \quad (2)$$

where q is EOQ and $E(s)$ is the expected shortage. EOQ is calculated using the formula $\sqrt{\frac{2KE(D)}{h}}$ where K is the annual order cost, h is the annual holding cost for one unit and $E(D)$ is the expected annual demand. Expected shortage can be calculated using (3).

$$E(s) = \sum_{x>s} (x-s)f(x) \quad (3)$$

IV. COMPUTATION OF REORDER POINT

In classical inventory theory, it is common to assume that the lead time demand follows normal distribution. However, when the demand is intermittent, the classical approach gives unsatisfactory results. Different approaches that estimate the lead time demand when it is intermittent are applied to the demand data. After modelling the lead time demand, the inventory policy is determined. The proposed inventory policies are compared in terms of the customer service level and the total costs. Determining an inventory policy involves determining inventory control parameters, such as reorder points and safety stocks. In order to do so, one needs to determine the lead time demand distribution. Four different approaches are used to model the lead time demand in this study. From the frequency distribution of lead time demand, the list of possible reorder point values r are obtained by setting $r=x$, where x are the lead time demand values and $f(x)$ are their corresponding probabilities. The probability distribution $f(x)$ is obtained from the empirical distribution in part A, from the Poisson distribution in part B, from the forecast values' empirical distribution in part C, and from the bootstrap sample's empirical distribution in part D.

In each approach, the reorder point that satisfies the given fill rate β is determined. $ES(r)$ is defined as the expected shortage for a given reorder point r which is evaluated as in (4).

$$ES(r) = \sum_{x>r} (x-r)f(x) \quad (4)$$

Then $\left(1 - \frac{ES(r)}{q}\right)$ is the calculated fill rate for a given r .

Then, the problem is to choose the smallest r satisfying (5) where q is the economic order quantity.

$$\beta \leq \left(1 - \frac{ES(r)}{q}\right) \quad (5)$$

A. Empirical Demand Distribution

An empirical model is implemented to estimate the distribution of lead time demand. Demands during the lead time are taken directly from the data set. After the determination of the lead time distribution, the fill rates calculated according to this distribution and the

corresponding reorder points are presented in Table II for each product that has been classified as intermittent.

B. Poisson demand

The mean and standard deviation of demand data for the selected products are calculated. For each product, the statistical hypothesis, where null hypothesis implies that data fits Poisson distribution (H_0 : Data follows Poisson distribution), is tested for $\alpha=0.01$. The p-values for the test results and the decisions are summarized in Table III. As it has been decided that the demand fits Poisson, the frequency distribution is constructed according to the Poisson probability distribution function using the parameters given in Table III. For each value of the lead time demand, fill rates are computed for each product. The calculated fill rates and the reorder points that satisfy these fill rates are represented in Table IV.

TABLE II
FILL RATES CALCULATED FOR EMPIRICAL DISTRIBUTION

Product Code	r	ES(r)	1-ES(r)/q
AZR2D09	2	0.136	0.864
AZR2D10	3	1.091	0.844
ML3D08	3	0.281	0.859
ML3D10	3	0.217	0.892
ML3D11	1	0.032	0.968
ML3D12	1	0.061	0.939
ML3D13	1	0.062	0.938

TABLE III
RESULTS OF THE HYPOTHESIS TESTING

Product Code	Mean	Standard Deviation	p-value	Decision ($\alpha=0.01$)
AZR2D09	2.38	2.05	0.471	Fail to reject H_0
AZR2D10	2.39	2.36	0.022	Fail to reject H_0
ML3D08	2.45	2.211	0.430	Fail to reject H_0
ML3D10	1.90	1.802	0.315	Fail to reject H_0
ML3D11	0.37	0.550	0.999	Fail to reject H_0
ML3D12	0.20	0.533	0.999	Fail to reject H_0
ML3D13	0.25	0.560	0.999	Fail to reject H_0

TABLE IV
FILL RATES CALCULATED FOR POISSON DISTRIBUTION

Product Code	r	ES(r)	1-ES(r)/q
AZR2D09	5	0.200	0.857
AZR2D10	4	0.393	0.869
ML3D08	4	0.261	0.870
ML3D10	3	0.313	0.844
ML3D11	1	0.133	0.867
ML3D12	2	0.061	0.939
ML3D13	2	0.061	0.939

C. Croston's Method

The Croston's method estimates the mean demand per period by applying exponential smoothing separately to the intervals between nonzero demands and their sizes [21]. The notation used is defined as follows:

$X(t)$: the observed demand in period t , $t = 1, \dots, T$.

