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Robust Stabilization of Jet Engine Compressor in the Presence of Noise and Unmeasured States

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***Abstract*— Compressors for jet engines in operation experience disturbances such as variations in the states of the system, mass flow, and pressure. These disturbances sometimes result in surge and stall instability problems, which adversely affects its performance. In this work, first we modify the Moore and Grietzer three-state model for compressors to include disturbance (noise signals) and then use the method of integrator backstepping, coupled with saturation functions to develop robust controllers for the stabilization of the compressor. Also, we develop robust observers for estimating the states of the system in situation where there exist some states that cannot be measured. Implementing the developed controllers on the system, simulation results showed that stability was achieved. Also, the observer designed for the jet compressor was able to provide accurate state estimates.**

***Index Terms*— Integrator backstepping, observer, robust control, stall and surge, unmeasured states**

I. Introduction

IN the operation of jet engine compressor, there is the need to control surge and stall, so as to ensure stability; and in turn reduce machine damage that may arise from excessive vibrations and high thermal loading.

A number of research works with regard to stability analysis of jet compressor engines have evolved over the years. Some of these works looked at modeling the system while others considered the stability dynamics.

Moore and Greitzer [12] proposed a three-state nonlinear model that characterized the dynamics of the behavior of a compressor. The control of surge and rotating stall in compressors has been investigated by a number of researchers see [13, 14]. Krstic et al [17] in their work, on jet engine compressor, used integrator backstepping to avoid cancellation of useful nonlinearities in the stabilization analysis.

Jan and Olgav [9] in their work used the backstepping method to design a closed couple valve for controlling surge and stall in compressors.

Feng and Shih-Chiang [3], developed an adaptive controller regulating for rotating stall and surge in Jet engines using a function approximation approach. The concept of integrator backstepping in stability analysis is well expounded in literature [4, 6, 17, 18, 19]. In actual operation of Jet compressors, the controllers developed in these models do not stabilize the system when it is subjected to uncertainties such as system modeling errors, in-service changes amongst others, and compressor disturbances (noise) such as speed fluctuations, combustion noise etc., which significantly reduces the efficiency of the compressor [10, 15, 16]. Consequently, in this work, the aim is to resolve these instability problems associated with compressors subjected to disturbances (noise).

In addressing these problems, we develop robust controllers for the stabilization of the compressors in the presence of disturbances (noise signals) using the integrator backstepping method, coupled with saturator. Also, we develop controllers for observer design for the compressor, in the situation where there is no stall, and the pressure rise is an unmeasured state (unmeasured state).

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II. Problem Formulation

We begin with the basic three state Moore and Grietzer model [12] representing the compressor dynamics for a Jet Engine. This is given as:

$$\dot{R} = -\sigma R^2 - \sigma R(2\phi + \phi^2) \tag{1}$$

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$$\dot{\phi} = -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R \qquad (2)$$

$$\dot{\psi} = -u \qquad (3)$$

where R is the normalized stall cell squared amplitude.

$\phi = \varphi - 1$

Where φ is the mass flow

$\psi = \Psi - \Psi_{co} - 2$

Ψ is the pressure rise and

Ψ_{co} is a constant.

u is the input or control

The system represented by equations 1-3 does not have noise signals. Using Integrator backstepping, the stabilized system is given as:

$$\dot{R} = -\sigma R^2 - \sigma R(2\phi + \phi^2) \qquad (4)$$

$$\dot{\phi} = -Z_3 - C_1\phi - \frac{1}{2}\phi^3 - 3R\phi \qquad (5)$$

$$\dot{Z}_3 = \phi - C_2 Z_3 \qquad (6)$$

A. Introduction of Disturbances (Noise signals)

In this section we modify the basic Moore and Greitzer model to include noise, and then develop stabilizing controller for it. Introducing noise as: $\theta_1, \theta_2, \theta_3$ to equations (1-3), the system is modified as:

$$\dot{R} = -\sigma R^2 - \sigma R(2\phi + \phi^2) + \theta_1 \qquad (7)$$

$$\dot{\phi} = -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R + \theta_2 \qquad (8)$$

$$\dot{\psi} = -u + \theta_3 \qquad (9)$$

B. Definitions

The following are some definitions needed for the development of the results:
Lyapunov Function [1, 5, 7, 8, 18, 19]
A Lyapunov function is defined as follows:
Let: $V : R^+ \times R^n \to R$ be a C^1 function defined in a domain $D \subset R^n$ that includes the origin. Then the Lyapunov function, $V(t, x)$, which must satisfy the following conditions:

1. V is proper at the equilibrium state x_e :

   $$x \in R^n \big| \ V(x) \le \varepsilon \qquad (10)$$

   that is, V is a compact subset of some neighbourhood O of x_e for each ε >0 small enough.
2. V is positive definite on O:

   $V(x_e) = 0$ and

   $$V(x) > 0 \ \forall x \in O, x \ne x_e \qquad (11)$$

   For $x \ne x_e$ in O there is some time $t_1 \in T, t_1 > 0$ and some control $u \in U^{(0,t_1)}$ admissible for x such that the trajectory $\xi = \varsigma(x,u)$ resulting from the control and this initial state,

   $V(\xi(t)) \le V(x)$ $V(\xi(t)) \le V(x)$ and
   $V(\xi(t)) \le V(x)$
3. $V(x,t) \to \infty$ as $\|x\| \to \infty$ (12)

This third property is referred to as radially unbounded or uniformly unbounded or weakly coercive.
Saturation Function
We define saturation function $\phi(\theta_i, \lambda)$ as follows:

$$\phi_i(\theta_i, \lambda_i) = \text{sgn}(\theta_i).\min\{|\theta_i|, \lambda_i\} \qquad (13)$$

Where λ_i is the saturation level
$\text{sgn}(.)$ is the signum function defined as:

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$$\operatorname{sgn}(\theta)=\begin{cases}1 & if \;\; \theta>0\\ 0 & if \;\; \theta=0\\ -1 & if \;\; \theta<0\end{cases} \tag{14}$$

III. THEOREM 1

Given a feedback control system with disturbance signal at each subsystem of the form:

$$\dot{x}=f(x,u,\theta) \tag{15}$$

$$x\in R^n, u\in R^m, \theta\in R^n$$

Then, the existence of a controller of the form: $u=k(x,\phi(\theta,\lambda))$, where $\phi(.)$ is a saturation function such that $\frac{dV}{dt}<0$ or $\frac{dV}{dt}<-\|x\|^2$ is a necessary and sufficient condition for the resulting closed loop control system to be robustly globally(locally) asymptotically stable.

Proof

The proof of the theorem requires both necessity and sufficiency conditions to be satisfied. Details of the proof of the theorem can be found in [2].

A. *Methodology for stabilization of the Compressor*

Using equation (14), we introduce saturators in to the system as follows:

$$\dot{R}=-\sigma R^2-\sigma R(2\phi+\phi^2)+\mu_1(\theta_1,\lambda_1) \tag{16}$$

$$\dot{\phi}=-\psi-\frac{3}{2}\phi^2-\frac{1}{2}\phi^3 -3R\phi-3R+\mu_2(\theta_2,\lambda_2) \tag{17}$$

$$\dot{\psi}=-u+\mu_3(\theta_3,\lambda_3) \tag{18}$$

IV. THEOREM 2

For the Compressor system bombarded with noise signals represented by equations (7-9), asymptotic stabilization of the system is obtained with the choice of control law:

$$\begin{aligned}u&=c_2z_3+k_0\dot{\mu}_1\\ &+k_1\begin{pmatrix}-\sigma R^2-k\sigma R+\sigma R\\ -z_2\sigma R-2k\sigma Rz_2\end{pmatrix}\\ &+k_2\left(-c_1z_2+\sigma R^2+2k\sigma R^2\right)+\ddot{k}-\ddot{\mu}_2\end{aligned} \tag{19}$$

where

$$k=\left(\frac{\mu_1(\theta_1,\lambda_1)}{\sigma R}\right)^{\frac{1}{2}} \tag{20}$$

$$\begin{aligned}k_0&=\frac{3\left(\frac{R^3\mu_1{}^3}{\sigma}\right)^{\frac{1}{2}}}{\mu_1}+\frac{1.5\left(\frac{R\mu_1}{\sigma}\right)^{\frac{1}{2}}}{\mu_1}\\ &-\frac{0.75\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}}{\mu_1}+\frac{1.5z_2}{\sigma R}\\ &+\frac{0.75\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}}{\sigma R}+\frac{0.75z_2{}^2\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}}{\mu_1}\end{aligned} \tag{21}$$

