Imbibition Phenomenon arising in Double Phase Flow through Porous Medium with Capillary Pressure

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***Abstract*- The present paper discusses the theoretical studies involved in the development of the differential equations which describe the spontaneous imbibition of water by an oil-saturated rock. The basic assumption underlying in the present investigation is that the oil and water form two immiscible liquid phases and the latter represents preferentially wetting phase. The Saturation of injected water is calculated by Adomian Decomposition Technique for Nonlinear differential equation of Imbibition phenomena under assumption that Saturation is decomposed in to saturation of different fingers. Graphical illustration has been done by Mathematica 7.0.**

***Index Terms*-Adomian Decomposition Method, Immiscible Fluid, Imbibition phenomena**

I.INTRODUCTION

This paper discusses mathematically phenomenon of imbibition in double phase flow of two immiscible fluids in a homogeneous porous media with capillary pressure. The phenomenon arises due to the difference in the wetting abilities of the phases. It is assumed that the injection of a preferentially wetting fluid into a porous medium saturated with another non-wetting fluid is initiated by imbibition and the injected fluid and the native fluid form immiscible fluid phases of different wetting abilities, with respect to the porous medium.

Imbibition due to capillary forces is known as spontaneous capillary imbibition or natural imbibition (Rose [8]). Displacement of oil from porous medium by an external force which gives rise to a pressure gradient is known as forced imbibition.

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Most of the experimental studies were primarily concerned with the imbibition aspect of the flow mechanism in the matrix compared to the total flow problem in the fracture–matrix system. Rangel-German and Kovscek [7]; Karimaie, Torsaeter, Esfahani, Dadashpour, Hashemi [4] and Graue, Moe, Baldwin[3] investigated the effects of injection rate, initial water saturation and gravity on water injection in slightly water-wet fractured porous media. On the other hand, (Yildiz, Gokmen, Cesur [10]) examined the effects of shape factor, characteristic length, and boundary conditions on the spontaneous imbibition phenomenon. Some has discussed the physics of oil-water motion in a cracked porous medium. Mehta has discussed analytically the phenomenon of imbibition in porous media under certain condition by using a singular perturbation approach. It has been discussed by (Mehta ([5] and [6]) from different point of view.

In the present paper, we have discussed the imbibition phenomenon arising in the flow of two immiscible fluid flows through homogeneous porous media with the effect of capillary pressure and obtained an approximate solution of the nonlinear differential system governing imbibition phenomena through Adomian Decomposition Technique.

II.MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

We consider here that a finite cylindrical piece of homogeneous porous medium of length L containing viscous oil is completely surrounded by an impermeable surface except for one end of the cylinder which is labelled as the Imbibition phase and this end is exposed to an adjacent formation of 'injected' water. It is assumed that the injected water and the viscous oil are two immiscible liquids of different salinities with small viscosity difference; the former represents the preferentially wetting and less viscous phase as in fig-2. Figure is adopted from [Dominique Salin], WRR 39 18, 1135-1143 (2003), Viscous fingering in porous media.

Fig-1-Imbibition Phenomena

By assuming the validity of Darcy's law for the double phase flow system, From (Scheidegger [1960]) the seepage velocities of wetting phase (v_w) and the non-wetting phase (v_o) as

$$v_w = -\frac{k_w}{\mu_w} K \frac{\partial p_w}{\partial x} \tag{1}$$

$$v_o = -\frac{k_o}{\mu_o} K \frac{\partial p_o}{\partial x} \tag{2}$$

Where k_w and k_o are relative permeability; p_w and p_o are pressures and μ_w and μ_o are kinematic viscosities (which are constants) of the wetting phase and non-wetting phase respectively and K is the permeability of homogeneous medium. The coordinate x is measured along the axis of the cylindrical medium, the origin being located at the imbibition face x = 0.