$I(t)$: the smoothed estimate of the mean interval between nonzero demands

$S(t)$: the smoothed estimate of the mean size of a nonzero demand

z : the time interval since the last nonzero demand

α : smoothing constant between 0 and 1.

The Croston's method works as follows:

If $X(t) = 0$ then

$$S(t) = S(t - 1)$$

$$I(t) = I(t - 1)$$

$$z = z + 1$$

or else

$$S(t) = \alpha X(t) + (1 - \alpha)S(t - 1)$$

$$I(t) = \alpha z + (1 - \alpha)I(t - 1)$$

$$z = 1$$

Considering the demand size and intervals together, the estimate of the mean demand per period can be calculated as in (6)

$$M(t) = \frac{S(t)}{I(t)} \quad (6)$$

In the literature it is recommended to use low values for the smoothing constant [11]. For each product analyzed, the smoothing constant α is assumed as 0.1 in calculations since this value minimizes MAPE (mean absolute percentage error). When demand occurs at every review interval, the Croston's method would be identical to conventional exponential smoothing. The fill rates calculated for the Croston's method are summarized in Table IV.

TABLE IV
FILL RATES CALCULATED FOR CROSTON'S METHOD

Product Code	r	ES(r)	1-ES(r)/q
AZR2D09	5	0.136	0.903
AZR2D10	2	0.075	0.975
ML3D08	4	0.381	0.810
ML3D10	1	0.682	0.659
ML3D11	2	0.500	0.500
ML3D12	1	0.273	0.727
ML3D13	2	0.182	0.818

D. Markov Bootstrap Method

The results of the implementation of bootstrap method introduced in [2] will be presented in this section. Since other methods concentrate on estimating the mean demand, they provide an inaccurate estimate of the lead time distribution. This method has the advantage of capturing the autocorrelations between demand realizations better, especially when dealing with intermittent demands with a high proportion of zero values. With this method, a two-state Markov chain is defined, namely the demand's being zero and nonzero. Considering the data set, the transition probabilities between states are computed. According to the transition probabilities computed, a sequence consisting of 0 and 1 is obtained. The zeros mean that there is no demand, while 1's mean that the demand is greater than zero. In the months when the demand is 1, previous data as regards the size of the demand are used and forecasting is performed with the bootstrap method. The 1's in the sequence are replaced by the obtained demand estimations and the forecasting procedure is completed. Finally, the lead time demand is computed and this procedure is repeated for a

number of times. The Willemain's Bootstrap Method can be briefly summarized with the following steps [2]:

Step 0 – Obtain historical demand data in the selected time buckets (e.g. days, weeks, months).

Step 1 – Estimate transition probabilities for the two – state (zero vs. nonzero) Markov model.

Step 2 – Under the condition of the last observed demand, use the Markov model to generate a sequence of zero / nonzero values over the forecast horizon.

Step 3 – Replace every nonzero state marker with a numerical value sampled at random with replacement from the set of observed nonzero demands.

Step 4 – Jitter the nonzero demand values.

Step 5 – Sum the forecast values over the horizon to get one predicted value of LTD.

Step 6 – Repeat steps 2 – 5 for many times.

Step 7 – Sort and use the resulting distribution of LTD values.

X^* is accepted as a random demand value and Z is accepted as a random deviated value. By using these notations, the process operates as follows:

$$JITTERED = 1 + INT(X^* + Z \sqrt{X^*})$$

$$IF JITTERED \leq 0, THEN JITTERED = X^*$$

The calculated fill rates and the reorder points that satisfy the predetermined fill rate are presented in Table V for each product, respectively.

