Proceedings of the World Congress on Engineering 2011 Vol I
WCE 2011, July 6 - 8, 2011, London, U.K.

Random Ishikawa Iteration Scheme of Two Random Operators

Abdul Rahim Khan and AbdulAziz Domlo *

Abstract—**In this paper, we prove the existence of a random common fixed point of two continuous contractive random operators on a separable Banach space. We introduce the random modified Ishikawa iteration scheme and use our existence result to study weak and strong convergence of this scheme to a unique random common fixed point of two random asymptotically quasi-nonexpansive operator. Our work extends and improves some results from the current literature.**

Keywords: Random Ishikawa iteration; random operators; random common fixed point; Banach space; measurable function.

1 Introduction

Ishikawa [8], in 1974, devised a two-step iteration scheme to establish convergence of a Lipschitzian pseudocontractive map in the context of a Hilbert space where the Mann iteration process failed to converge. Tan and Xu [13] introduced and studied a modified Ishikawa iteration for an asymptotically quasi-nonexpansive map. Qihou [10], in 2001, proved the convergence of this scheme to a fixed point of an asymptotically quasi-nonexpansive map on a Banach space. Shahzad and Udomene [12] considered the modified Ishikawa iteration for two asymptotically quasi-nonexpansive maps and proved its convergence to a common fixed point of the maps in a Banach space. Recently, iterative approximation of common fixed points of two or more maps has appeared in the literature (see, e.g., [3-6], [9-10], [12-13]).

Random iteration schemes have been introduced and studied by a number of authors (see, e.g., [3-6]); in particular, Choudhury [5] constructed a random Mann iteration scheme in a separable Hilbert space and proved its convergence to a random fixed point under a contractive condition and Duan and Li [6] proved some convergence theorems about a random Mann iteration scheme of a continuous random operator on a separable Banach space.

*Department of Mathematics and Statistics, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia and Department of Mathematics,Taibah University, Madinah Munawrah 30002, Saudi Arabia Email: arahim@kfupm.edu.sa and azizdomlo@gmail.com

The existence of common fixed points of two commuting and noncommuting maps has been studied by several authors (see, for example, [1] and [7]). Unfortunately, the existence results of common fixed points are not known in many situations even in deterministic case; of course, this problem becomes more severe for random operators. Therefore, approximation of common fixed points of maps by iterative methods is very much needed.

The purpose of this paper is to extend the results of [1], [5-6] and [12-13] for random modified Ishikawa iteration scheme of two asymptotically quasi-nonexpansive random operators on a separable Banach space.

2 Preliminaries

Let C be a nonempty subset of a Banach space X and S, $T : C \to C$. We say T is: (i) $(L-\gamma)$ uniform Lipschitz if there exist constants $L > 0$ and $\gamma > 0$ such that $\|T^n x - T^n y\| \leq L\|x-y\|^\gamma$, for all $x, y \in C$ and $n = 1, 2, 3, \ldots$; (ii) semi-compact if for a sequence $\{x_n\}$ in C with $\lim\limits_{n\to\infty} \|x_n - Tx_n\| = 0$, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \to p \in C$; (iii) demiclosed at 0 if for each sequence $\{x_n\}$ in C converging weakly to x and $\{Tx_n\}$ converging strongly to 0, we have $Tx = 0$. The maps S and T: (i) satisfy the property (E.A) (cf. [1]) if there exists a sequence $\{x_n\}$ in C such that $\lim\limits_{n\to\infty} Sx_n = \lim\limits_{n\to\infty} Tx_n = t$, for some $t \in C$; (ii) are 1-subcommutative (cf. [7]) if C is q-starspahed and $\|TSx - STx\| \leq \dfrac{1}{k}\|((1-k)q + kTx) - Sx\|$ for all $x \in C$ and $k \in (0, 1)$, (iii) weakly compatible (cf. [1]) if $STu = TSu$ whenever $Su = Tu$.

