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Multiple Target Tracking with C^2 Symmetric Measurements

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Abstract— **In this paper a simple tracking problem for multiple targets moving in a straight line with constant velocities has been formulated. It is assumed that, although sensor data association problem is present, the measurements are symmetrical function of targets' position. Two types of C^2 symmetrical measurements namely sum of product and sum of power form have been considered. The resultant measurement noise covariance arises due to symmetrical association has been calculated analytically and verified by Monte Carlo run. The problem has been solved using Extended Kalman Filter. As targets' identity may be lost or interchanged during tracking, tracks have been labeled using permutation over the states for lowest sum of square of error with the truth value. After the track labeling, root mean square error has been compared with linear measurement. It is observed that RMSE in sum of power is higher than sum of product form of measurement association.**

Index Terms—**Kalman Filter, Nonlinear filtering. Multiple target tracking**

I. INTRODUCTION

CONSIDERABLE amount of research [1-5] has been carried out on multiple target tracking during last three decades. The research interest on this particular topic is increasing rapidly as simultaneous tracking and managing identity of the targets find many real life applications such as air traffic control, military air and underwater surveillance system, people tracking to name a few.

In multiple target tracking scenario same sensor is used to measure all the targets' position, velocity etc. Difficulties in processing the sensor data arise as almost all the cases correct association between measurements and targets is unknown. Situation becomes worse when number of target is unknown and they originate and terminate at random time [6]. This type of generalized problem has been solved using data association hypothesis [6] and also using random finite set [7]. The method is computationally inefficient and may not be very useful when the number of targets to be tracked is fixed. Even when the number of targets is not varying still the problem remains challenging enough due to data association between measurements. In fact if we consider nonlinear data association there are infinite number of possible association between targets and measurements. For further simplification many times we assume data association is based on some subset of the set of all possible associations. In this paper data association is restricted as symmetrical function of class C^2, i.e. the function is twice continuously differentiable. Among all possible C^2 type of symmetry only two types namely sum of product and sum of power have been taken for study and comparison. Other kinds of symmetrical measurement association have been kept aside for future work.

The similar kind of problem with symmetrical measurement was formulated in earlier literatures [2- 5] and solved with different nonlinear filters such as extended Kalman filter [2, 5], unscented Kalman filter [3, 4] and particle filter [4] etc. But the following aspects of the problem had not been considered.

(i) The aspect of track label and track exchange has not been mentioned and considered in any of the earlier publications.

(ii) The results obtained from nonlinear filters are not compared using RMSE plot with the Kalman filter (KF), termed as *associated filter* by Kamen [2], which is used when data association is absent and system is linear.

(iii) The modified measurement noise covariance arises due to association of measurement was not calculated theoretically.

In this paper a simple problem of N number of particle moving in a straight line has been reformulated. The data association between the measurements has been assumed as stated earlier. It has been argued that the Gaussian measurement noise will remain Gaussian even after the sum of product and sum of power kind of symmetrical data association. The resultant covariance matrix arises after data association has been derived for two as well as three particles in motion. The derived expressions have been verified using Monte Carlo run. The position and velocity of the particles has been estimated using extended Kalman filter (EKF). The results are compared with KF [8, 9] using RMSE plot. The aspects of track labeling and exchange have been discussed.

This paper is organized as follows: Section 2 presents the formulation of target tracking problem of N number of particles. Next section deals with calculation of measurement noise covariance due to data association. Simulation results are discussed on section 4. Concluding remarks are in section 5.