The flow is counter current so for the imbibition phenomenon

$$v_w = -v_o \tag{3}$$

Therefore from equation (1) and (2) we may write

$$\frac{k_w}{\mu_w}\frac{\partial p_w}{\partial x} + \frac{k_o}{\mu_o}\frac{\partial p_o}{\partial x} = 0 \tag{4}$$

The definition of capillary pressure p_c gives

$$p_c = p_o - p_w \tag{5}$$

Combining equation (4) and (5), we get

$$\left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}\right)\frac{\partial p_w}{\partial x} + \frac{k_o}{\mu_o}\frac{\partial p_c}{\partial x} = 0 \tag{6}$$

Substituting the value of $\frac{\partial p_w}{\partial x}$ from the equation (6) into the equation (1), we obtain

$$v_w = \frac{K\left(\frac{k_w}{\mu_w}\right)\left(\frac{k_o}{\mu_o}\right)\frac{\partial p_c}{\partial x}}{\left(\frac{k_w}{\mu_w} + \frac{k_o}{\mu_o}\right)} \tag{7}$$

The equations of continuity for the wetting phase is given by

$$\varphi \frac{\partial S_w}{\partial t} + \frac{\partial v_w}{\partial x} = 0 \tag{8}$$

Where 'φ' is the porosity of the medium and S_w is the saturation of the wetting phase.

Substituting the value of v_w from equation (7) in to equation (8), it becomes

$$\phi \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x}\left[K \frac{k_w k_0}{k_w \mu_0 + k_0 \mu_w} \frac{dp_c}{dS_w} \frac{\partial S_w}{\partial x}\right] = 0 \tag{9}$$

This equation (9) is a non-linear partial differential equation, which describes the linear counter current imbibition phenomenon of two immiscible fluids flow through homogeneous porous cylindrical medium with impervious bounding surfaces on three sides.

It is well known that, fictitious relative permeability is the function of displacing fluid saturation. Then at this stage for definiteness of the mathematical analysis, we assume standard forms of (Scheidegger and Johnson [1960]) for the analytical relationship between the relative permeability, phase saturation and capillary pressure phase saturation as

$$k_w = S_w \tag{10}$$

$$k_o = 1 - \alpha S_w \qquad (\text{where } \alpha = 1.11) \tag{11}$$

$$p_c = -\beta S_w \tag{12}$$

Since the present investigation involves water and viscous oil, therefore according to (Scheidegger [1960]), we have

$$\frac{k_o k_w}{k_w \mu_o + k_o \mu_w} \approx \frac{k_o}{\mu_o} = \frac{1 - \alpha S_w}{\mu_o} = \frac{S^*{}_w}{\mu_o} \tag{13}$$

$$\text{where } S = 1 - \alpha S_w \quad \text{say}$$

Substituting the values of k_o and p_c from equations (11) and (12) into the equation (9) and using (13) we get

$$\frac{\partial S_w}{\partial t} = \frac{K\beta}{\mu_0 \phi}\frac{\partial}{\partial x}\left[(1 - \alpha S_w)\frac{\partial S_w}{\partial x}\right] \tag{14}$$

We now choose new variables to convert (14) into dimensionless form

$$X = \frac{x}{L}, \quad T = \frac{k\beta}{\phi \mu_0 L^2} t$$

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$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X}\left[(1-\alpha S_w)\frac{\partial S_w}{\partial X}\right] \tag{15}$$

$$S_W(X,0) = f(X) \quad = e^{-X} \tag{16}$$

As Saturation of injected water decreases exponentially when X increases, this is significant with phenomenon of imbibition (Mehta [2008]).

For $1-\alpha S_w$ =S

Equation (15) and (16) takes the form

$$\frac{\partial S}{\partial T} = \frac{\partial}{\partial X}\left[S\frac{\partial S}{\partial X}\right] \tag{17}$$

having boundary conditions

$$S(X,0) = f(X) \quad = 1-\alpha e^{-X} \text{ ‘say’} \tag{18}$$

This is the desired non-linear partial differential equation describing the linear counter current imbibition phenomenon with capillary pressure.