Clearly, commuting maps are 1-subcommutative. We provide an example of two maps which are weakly compatible and satsify the propery $(E \cdot A)$ but are not 1-subcommutative: Let $X = \mathbb{R}$ with usual norm and $C = [1, \infty)$. Let $Sx = 2x - 1$ and $Tx = x^2$ for all $x \in C$. Then C is 1-starshaped. S and T are weakly compatible maps and satisfy the property $(E \cdot A)$ but S and T are not 1-subcommutative.

Let (Ω, Σ) be a measurable space ($\Sigma - \sigma$-algebra) and C a nonempty subset of a Banach space X. Let $\xi : \Omega \to C$ and $S, T : \Omega \times C \to C$. Then: (i) ξ is measurable if $\xi^{-1}(U) \in \Sigma$, for each open subset U of C; (ii) T is

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

a random operator if for each fixed $x \in C$, the map $T(.,x) : \Omega \to X$ is measurable; (iii) ξ is a random fixed point of the random operator T if ξ is measurable and $T(\omega, \xi(\omega)) = \xi(\omega)$, for each $\omega \in \Omega$; (iv) ξ is a random coincidence (resp., random common fixed) point of S and T if ξ is measurable and $S(\omega, \xi(\omega)) = T(\omega, \xi(\omega))$ (resp., $\xi(\omega) = S(\omega, \xi(\omega)) = T(\omega, \xi(\omega))$), for each $\omega \in \Omega$; (v) S and T are weakly compatible if $T(\omega, S(\omega, \xi(\omega))) = S(\omega, T(\omega, \xi(\omega)))$, for every $\omega \in \Omega$ whenever $T(\omega, \xi(\omega)) = S(\omega, \xi(\omega))$ where ξ is measurable.

Throughout this paper, we employ the following notations: $RF(T)$, the set of random fixed points of T; $RF(S,T)$, the set of random common fixed points of S and T; $RC(S,T)$, the set of random coincidence points of S and T; $T^n(\omega, x)$, the n-th iterate of T, $T(\omega, T(\omega, T(\omega, \ldots, T(\omega, x))))$.

A random operator $T : \Omega \times C \to C$ is called: (i) continuous (resp. demiclosed, $(L-\gamma)$ uniform Lipschitz) if the map $T(\omega, .)$ is continuous (resp. demiclosed, $(L-\gamma)$ uniform Lipschitz); (ii) asymptotically nonexpansive random operator if there exists a sequence of measurable maps $u_n : \Omega \to [0, \infty)$ with $\lim_{n\to\infty} u_n(\omega) = 0$, for each $\omega \in \Omega$, such that for arbitrary $x, y \in C$, we have $\|T^n(\omega, x) - T^n(\omega, y)\| \le (1 + u_n(\omega))\|x - y\|$, for each $\omega \in \Omega$; (iii) asymptotically quasi-nonexpansive random operator if there exists a sequence of measurable maps $u_n : \Omega \to [0, \infty)$ with $\lim_{n\to\infty} u_n(\omega) = 0$, for each $\omega \in \Omega$, such that $\|T^n(\omega, \eta(\omega)) - \xi(\omega)\| \le (1 + u_n(\omega))\|\eta(\omega) - \xi(\omega)\|$, for each $\omega \in \Omega$, $\xi \in RF(T)$ and $\eta : \Omega \to C$ is any measurable map; (iv) semi-compact random operator if for a sequence of measurable maps $\{\xi_n\}$ from Ω to C with $\lim_{n\to\infty} \|\xi_n(\omega) - T(\omega, \xi_n(\omega))\| = 0$, for every $\omega \in \Omega$, there exists a subsequence $\{\xi_{n_i}\}$ of $\{\xi_n\}$ such that $\xi_{n_i}(\omega) \to \xi(\omega)$, for each $\omega \in \Omega$, where $\xi : \Omega \to C$ is measurable.