TABLE V
FILL RATES CALCULATED FOR BOOTSTRAP METHOD

Product Code	r	ES(r)	1-ES(r)/q
AZR2D09	8	0.329	0.835
AZR2D10	5	0.529	0.824
ML3D08	7	0.239	0.880
ML3D10	6	0.488	0.874
ML3D11	3	0.084	0.916
ML3D12	1	0.079	0.921
ML3D13	1	0.082	0.918

V. INVENTORY POLICY

The continuous review (r,q) inventory policy is implemented for the system. The reorder point r is evaluated using the LTD distribution according to the modelling methods described above. The order size q is calculated according to the economic order quantity using the average annual demand. For the evaluated products, the proposed policies are compared with the current policy in terms of the customer service level and total costs. The aim is to implement the policy that optimizes the system.

VI. CONCLUSION

Using the calculated reorder points, the inventory costs for all demand forecasting methods are presented in Table VI and compared with the currently applied state. The inventory cost (IC) is obtained as the sum of the holding cost (HC) and the ordering cost (OC). The difference between IC, which is obtained with every method, and current IC has been given proportionally. This difference has been found according to the principle given with (7).

TABLE VI
COMPARISON

Product Code	Current System		Empirical model			Poisson model			Croston's Model			Bootstrap Model		
	Inv. costs	Service level	Inv. costs	Service level	Diff (%)	Inv. costs	Service level	Diff (%)	Inv. costs	Service level	Diff (%)	Inv. costs	Service level	Diff (%)
AZR2D09	173,82	0,70	80,42	0,864	53,73	92,42	0,90	46,83	43,82	0,975	74,79	125,1	0,838	28,02
AZR2D10	60,57	0,70	20,22	0,844	66,65	40,36	0,886	33,36	31,43	0,975	48,10	42,83	0,838	29,28
ML3D08	79,38	0,70	46,23	0,859	41,76	47,76	0,833	45,19	43,51	0,810	45,19	56,77	0,805	28,48
ML3D10	147,96	0,70	88,06	0,892	40,48	55,36	0,804	62,58	-	0,650	-	91,96	0,814	37,85
ML3D11	143,19	0,70	36,45	0,968	74,54	36,99	0,844	74,17	-	0,650	-	3,24	0,884	97,74
ML3D12	112,55	0,70	36,74	0,939	67,35	36,55	0,984	67,52	-	0,727	-	37,69	0,921	66,51
ML3D13	101,67	0,70	29,37	0,938	71,11	30,45	0,984	70,04	25,59	0,818	74,83	30,27	0,918	70,22

$$diff (\%) = \frac{current\ IC - obtained\ IC}{current\ IC} \times 100\% \quad (7)$$

IC has not been computed for those states in which the service level is not met. This state has been encountered during modelling with the Croston's method. IC has not been computed because the estimation, obtained with the Croston's method for the products with Code No. ML3D10, ML3D11 and ML3D12, has not met the service fill rate. Although the lead time of the product with Code No. ML3D11 is 5 months, its mean lead time demand has been found as 1.66. The Croston's method has not yielded any good results in cases with a lead time demand lower than 2. It might be stated that this method will yield a better result in those intermittent data sets where the demand data contain different values from 0 and 1 and the months with no observation of any demands are relatively fewer.

When a comparison is made according to the current system, the empirical forecasting method has yielded a better result only for the product AZR2D10. When the empirical model is considered in general, it has displayed a better performance than the Markov bootstrap method. Even though the Markov bootstrap method has seemed to have yielded a better result in only one product (ML3D11) than the others, the differences are quite close to each other with the current system in the product ML3D12. When the difference of cost is considered, the estimation obtained with the Croston's method for the product ML3D13 is the best; however, when evaluated together with the fill rate, the empirical and bootstrap approaches yield approximately the same results.

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