III.ADOMIAN DECOMPOSITION METHOD THEORETIC APPROACH

We solve equation (17) for $L_T S$ separately and we get

$$L_T S(X,T) = NS(X,T) \tag{19}$$

Let L_T^{-1} be the inverse operators of $L_T S$ given by the form:

$$L_T^{-1} = \int (.)\, dT \tag{20}$$

Then operating both sides of equation (19) with the inverse operators (20) we obtain

$$S(X,T) = A(X) + L_T^{-1}\left(S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right)^2\right) \tag{21}$$

Where A(X) can be solved subjected to the corresponding initial condition (18) and we obtain:

$$A(X) = 1-\alpha e^{-X} \tag{22}$$

Now from (22) we get the following form

$$S(X,T) = 1-\alpha e^{-X} + \left[L_T^{-1}\left(S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right)^2\right)\right] \tag{23}$$

We write the parameterized form of (23)

$$S(X,T) = S_0 + \lambda\left[L_T^{-1}\left(S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right)^2\right)\right] \tag{24}$$

and the parameterized decomposition forms of S(X, T) and NS(X, T) as

$$S(X,T) = \sum_{n=0}^{\infty} \lambda^n S_n(X,T) \tag{25}$$

Where S_0, S_1, S_2----------are saturation at x= 0, 0.1, 0.2 …for any T>0 in different fingers. The saturation at X for any T>0 is assumed to be decomposition of saturation of different fingers for given X, T>0 because the no. of interconnected fingers are very large at infinite strip in practical. Hence we may choose equation (25). (See Fig.2)

and $S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial X}\right)^2$ =NS(X,T)= $\sum_{n=0}^{\infty} \lambda^n A_n$ (26)

Where A_n's are the Adomian's special polynomials are to be determined. Here the parameter λ looks like a perturbation parameter; but actually is not a perturbation parameter; it is used only for grouping the terms.

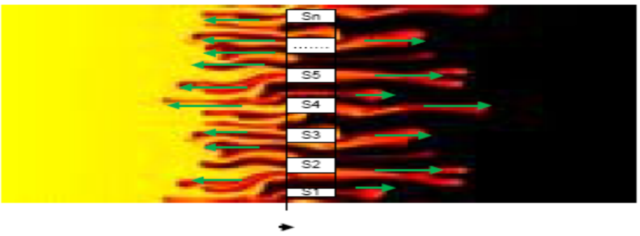

Fig-2: Schematics decomposition of saturation of different fingers

Now substitution of (26) and (27) into (25) gives

$$\sum_{n=0}^{\infty} \lambda^n S_n(X,T) =$$
$$S_0(X,T) + \lambda\left[L_T^{-1}\left(\sum_{n=0}^{\infty} \lambda^n A_n\right)\right] =$$
$$S_0(X,T) + \lambda\left[L_T^{-1}\left(\sum_{n=0}^{\infty} \lambda^n S_n(X,T)\frac{\partial^2 \sum_{n=0}^{\infty} \lambda^n S_n(X,T)}{\partial X^2} + \left(\frac{\partial \sum_{n=0}^{\infty} \lambda^n S_n(X,T)}{\partial X}\right)^2\right)\right] \tag{27}$$

If we compare the like power terms of λ from both sides of equation (27), we get

$$S_0(X,T) = 1-\alpha e^{-X}$$

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$$S_1(X,T) = \left[L_T^{-1}(A_0)\right]$$

$$S_2(X,T) = \left[L_T^{-1}(A_1)\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

....

$$S_{n+1}(X,T) = \left[L_T^{-1}(A_n)\right], \tag{28}$$

Next we determine Adomian's special Polynomials A_n.

IV.DETERMINATION OF ADOMIAN'S SPECIAL POLYNOMIALS

The $A_n{}'s$ polynomials are determined in such a way that each A_n depend only on $S_0, S_1, \ldots\ldots\ldots S_n$ for $A_0 = A(S_0), A_1 = A_1(S_0, S_1), A_2 = A_2(S_0, S_1, S_2)$ etc. In order to do this we substitute (28) in to (29) and we have

$$NS(X,T) = S\frac{\partial^2 S}{\partial X^2} + \left(\frac{\partial S}{\partial x}\right)^2 =$$

$$S_0\frac{\partial^2 S_0}{\partial X^2} + \left(\frac{\partial S_0}{\partial X}\right)^2 +$$

$$\lambda(S_0\frac{\partial^2 S_1}{\partial X^2} + S_1\frac{\partial^2 S_0}{\partial X^2} + 2\frac{\partial S_1}{\partial X}\frac{\partial S_0}{\partial X}) +$$

$$\lambda^2(S_0\frac{\partial^2 S_2}{\partial X^2} + S_1\frac{\partial^2 S_1}{\partial X^2} + S_2\frac{\partial^2 S_0}{\partial X^2} + 2\frac{\partial S_0}{\partial X}\frac{\partial S_2}{\partial X} + \left(\frac{\partial S_1}{\partial X}\right)^2)$$