Let $S, T : \Omega \times C \to C$ be random operators. The random modified Ishikawa iteration for two maps is defined as:

$$\begin{aligned} \xi_{n+1}(\omega) &= (1-\alpha_n)\xi_n(\omega) + \alpha_n S^n(\omega, \eta_n(\omega)), \\ \eta_n(\omega) &= (1-\beta_n)\xi_n(\omega) + \beta_n T^n(\omega, \xi_n(\omega)), \quad (2.1) \end{aligned}$$

for each $\omega \in \Omega$, $n = 1, 2, 3, \ldots$, where $\alpha_n, \beta_n \in [\delta, 1-\delta]$ for some $\delta \in (0, \frac{1}{2})$, $\xi_1 : \Omega \to C$ is an arbitrary measurable map, and $(\xi_n\}$ and $\{\eta_n\}$ are sequences of maps from Ω to C. If $\beta_n = 0$, for all n in (2.1), then it reduces to the random modified Mann iteration (see [5-6]).

Let C be a closed convex subset of a separable Banach space X, and $\{\xi_n\}$ is pointwise convergent; that is, $\xi_n(\omega) \to \xi(\omega)$, for each $\omega \in \Omega$. Then closedness of C implies that ξ is a map from Ω to C. Moreover, if T is a continuous random operator, then by [2, Lemma 8.2.3], the map $\omega \to T(\omega, f(\omega))$ is measurable for any measurable map f from Ω to C. Thus $\{\xi_n\}$ is a sequence of measurable maps and $\xi : \Omega \to C$, being the limit of the sequence of measurable maps, is also measurable.

We need the following useful known results.

Lemma 1 ([10, Lemma 2]). *Let the sequences $\{a_n\}$ and $\{u_n\}$ of real numbers be such that $a_{n+1} \le (1 + u_n)a_n$, where $a_n \ge 0, u_n \ge 0$, for all $n = 1, 2, 3, \ldots$ and $\sum_{n=1}^{\infty} u_n < +\infty$. Then: (i) $\lim_{n\to\infty} a_n$ exists; (ii) if $\liminf_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} a_n = 0$.*

Theorem 2 [11, Theorem 3.1]. *Let X and Y be separable Banach spaces. Let C be a nonempty weakly compact subset of X, and $f, T : \Omega \times C \to Y$ be continuous random operators such that, for each $\omega \in \Omega, T(\omega, C)$ is bounded and $(f - T)(\omega, .)$ is demiclosed at 0. If the set $\{x \in C : f(\omega, x) - T(\omega, x) = 0\}$ is nonempty for each $\omega \in \Omega$, then there exists a measurable map $\xi : \Omega \to C$ such that $f(\omega, \xi(\omega)) - T(\omega, \xi(\omega)) = 0$, for each $\omega \in \Omega$.*

Lemma 3 (cf.[13]). *Let C be a nonempty bounded closed convex subset of a uniformly convex Banach space X, and $T : C \to C$ be asymptotically nonexpansive. Then $I - T$ is demiclosed at 0.*

Lemma 4 [13]. *Let X be a uniformly convex Banach space. Assume that $0 < b \le t_n \le c < 1$, $n = 1, 2, 3, \ldots$ Let the sequences $\{x_n\}$ and $\{y_n\}$ in X be such that $\limsup_{n\to\infty} \|x_n\| \le a$, $\limsup_{n\to\infty} \|y_n\| \le a$, and $\lim_{n\to\infty} \|t_n x_n + (1 - t_n)y_n\| = a$, for some $a \ge 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

3 Main Results

We begin with the existence of a unique random common fixed point of two continuous and contractive random operators. For this, we follow the arguments of Theorem 1 due to Aamri and El Moutawakil [1] to establish its random version in the following result.