$$\begin{aligned}k_1&=3z_2+2\sigma R+\frac{3\mu_1}{2\sigma}\left(\frac{R\mu_1}{\sigma}\right)^{\frac{1}{2}}\\ &+\frac{3R^2\mu_1{}^3}{\sigma}\left(\frac{R^3\mu_1{}^3}{\sigma}\right)^{-\frac{1}{2}}-\frac{1.5\mu_1z_2}{\sigma R^2}\\ &+\frac{0.75\mu_1}{\sigma R^2}\left(\frac{\mu_1}{\sigma R}\right)^{-\frac{1}{2}}-\frac{0.75\mu_1}{\sigma R^2}\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}\\ &-\frac{0.75\mu_1z_2{}^2}{\sigma R^2}\left(\frac{\mu_1}{\sigma R}\right)^{-\frac{1}{2}}\end{aligned} \tag{22}$$

$$\begin{aligned}k_2&=3R-c_1+3z_2\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}\\ &+1.5z_2{}^2+1.5\left(\frac{\mu_1}{\sigma R}\right)+1.5\end{aligned} \tag{23}$$

$$z_2=\phi-k+1 \tag{24}$$

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$$z_3 = \begin{bmatrix} \psi - c_1 z_2 + \sigma R^2 + 2k\sigma R^2 + 1.5 z_2 \\ +0.5 z_2{}^3 + 1.5 k z_2{}^2 + 1.5 k^2 z_2 \\ +0.5 k^3 - 1.5k + 3R z_2 + 3kR \\ +1 - \mu_2(\theta_2, \lambda_2) + \dot{k} \end{bmatrix} \quad (25)$$

and c_1, c_2 are constants with $c_1, c_2 > 0$

A. Proof of theorem 2

$$\phi = \gamma_1\left(R, \mu_1(\theta_1, \lambda_1)\right) =$$

Let $$\left(\left(\frac{\mu_1(\theta_1, \lambda_1)}{\sigma R} \right)^{\frac{1}{2}} - 1 \right) = k - 1 \quad (26)$$

Where $k = \left(\dfrac{\mu_1(\theta_1, \lambda_1)}{\sigma R} \right)^{\frac{1}{2}}$

Therefore, substituting into equation (19)

$$\begin{aligned} \dot{R} &= -\sigma R^2 - 2\sigma R(k-1) \\ &\quad - \sigma R(k^2 - 2k + 1) + \mu_1(\theta_1, \lambda_1) \\ &= -\sigma R^2 - 2k\sigma R + 2\sigma R - k^2 \sigma R \\ &\quad + 2k\sigma R - \sigma R + \mu_1(\theta_1, \lambda_1) \\ &= -\sigma R^2 + \sigma R - k^2 \sigma R + \mu_1(\theta_1, \lambda_1) \end{aligned} \quad (27)$$

$$k^2 = \frac{\mu_1(\theta_1, \lambda_1)}{\sigma R}; \quad \therefore k^2 \sigma R = \mu_1(\theta_1, \lambda_1) \quad (28)$$

,
Hence

$$\begin{aligned} \dot{R} &= -\sigma R^2 + \sigma R - \mu_1(\theta_1, \lambda_1) + \mu_1(\theta_1, \lambda_1) \\ \dot{R} &= -\sigma R^2 + \sigma R \\ &= -\sigma R\left(R - 1\right) \end{aligned} \quad (29)$$

Using the Lyapunov function,

$$V(R) = \frac{R^2}{2} \quad (30)$$

We have:

$$\begin{aligned} \dot{V}(R) &= \frac{\partial V(R)}{\partial R} . \dot{R} \\ &= R.\left(-\sigma R\left(R - 1\right)\right) \\ &= \left(-\sigma R^2\left(R - 1\right)\right) \end{aligned} \quad (31)$$

This is negative definite $\forall\ R > 1$ and $\sigma > 0$. Hence there would be Local asymptotic stability. But ϕ is not the control so we introduce z_2 to track error.