II. PROBLEM FORMULATION

A simple problem of N number of particles moving in a straight line in single dimension has been formulated in this section. This formulation can simply be extended to three dimensional motions. The data association among the measurements of the individual particles is assumed as the symmetrical function of class C^2, i.e. the function is twice continuously differentiable. Similar type of problem has been formulated earlier in [2, 5]. For N number of targets moving in space, state vector can be assumed as $X_k = \begin{bmatrix} x_{1k} & x_{2k} & . & . & . & x_{Nk} & v_{1k} & v_{2k} & . & . & . & v_{Nk} \end{bmatrix}^T$

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Where x_{ik} and v_{ik} represent the respective x coordinate position and velocity of ith target at time kT and i=1, 2… N. The evolution of position and velocity with time in state space form can be written as

$$X_{k+1} = FX_k + Bw_k \tag{1}$$

Where, $F = \begin{bmatrix} I_N & TI_N \\ 0_N & I_N \end{bmatrix}$, $B = \begin{bmatrix} (T^2/2)I_N & 0_N \\ 0_N & I_N \end{bmatrix}$ and process noise w_k is zero-mean Gaussian white noise with Q_k covariance $(w_k \sim N(0, Q_k))$. Assuming the particles are moving in constant velocity and there is no acceleration, process noise can be considered as zero similar to [2].

Linear Measurement: Now suppose the sensor that provides noisy measurement of the position of particles is located at the origin of the co ordinate system. If we assume the ideal case where sensor output is only the individual position of the targets, the measurement equations become linear and can be written as

$$Y_k = HX_k + u_k \tag{2}$$

Here $Y_k = \begin{bmatrix} y_{1k} & y_{2k} & . & . & . & y_{Nk} \end{bmatrix}^T$ where y_{ik} is the ith sensor measurement data at time instance kT, H is measurement matrix and $u_k = \begin{bmatrix} u_{1k} & u_{2k} & . & . & . & u_{Nk} \end{bmatrix}^T$ is the measurement noise. As stated earlier considering only the targets' position as measurements; measurement matrix becomes $H = \begin{bmatrix} I_N & 0_N \end{bmatrix}$. We also assume the measurement noise or sensor noise u_k is white Gaussian with zero mean and σ_k^2 covariance $(u_k \sim N(0, \sigma_k^2))$. As in this case both the process and measurement equations are linear the problem can be solved using KF and the solution would be the best possible estimate of the states.

Symmetrical Measurement: In non ideal situation or real life situation, association among the measurements is present. In that case users receive data which is association of the individual target's position. So knowledge about the association of data is necessary to solve the problem. In this paper we assume that the association among the measurements is symmetrical function of class C^2. Two types of symmetry such as sum of product and sum of power have been considered. Due to data association the measurement equations are no longer linear hence no optimal solution is available. Also due to association between measurements the characteristics of resultant noise may change and will be discussed later.

The association between the measurements could be sum of power form for which the measurement equations can be written as

$$Y_k = \begin{bmatrix} \sum_{i=1}^{N} y_{ik} & \sum_{i=1}^{N} y_{ik}^2 & \dots & \sum_{i=1}^{N} y_{ik}^N \end{bmatrix}^T \tag{3}$$

Another way of representing symmetry could be sum of product form in which the measurement for N particle can be written as

$$Y_k = \begin{bmatrix} \sum_{i=1}^{N} y_{ik} & \sum_{i=1}^{N-1}\sum_{j=i+1}^{N} y_{ik}y_{jk} & \sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{l=j+1}^{N} y_{ik}y_{jk}y_{lk} & \dots & \prod_{i=1}^{N} y_{ik} \end{bmatrix}^T$$

There are also other possible forms of this type of symmetry but here we shall not consider them. Total number of individual measurements associated in (mx1)th element of Y matrix can easily be calculated as $N!/m!(N-m)!$. The total number of terms in Y matrix would be $2^N - 1$. It has been proved that for both type of measurement equations the system is observable [5].