Theorem 5 *Let C be a nonempty weakly compact subset of separable Banach space X and $S, T : \Omega \times C \to C$ be continuous random operators such that, for each $\omega \in \Omega$, the following conditions hold:*

(i) the maps $S(\omega, .)$ and $T(\omega, .)$ satisfy the property (E.A);

(ii) $T(\omega, C)$ is complete or $S(\omega, C)$ is complete with $S(\omega, C) \subseteq T(\omega, C)$;

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

(iii) *for all* $x \neq y$ *in* C, $r \in [0, +\infty)$ *and* $\alpha \in [0, 1)$,

$$\begin{aligned} d(S(\omega, x), S(\omega, y)) \quad < \quad & \max\{d(T(\omega, x), T(\omega, y)), \\ & rd(S(\omega, x), T(\omega, x)) \\ & +\alpha d(S(\omega, y), T(\omega, y)), \\ & \frac{1}{2}[d(S(\omega, x), T(\omega, y)) \\ & +d(S(\omega, y), T(\omega, x))]\}. \end{aligned}$$

(iv) $(S - T)(\omega, .)$ *is demiclosed at* 0 *and* $T(\omega, C)$ *is bounded.*

Then $RC(S, T) \neq \phi$. *Further, if* S *and* T *are weakly compatible, then* $RF(S, T)$ *is singleton.*

Proof. By (i), there exists a sequence $\{x_n\}$ in C such that, for each $\omega \in \Omega$, $\lim_{n\to\infty} S(\omega, x_n) = \lim_{n\to\infty} T(\omega, x_n) = t$, for some $t \in C$. Suppose that $T(\omega, C)$ is complete for every $\omega \in \Omega$, then there exists a point $u \in C$ such that $T(\omega, u) = t$, for each $\omega \in \Omega$. We show that $S(\omega, u) = T(\omega, u)$, for every $\omega \in \Omega$. By (iii), we have

$$\begin{aligned} d(S(\omega, x_n), S(\omega, u)) \quad < \quad & \max\{d(T(\omega, x_n), T(\omega, u)), \\ & rd(S(\omega, x_n), T(\omega, x_n)) \\ & +\alpha d(S(\omega, u), T(\omega, u)), \\ & \frac{1}{2}[d(S(\omega, x_n), T(\omega, u)) \\ & +d(S(\omega, u), T(\omega, x_n))]\}, \end{aligned}$$

for every $\omega \in \Omega$. Taking the limit as $n \to \infty$, we get

$$\begin{aligned} & d(T(\omega, u), S(\omega, u)) \\ & \leq \max\{\alpha d(S(\omega, u), T(\omega, u)), \frac{1}{2} d(S(\omega, u), T(\omega, u))\}, \end{aligned}$$

for every $\omega \in \Omega$. This is possible only if $d(T(\omega, u), S(\omega, u)) = 0$, for every $\omega \in \Omega$. Thus, by Theorem 2, there exists a measurable map $\zeta : \Omega \to C$ such that $S(\omega, \zeta(\omega)) = T(\omega, \zeta(\omega))$, for each $\omega \in \Omega$; that is, $\zeta \in RC(S, T)$.

If S and T are weakly compatible, then

$$\begin{aligned} T(\omega, T(\omega, \zeta(\omega))) &= T(\omega, S(\omega, \zeta(\omega))) \\ &= S(\omega, T(\omega, \zeta(\omega))) \\ &= S(\omega, S(\omega, \zeta(\omega))), \end{aligned}$$

for every $\omega \in \Omega$. We show that $S(\omega, S(\omega, \zeta(\omega))) = S(\omega, \zeta(\omega))$, for every $\omega \in \Omega$. Suppose not, then for some $\omega \in \Omega$, we get

$$\begin{aligned} & d(S(\omega, S(\omega, \zeta(\omega))), S(\omega, \zeta(\omega))) \\ & < \max\{d(T(\omega, S(\omega, \zeta(\omega))), T(\omega, \zeta(\omega))), \\ & rd(S(\omega, S(\omega, \zeta(\omega))), T(\omega, S(\omega, \zeta))), \\ & +\alpha d(S(\omega, \zeta(\omega)), T(\omega, \zeta(\omega))), \\ & \frac{1}{2}[d(S(\omega, S(\omega, \zeta(\omega))), T(\omega, \zeta(\omega))) \\ & +d(S(\omega, \zeta(\omega)), T(\omega, S(\omega, \zeta(\omega))))]\} \\ = \; & d(S(\omega, S(\omega, \zeta(\omega))), S(\omega, \zeta(\omega))) \end{aligned}$$