$$+\lambda^3(\ldots\ldots.) \tag{29}$$

From (29) we conclude that the Adomian polynomials have the following form:

$$A_0 = S_0\frac{\partial^2 S_0}{\partial X^2} + \left(\frac{\partial S_0}{\partial X}\right)^2$$

$$A_1 = S_0\frac{\partial^2 S_1}{\partial X^2} + S_1\frac{\partial^2 S_0}{\partial X^2} + 2\frac{\partial S_1}{\partial X}\frac{\partial S_0}{\partial X}$$

$$A_2 = S_0\frac{\partial^2 S_2}{\partial X^2} + S_1\frac{\partial^2 S_1}{\partial X^2} + S_2\frac{\partial^2 S_0}{\partial X^2} + 2\frac{\partial S_0}{\partial X}\frac{\partial S_2}{\partial X} + \left(\frac{\partial S_1}{\partial X}\right)^2$$

....

Hence, the polynomial A_0 has the following form:

$$A_0 = -\alpha e^{-X} + 2\alpha^2 e^{-2X}$$

$$S_1(X,T) = -\alpha T e^{-X} + 2\alpha^2 T e^{-2X}$$

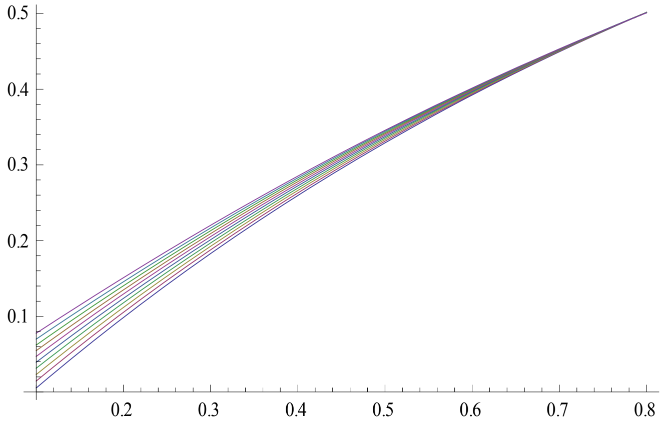
....

If we approximate the Saturation of Water 'S' up to four terms, it gives

$$S(X,T) = S_0(X,T) + S_1(X,T) + S_2(X,T) + S_3(X,T) \tag{30}$$

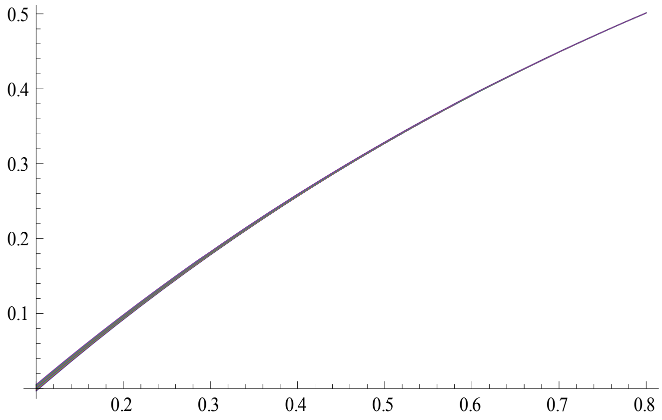
A. Graph -1

For 0<X<1, and For fix T=.01,.02…..1



B.Graph -2

For 0<X<1, and For fix T=.001,.002----.01



V.CONCLUSION

An oil water imbibition problem in a homogenous porous medium has been analytically discussed under special conditions, by using Adomian Decomposition approach. Some specific results (which are quoted in standard literature) for relative permeability, capillary pressure and oil-water viscosity ratio have been considered. In the present discussion an approximate analytical expression for the wetting phase saturation has been obtained. Mathematical and graphical solutions has been developed for predicting the possible Imbibition of oil and water in unsteady unidirectional flows through semi-infinite, homogeneous, isotropic porous media subject to the source of Saturation that vary exponentially with distance. Equation (30) represents the Saturation of

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water that Increases exponentially with distance X for any time T >0 which is physically consistent with the real phenomena.

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