Two particles motion: The simplest estimation problem where data association plays a role is due to two particles in motion. In that case, measurement equations for sum of power form can be written as:

$$Y_k = g(x_{1k}, x_{2k}) + \eta_k \tag{4}$$

Where, $g(x_{1k}, x_{2k}) = \begin{bmatrix} x_{1k} + x_{2k} & x_{1k}^2 + x_{2k}^2 \end{bmatrix}^T$ and $\eta_k = \begin{bmatrix} u_{1k} + u_{2k} & u_{1k}^2 + u_{2k}^2 + 2x_{1k}u_{1k} + 2x_{2k}u_{2k} \end{bmatrix}^T$, is the new noise vector. Here it should be noted that if u_{ik}, x_{ik} are Gaussian, the resultant noise, η_k arises due to association among the measurements, also remains Gaussian due to central limit theorem. It has also been found that $E[\eta_k] = \begin{bmatrix} 0 & 2\sigma^2 \end{bmatrix}^T$ where $u_{ik} \sim N(0, \sigma^2)$. So to obtain zero mean noise sequence η_k could be modified as $\eta_k = \begin{bmatrix} u_{1k} + u_{2k} & u_{1k}^2 + u_{2k}^2 + 2x_{1k}u_{1k} + 2x_{2k}u_{2k} - 2\sigma^2 \end{bmatrix}^T$.

Similarly for two particles, in sum of product form, the measurement equation can be written in the form of (4) Where, $g(x_{1k}, x_{2k}) = \begin{bmatrix} x_{1k} + x_{2k} & x_{1k}x_{2k} \end{bmatrix}^T$ and $\eta_k = \begin{bmatrix} u_{1k} + u_{2k} & x_{1k}u_{2k} + x_{2k}u_{1k} + u_{1k}u_{2k} \end{bmatrix}^T$. It can easily be shown that the mean of noise sequence, η_k, is zero.

Three particles motion: For three particles motion in sum of power form $g(x_{1k}, x_{2k}, x_{3k})$ and η_k would be

$$g(x_{1k}, x_{2k}, x_{3k}) = \begin{bmatrix} x_{1k} + x_{2k} + x_{3k} & x_{1k}^2 + x_{2k}^2 + x_{3k}^2 & x_{1k}^3 + x_{2k}^3 + x_{3k}^3 \end{bmatrix}^T$$

$$\eta_k = \begin{bmatrix} (u_{1k} + u_{2k} + u_{3k}) \\ (u_{1k}^2 + u_{2k}^2 + u_{3k}^2 + 2x_{1k}u_{1k} + 2x_{2k}u_{2k} + 2x_{3k}u_{3k} - 3\sigma^2) \\ \begin{pmatrix} u_{1k}^3 + u_{2k}^3 + u_{3k}^3 + 3x_{1k}^2u_{1k} + 3x_{2k}^2u_{2k} + 3x_{3k}^2u_{3k} + \\ 3x_{1k}u_{1k}^2 + 3x_{2k}u_{2k}^2 + 3x_{3k}u_{3k}^2 - 3\sigma^2(\hat{x}_{1k|k-1} + \\ \hat{x}_{2k|k-1} + \hat{x}_{3k|k-1}) \end{pmatrix} \end{bmatrix}$$

For sum of product form $g(x_{1k}, x_{2k}, x_{3k})$ and η_k would be

$$g(x_{1k}, x_{2k}, x_{3k}) = \begin{bmatrix} x_{1k} + x_{2k} + x_{3k} \\ x_{1k}x_{2k} + x_{2k}x_{3k} + x_{3k}x_{1k} \\ x_{1k}x_{2k}x_{3k} \end{bmatrix}$$

$$\eta_k = \begin{bmatrix} (u_{1k} + u_{2k} + u_{3k}) \\ \begin{pmatrix} (x_{1k}u_{2k} + x_{2k}u_{1k} + u_{1k}u_{2k} + x_{2k}u_{3k} + x_{3k}u_{2k} + \\ u_{2k}u_{3k} + x_{1k}u_{3k} + x_{3k}u_{1k} + u_{1k}u_{3k}) \end{pmatrix} \\ \begin{pmatrix} (u_{1k}x_{2k}x_{3k} + u_{2k}x_{1k}x_{3k} + u_{3k}x_{1k}x_{2k} + x_{1k}u_{2k}u_{3k} + \\ x_{2k}u_{3k}u_{1k} + x_{3k}u_{1k}u_{2k} + u_{1k}u_{2k}u_{3k}) \end{pmatrix} \end{bmatrix}$$

In all the cases the noise sequence η_k would be zero mean Gaussian. The covariance of the η_k will be calculated in the next section.