a contradiction. Thus $T(\omega, S(\omega, \zeta(\omega))) = S(\omega, S(\omega, \zeta(\omega)) = S(\omega, \zeta(\omega))$, for every $\omega \in \Omega$. Similarly, we can prove the case, $S(\omega, C)$ is complete with $S(\omega, C) \subseteq T(\omega, C)$, for every $\omega \in \Omega$. The uniqueness can be easily verified by using (iii). ■

We establish a pair of lemmas to prove our convergence theorems.

Lemma 6 *Let* C *be a nonempty closed convex subset of a separable Banach space* X, *and* S *and* T *be two asymptotically quasi-nonexpansive continuous random operators from* $\Omega \times C$ *to* C, *with the sequences of measurable maps* $u_n, v_n : \Omega \to [0, \infty)$ *corresponding to* S *and* T, *respectively. Assume that* $\sum_{n=1}^{\infty} u_n(\omega) < \infty$ *and* $\sum_{n=1}^{\infty} v_n(\omega) < \infty$, *for each* $\omega \in \Omega$, *and* $RF(S, T) \neq \phi$. *If* $\{\xi_n\}$ *is defined by (2.1) and* $\zeta \in RF(S, T)$, *then for each* $\omega \in \Omega$:

(i) *there exists a sequence of measurable maps* $t_n : \Omega \to [0, \infty)$ *such that* $\sum_{n=1}^{\infty} t_n(\omega) < \infty$ *and* $\|\xi_{n+1}(\omega) - \zeta(\omega)\| \leq (1 + t_n(\omega))^k \|\xi_n(\omega) - \zeta(\omega)\|$, *for all* $n = 1, 2, 3, \ldots$;

(ii) *there exists* $M > 0$ *(depending on* ω*) such that* $\|\xi_{n+m}(\omega) - \zeta(\omega)\| \leq M \|\xi_n(\omega) - \zeta(\omega)\|$, *for all* $n, m = 1, 2, 3, \ldots$.

Proof. Similar to that of Theorem 3.1 of [12] and is omitted. ■

Lemma 7 *Let* C *be a nonempty closed convex subset of a separable uniformly convex Banach space* X, *and* S *and* T *be two* $(L - \gamma)$ *uniform Lipschitz and asymptotically quasi-nonexpansive continuous random operators from* $\Omega \times C \to C$ *with the corresponding sequences of measurable maps* $u_n, v_n : \Omega \to [0, \infty)$ *satisfying* $\sum_{n=1}^{\infty} u_n(\omega) < \infty$ *and* $\sum_{n=1}^{\infty} v_n(\omega) < \infty$, *for each* $\omega \in \Omega$. *Suppose that* $RF(S, T) \neq \phi$ *and the sequence* $\{\xi_n\}$ *is as in (2.1). Then for each* $\omega \in \Omega$:

(i) $\lim_{n\to\infty} \|\xi_n(\omega) - \zeta(\omega)\|$ *exists for each* $\zeta \in RF(S, T)$;

(ii) $\lim_{n\to\infty} \|\xi_n(\omega) - S^n(\omega, \xi_n(\omega))\| = \lim_{n\to\infty} \|\xi_n(\omega) - T^n(\omega, \xi_n(\omega))\| = 0$;

(iii) $\lim_{n\to\infty} \|\xi_n(\omega) - S(\omega, \xi_n(\omega))\| = \lim_{n\to\infty} \|\xi_n(\omega) - T(\omega, \xi_n(\omega))\| = 0$;

Proof. (i) Follows from Lemma 6 and Lemma 1.