III. CALCULATION OF COVARIANCE

How the noise sequences change due to symmetrical association of measurements have been discussed on the previous section. It has also been argued that the resultant noise would be zero mean Gaussian in nature. In this section covariance of the resultant noise R_k has been calculated analytically for two and three particles cases with both sum of power and sum of product symmetry. The calculation of

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covariance for symmetrical measurement has not been explicitly appeared in the literature earlier. Approximated expression of R_k for three particles in motion and only sum of power form symmetry case has been found in [3].

Two particles motion: Considering two particles moving in a straight line, the covariance R_k of resultant noise can be obtained by $R_k = E[\eta_k \eta_k{}^T]$. As the association of noise is known, the covariance R_k of η_k has been calculated assuming x_{ik} and u_{ik} as independent. For sum of power type of symmetry,

$$R_k = \begin{bmatrix} R_{11k} & R_{12k} \\ R_{21k} & R_{22k} \end{bmatrix}, \text{ where } R_{11k} = 2\sigma^2,$$

$$R_{12k} = R_{21k} = 2\sigma^2 (\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1}) \text{ and}$$

$$R_{22k} = 4\sigma^2 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + P_{11k} + P_{22k}) + 4\sigma^4$$

Where P is prior error covariance matrix. Similarly for sum of product type of symmetry R_k matrix will be

$$R_{11k} = 2\sigma^2, R_{12k} = R_{21k} = \sigma^2 (\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1})$$

$$R_{22k} = \sigma^2 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + P_{11k} + +P_{22k}) + \sigma^4$$

Three particles motion: Similar to above measurement noise covariance matrix R_k can be calculated for three particles scenario. In sum of power symmetry form the measurement noise covariance can be calculated as

$$R_{11k} = 3\sigma^2;\ R_{12k} = R_{21k} = 2\sigma^2 (\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1} + \hat{x}_{3k|k-1})$$

$$R_{13k} = R_{31k} = 9\sigma^4 + 3\sigma^2 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + \hat{x}^2_{3k|k-1} + P_{11k} + P_{22k} + P_{33k})$$

$$R_{22k} = 6\sigma^4 + 4\sigma^2 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + \hat{x}^2_{3k|k-1} + P_{11k} + P_{22k} + P_{33k})$$

$$\begin{aligned} R_{23k} = R_{32k} = {} & 12\sigma^4 (\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1} + \hat{x}_{3k|k-1}) + \\ & 6\sigma^2 (\hat{x}^3_{1k|k-1} + \hat{x}^3_{2k|k-1} + \hat{x}^3_{3k|k-1} + 3(\hat{x}_{1k|k-1} P_{11k} + \hat{x}_{2k|k-1} P_{22k} + \hat{x}_{3k|k-1} P_{33k})) \end{aligned}$$

$$\begin{aligned} R_{33k} = {} & 45\sigma^6 + 9\sigma^2 (\hat{x}^4_{1k|k-1} + \hat{x}^4_{2k|k-1} + \hat{x}^4_{3k|k-1} + \\ & 6(x^2_{1k|k-1} P_{11k} + x^2_{2k|k-1} P_{22k} + \hat{x}^2_{3k|k-1} P_{33k}) + 3(P^2_{11k} + P^2_{22k} + P^2_{33k})) \\ & + 36\sigma^4 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + \hat{x}^2_{3k|k-1}) + 45\sigma^4 (P_{11k} + P_{22k} + P_{33k}) \\ & + 18\sigma^4 (P_{12k} + P_{23k} + P_{13k}) \end{aligned}$$

For sum of product kind of symmetry (R_k) can be calculated

$$R_{11k} = 3\sigma^2\ \ R_{12k} = R_{21k} = 2\sigma^2 (\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1} + \hat{x}_{3k|k-1})$$

$$R_{13k} = \sigma^2 (\hat{x}_{1k|k-1}\hat{x}_{2k|k-1} + \hat{x}_{2k|k-1}\hat{x}_{3k|k-1} + \hat{x}_{1k|k-1}\hat{x}_{3k|k-1} + P_{12k} + P_{23k} + P_{13k})$$

$$\begin{aligned} R_{22k} = {} & 3\sigma^4 + 2\sigma^2 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + \hat{x}^2_{3k|k-1} + \hat{x}_{1k|k-1}\hat{x}_{2k|k-1} + \hat{x}_{2k|k-1}\hat{x}_{3k|k-1} \\ & + \hat{x}_{1k|k-1}\hat{x}_{3k|k-1} + P_{11k} + P_{22k} + P_{33k} + P_{12k} + P_{23k} + P_{13k}) \end{aligned}$$

$$\begin{aligned} R_{23k} = {} & \sigma^2 ((\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1})(\hat{x}^2_{3k|k-1} + P_{33} + 2P_{12}) + \\ & (\hat{x}_{2k|k-1} + \hat{x}_{3k|k-1})(\hat{x}^2_{1k|k-1} + P_{11} + 2P_{23}) + \\ & (\hat{x}_{3k|k-1} + \hat{x}_{1k|k-1})(\hat{x}^2_{2k|k-1} + P_{22} + 2P_{13})) + \sigma^4 (\hat{x}_{1k|k-1} + \hat{x}_{2k|k-1} + \hat{x}_{3k|k-1}) \end{aligned}$$

$$\begin{aligned} R_{33k} = {} & \sigma^2 [\{(\hat{x}^2_{1k|k-1} + P_{11k})(\hat{x}^2_{2k|k-1} + P_{22k}) + 4\hat{x}_{1k|k-1}\hat{x}_{2k|k-1} P_{12k} + 2P^2_{12k}\} \\ & \{(\hat{x}^2_{2k|k-1} + P_{22k})(\hat{x}^2_{3k|k-1} + P_{33k}) + 4\hat{x}_{2k|k-1}\hat{x}_{3k|k-1} P_{23k} + 2P^2_{23k}\} + \\ & + \{(\hat{x}^2_{1k|k-1} + P_{11k})(\hat{x}^2_{3k|k-1} + P_{33k}) + 4\hat{x}_{3k|k-1}\hat{x}_{1k|k-1} P_{13k} + 2P^2_{13k}\}] + \\ & \sigma^4 (\hat{x}^2_{1k|k-1} + \hat{x}^2_{2k|k-1} + \hat{x}^2_{3k|k-1} + P_{11k} + P_{22k} + P_{33k}) + \sigma^6 \end{aligned}$$

During estimation the measurement noise covariance is taken as calculated from the above expressions. Also it should be noted that without using the derived expressions, measurement noise covariance matrix could be generated using Monte Carlo run. All the above derived expressions are verified with that of obtained from Monte Carlo results in MATLAB environment.

IV. SIMULATION RESULTS

As the formulated problem is nonlinear in nature, it has been solved using extended Kalman filter (EKF) for both type of symmetry. The truth model as well as filter has been simulated in MATLAB environment. The initial state values for truth have been taken as $X_0 = [5 \ \ -10 \ \ 20 \ \ -1.5 \ \ 2 \ \ -3]^T$. Measurement noise before association has been assumed to be white Gaussian with zero mean and covariance σ_k^2 =diag (25 25 25). As the process noise covariance (Q) is zero, velocity of the particles remains constant in their respective initial values during the simulation which has been carried out for 20 seconds with the sampling time 0.01second. The estimated values of states have been initialized with $\hat{X}_0 = [0 \ \ 0 \ \ 0 \ \ 0 \ \ 0 \ \ 0]^T$ along with the error covariance $P_0 = diag[9 \ \ 25 \ \ 64 \ \ 2 \ \ 0.25 \ \ 16]$. The estimated and truth values of position of three targets for a single representative run have been plotted in figure 1 for sum of power symmetry. It should be noted from the figure that all the particles are moving from left to right and crosses each other within 20 seconds. From the repeated simulation interestingly it has been found out that in some cases the filter tracks the positions of the particles very well but without identifying the particle (as shown in figure 1). In this particular run track exchange may also occurs during the crossover. So we can say that although the estimator is following the truth, target tracks are not been labeled. Similar kinds of results are obtained for sum of product symmetry and are not shown here.