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

(ii) Let $\zeta \in RF(S,T)$. Then, for each $\omega \in \Omega$,

$$\begin{aligned} & \|\eta_n(\omega) - \zeta(\omega)\| \\ \leq\ & (1-\beta_n)\|\xi_n(\omega) - \zeta(\omega)\| + \beta_n\|T^n(\omega, \xi_n(\omega)) - \zeta(\omega)\| \\ \leq\ & (1-\beta_n)\|\xi_n(\omega) - \zeta(\omega)\| \\ & + \beta_n(1+v_n(\omega))\|\xi_n(\omega) - \zeta(\omega)\| \\ \leq\ & (1+t_n(\omega))\|\xi_n(\omega) + \zeta(\omega)\|. \end{aligned}$$

Thus, for each $\omega \in \Omega$, we obtain

$$\begin{aligned} & \|S^n(\omega, \eta_n(\omega)) - \zeta(\omega)\| \\ \leq\ & (1+u_n(\omega))\|\eta_n(\omega) - \zeta(\omega)\| \\ \leq\ & (1+t_n(\omega))^2\|\xi_n(\omega) - \zeta(\omega)\| \end{aligned}$$

and so,

$$\limsup_{n\to\infty} \|S^n(\omega, \eta_n(\omega)) - \zeta(\omega)\| \leq \lim_{n\to\infty} \|\xi_n(\omega) - \zeta(\omega)\|.$$

Moreover, for each $\omega \in \Omega$, we have

$$\begin{aligned} & \lim_{n\to\infty} \|(1-\alpha_n)(\xi_n(\omega) - \zeta(\omega)) \\ & + \alpha_n(S^n(\omega, \eta_n(\omega)) - \zeta(\omega))\| \\ & = \lim_{n\to\infty} \|\xi_{n+1}(\omega) - \zeta(\omega)\|. \end{aligned}$$

By Lemma 4, $\lim_{n\to\infty} \|S^n(\omega, \eta_n(\omega)) - \xi_n(\omega)\| = 0$, for each $\omega \in \Omega$. Similarly, we can prove that $\lim_{n\to\infty} \|T^n(\omega, \xi_n(\omega)) - \xi_n(\omega)\| = 0$, for each $\omega \in \Omega$. Now, for each $\omega \in \Omega$, we get

$$\begin{aligned} & \|S^n(\omega, \xi_n(\omega)) - \xi_n(\omega)\| \\ \leq\ & \|S^n(\omega, \xi_n(\omega)) - S^n(\omega, \eta_n(\omega))\| \\ & + \|S^n(\omega, \eta_n(\omega)) - \xi_n(\omega)\| \\ \leq\ & L(\alpha_n\|\xi_n(\omega) - T^n(\omega, \xi_n(\omega))\|)^\gamma \\ & + \|S^n(\omega, \eta_n(\omega)) - \xi_n(\omega)\|. \end{aligned}$$

Hence, $\lim_{n\to\infty} \|S^n(\omega, \xi_n(\omega)) - \xi_n(\omega)\| = 0$, for each $\omega \in \Omega$.

(iii) By (ii), we obtain for each $\omega \in \Omega$,

$$\begin{aligned} & \|\xi_n(\omega) - S(\omega, \xi_n(\omega))\| \\ \leq\ & \|\xi_n(\omega) - \xi_{n+1}(\omega)\| + \|\xi_{n+1}(\omega) - S^{n+1}(\omega, \xi_{n+1}(\omega))\| \\ & + \|S^{n+1}(\omega, \xi_{n+1}(\omega)) - S^{n+1}(\omega, \xi_n(\omega))\| \\ & + \|S^{n+1}(\omega, \xi_n(\omega)) - S(\omega, \xi_n(\omega))\| \\ \leq\ & \alpha_n\|\xi_n(\omega) - S^n(\omega, \eta_n(\omega))\| \\ & + \|\xi_{n+1}(\omega) - S^{n+1}(\omega, \xi_{n+1}(\omega))\| \\ & + L[\alpha_n\|\xi_n(\omega) - S^n(\omega, \eta_n(\omega))\|]^\gamma \\ & + L\|S^n(\omega, \xi_n(\omega)) - \xi_n(\omega)\| \to 0, \text{ as } n \to \infty. \end{aligned}$$