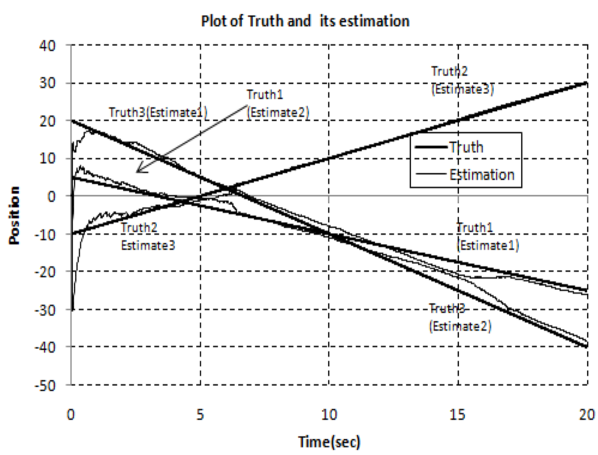

Fig 1. Truth and estimated values for a single representative run

If there is no data association, the measurement follows the equation 2. In that case the system is linear and the Kalman Filter can be applied to obtain optimal estimation. In figure 2 root mean square error (RMSE) obtained from EKF for 100 Monte Carlo runs has been compared with that of KF for first target. The RMSE for EKF has been calculated after permutation of state to obtain minimum sum of square error. It has been observed that the RMSE of KF is less than that of obtained from EKF. From the figure it can also be seen that the RMSE obtained from EKF in sum of product form is lower than that of sum of power form. It may be due to less value of measurement noise covariance

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in sum of product form. It can also be observed that there are kinks near 5th and 10th second which are due to the exchange of track labeling. Similar results obtained for second and third targets are not shown here. As RMSE of KF is much lower than EKF, more advanced nonlinear filter may help better estimation of targets' position.

V. DISCUSSIONS AND CONCLUSION

Using EKF a tracking problem for multiple targets with symmetrical data association has been solved. RMSE of the estimation has been calculated and compared with that obtained from associated filter. Different aspects of the problem such as track labeling and track exchange have been discussed. The target tracks have been labeled by minimizing sum of the squared errors over the permutation of the states of the estimator. The resultant covariance matrix has been derived and verified with that of obtained from Monte Carlo run. The RMSE of the estimator has been compared between Sum of power and Sum of product symmetrical measurement. It has been observed that RMSE of estimator for sum of product form is lower than that of obtained from sum of power form. It has been concluded that there is scope to improve the performance of the estimator by using more advanced nonlinear filtering techniques. Defining measurement association in terms of other types of symmetric functions of target positions and their analysis for larger number of targets following different kind of trajectories are under the scope of the future work.

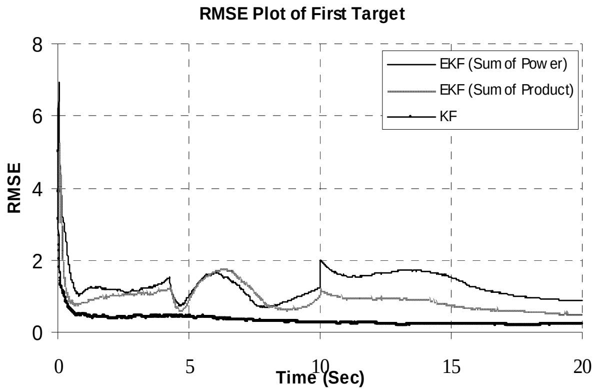

Fig 2. RMSE of KF and EKF sum of power and product form

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