Similarly, we can prove that $\lim_{n\to\infty} \|\xi_n(\omega) - T(\omega, \xi_n(\omega))\| = 0$, for each $\omega \in \Omega$. ■

Theorem 8 *Let C be a nonempty weakly compact convex subset of a separable uniformly convex Banach space X, and S and T be as in Lemma 7. Suppose that S and T are weakly compatible. If the conditions (i)–(iv) of Theorem 5 are satisfied, and $(I-S)(\omega,\cdot)$ and $(I-T)(\omega,\cdot)$ are demiclosed at 0, then the sequence $\{\xi_n\}$, defined by (2.1), converges weakly to a unique random common fixed point of S and T.*

Proof. By Theorem 5, $RF(S,T)$ is singleton. Let $RF(S,T) = \{\zeta\}$. Then, by Lemma 7 (i), $\lim_{n\to\infty} \|\xi_n(\omega) - \zeta(\omega))\|$ exists for each $\omega \in \Omega$, and hence $\{\xi_n\}$ is bounded. Since X is reflexive, there exists a subsequence $\{\xi_{n_j}\}$ of $\{\xi_n\}$ converging weakly to a measurable map $\eta : \Omega \to C$, for each $\omega \in \Omega$. By Lemma 7 (iii) and the demiclosedness of $I-S$ and $I-T$, $S(\omega, \eta(\omega)) = T(\omega, \eta(\omega)) = \eta(\omega)$, for each $\omega \in \Omega$. Thus, $\eta(\omega) = \zeta(\omega)$, for each $\omega \in \Omega$. In order to show that $\{\xi_n\}$ converges weakly to ζ, take another subsequence $\{\xi_{n_k}\}$ of $\{\xi_n\}$ converging weakly to a measurable map $\psi : \Omega \to C$, for each $\omega \in \Omega$. Again, as above, $\psi(\omega) = \zeta(\omega)$, for each $\omega \in \Omega$, so $\{\xi_n\}$ converges weakly to ζ. ■

On the lines of the proof of the above theorem and using Lemma 3, we can prove the following:

Corollary 9 *Let C and X be as in Theorem 8, and S and T be two asymptotically nonexpansive random operators from $\Omega \times C \to C$. Suppose that S and T are weakly compatible and the conditions (i)-(iv) of Theorem 5 are satisfied. Then the sequence $\{\xi_n\}$, defined by (2.1), converges weakly to a unique random common fixed point of S and T.*

Theorem 10 *Let C be a nonempty weakly compact convex subset of a separable uniformly convex Banach space X, and S and T be as in Lemma 7. Suppose that S and T are weakly compatible, the conditions (i)-(iv) of Theorem 5 hold and for some integer m, T^m or S^m is semi-compact. Then $\{\xi_n\}$, defined by (2.1), converges strongly to a unique random common fixed point of S and T.*

Proof. By Theorem 5, $RF(S,T)$ is singleton. Let T^m be semi-compact (the proof is similar if S^m is semi-compact). By Lemma 7, we obtain

$$\begin{aligned} & \|T^m(\omega, \xi_n(\omega)) - \xi_n(\omega)\| \\ \leq\ & \|T^m(\omega, \xi_n(\omega)) - T^{m-1}(\omega, \xi_n(\omega))\| \\ & + \|T^{m-1}(\omega, \xi_n(\omega)) - T^{m-2}(\omega, \xi_n(\omega))\| \\ & + \ldots + \|T^2(\omega, \xi_n(\omega)) - T(\omega, \xi_n(\omega))\| \\ & + \|T(\omega, \xi_n(\omega)) - \xi_n(\omega)\| \\ \leq\ & (m-1)L\|T(\omega, \xi_n(\omega)) - \xi_n(\omega)\|^\gamma \\ & + \|T(\omega, \xi_n(\omega)) - \xi_n(\omega)\| \to 0, \quad \text{as } n \to \infty. \end{aligned}$$

Since $\{\xi_n\}$ is bounded and T^m is semi-compact, $\{\xi_n\}$ has a convergent subsequence $\{\xi_{n_k}\}$ converging to a measurable map $\eta : \Omega \to C$. Hence, again by Lemma 7 (iii), we

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

have

$$\begin{aligned}\|\zeta(\omega)-S(\omega,\zeta(\omega))\| &= \|\zeta(\omega)-T(\omega,\zeta(\omega))\| \\ &= \lim_{n\to\infty}\|\xi_{n_k}(\omega)-T(\omega,\xi_{n_k}(\omega))\| \\ &= 0,\end{aligned}$$

for each $\omega\in\Omega$. Thus $RF(S,T)=\{\zeta\}$. As $\lim_{n\to\infty}\|\xi_n(\omega)-\zeta(\omega)\|$ exists, so $\{\xi_n\}$ converges strongly to ζ. ■

Remark 11 (i) Following the arguments of the proofs of Lemma 7 and Theorem 10, we can prove analogues of these results for two asymptotically nonexpansive random operators instead of $(L-\gamma)$ uniform Lipschitz and asymptotically quasi-nonexpansive random operators.

(ii) Corollary 9 extends Theorems 3.1-3.2 of Tan and Xu [13] to the case of two asymptotically nonexpansive random operators.

(iii) Theorem 2.1 [5] and Theorems 3.1-3.3 [6] deal with one continuous random operator whereas Corollary 9 gets hold of two continuous random operators.

(iv) Corollary 9 is a random version of Theorem 3.4 of Shahzad and Udomene [12] without the assumption that the set of common fixed point is nonempty.

(v) Theorem 2.2 in [7] concerns existence of deterministic common fixed point of 1-subcommutative maps while Theorem 5 provides existence of a unique random common fixed point of weakly compatible random operators.

Acknowledgment

The author A.R. Khan is grateful to King Fahd University of Petroleum & Minerals and SABIC for supporting research project SB100012.

References

[1] M. Aamri and D. El Moutawakil, Some new common fixed point theorems under strict contractive conditions, J. Math. Anal. Appl. 270(2002), 181–188.

[2] J.P. Aubin and H. Frankowska, Set-Valued Analysis, Birkhäuser, Boston, 1990.

[3] I. Beg and M. Abbas, Iterative procedures for solutions of random operator equations in Banach spaces, J. Math. Anal. Appl. 315(2006), 181-201.

[4] B.S. Choudhury, Convergence of a random iteration scheme to a random fixed point, J. Appl. Math. Stochastic Anal. 8(1995), 139–142.

[5] B.S. Choudhury, Random Mann iteration scheme, Appl. Math. Lett. 16(2003), 93–96.

[6] H. Duan and G. Li, Random Mann iteration scheme and random fixed point theorems, Appl. Math. Lett. 18(2005), 109-115.

[7] N. Hussain and A.R. Khan, Common fixed-point results in best approximation theory, Appl. Math. Lett. 16(2003), 575–590.

[8] S. Ishikawa, Fixed points by a new iteration method, Proc. Amer. Math. Soc. 44(1974), 147–150.

[9] A.R. Khan and N. Hussain, Iterative approximation of fixed points of nonexpansive maps, Sci. Math. Japon. 54(3) (2001), 503–511.

[10] L. Qihou, Iterative sequences for asymptotically quasi-nonexpansive mappings, J. Math. Anal. Appl. 259(2001), 1–7.

[11] N. Shahzad, Random fixed points of K-set-and pseudo-contractive random maps, Nonlinear Anal. 57(2004), 173-181.

[12] N. Shahzad and A. Udomene, Approximating common fixed points of two asymptotically quasi-nonexpansive mappings in Banach spaces, Fixed Point Theory Appl.(2006), 1-10.

[13] K.-K. Tan and H.-K. Xu, Fixed point iteration process for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc. 122(3)(1994), 733–739.