Set $z_2 = \phi - \gamma_1\left(R, \mu_1(\theta_1, \lambda_1)\right)$ (32)

$$\phi = z_2 + \gamma_1\left(R, \mu_1(\theta_1, \lambda_1)\right) \quad (33)$$

Substituting into equation (16),

$$\begin{aligned} \therefore \dot{R} &= -\sigma R^2 - 2\sigma R(z_2 + k - 1) \\ &\quad - \sigma R(z_2 + k - 1)^2 + \mu_1(\theta_1, \lambda_1) \end{aligned}$$

$$(z_2 + k - 1)^2 = z_2{}^2 + 2kz_2 - 2z_2 + k^2 - k + 1$$

$$\begin{aligned} \therefore\ \dot{R} &= -\sigma R^2 - \sigma R(2z_2 + 2k - 2 \\ &\quad + z_2 + 2kz_2 - 2z_2 + k^2 - k + 1) \\ &\quad + \mu_1(\theta_1, \lambda_1) \end{aligned}$$

$$\begin{aligned} \dot{R} &= -\sigma R^2 - k\sigma R + \sigma R \\ &\quad - z_2 \sigma R - 2k\sigma R z_2 \end{aligned} \quad (34)$$

From $z_2 = \phi - \gamma_1\left(R, \mu_1(\theta_1, \lambda_1)\right)$

$$z_2 = \phi - k + 1$$

Which gives:

$$\dot{z}_2 = \dot{\phi} - \dot{k} \quad (35)$$

From $$\begin{aligned} \dot{\phi} &= -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 \\ &\quad - 3R\phi - 3R + \mu_2(\theta_2, \lambda_2) \end{aligned}$$

$$\begin{aligned} \therefore\ \dot{z}_2 &= -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi \\ &\quad - 3R + \mu_2(\theta_2, \lambda_2) - \dot{k} \end{aligned} \quad (36)$$

Substituting for ϕ, we have:

$$\begin{aligned} \dot{z}_2 &= -\psi - \frac{3}{2}\left(z_2{}^3 + 2kz_2 - 2z_2 + k^2 - k + 1\right) \\ &\quad - \frac{1}{2}\begin{pmatrix} z_2{}^3 - 6kz_2 - 3z_2{}^2 + 3k^2 z_2 + 3kz_2{}^2 - 5kz_2 \\ +3z_2{}^2 + 3z_2 + k^3 - 3k^2 + 3k - 1 \end{pmatrix} \\ &\quad - 3R\left(z_2 + k - 1\right) - 3R + \mu_2(\theta_2, \lambda_2) - \dot{k} \end{aligned}$$

$$\begin{aligned} \dot{z}_2 &= -\psi - 1.5z_2 - 0.5z_2{}^3 \\ &\quad - 1.5kz_2{}^2 - 1.5k^2 z_2 - 0.5k^3 \\ &\quad + 1.5k - 3Rz_2 - 3kR - 1 \\ &\quad + \mu_2(\theta_2, \lambda_2) - \dot{k} \end{aligned} \quad (37)$$

Define $V_2(R,z_2)=\dfrac{R^2}{2}+\dfrac{{z_2}^2}{2}$

$$\dot{V}_2=R\dot{R}+Z_2\dot{Z}_2$$
$$=R\left(-\sigma R^2-k\sigma R+\sigma R-z_2\sigma R-2k\sigma Rz_2\right)$$
$$+z_2\begin{pmatrix}-\psi-1.5z_2-0.5{z_2}^3-1.5k{z_2}^2\\ -1.5k^2z_2-0.5k^3+1.5k-3Rz_2\\ -3kR-1+\mu_2(\theta_2,\lambda_2)-\dot{k}\end{pmatrix}$$
$$=R\left(-\sigma R^2-k\sigma R+\sigma R\right)$$
$$+z_2\begin{pmatrix}-\sigma R^2-2k\sigma R^2-\psi-1.5z_2\\ -0.5{z_2}^3-1.5k{z_2}^2-1.5k^2z_2\\ -0.5k^3+1.5k-3Rz_2-3kR\\ -1+\mu_2(\theta_2,\lambda_2)-\dot{k}\end{pmatrix} \qquad (38)$$

Selecting the control law using

$$\psi\le\frac{\delta V}{\delta R}g(R)+ke_1 \qquad (39)$$

Where $g(R)$ = all terms in R, that are multiplied by Z_2

$e_1=Z_2$ and k is a constant, $k>0$

$$\psi\le c_1z_2-\sigma R^2-2k\sigma R^2-1.5z_2$$
$$-0.5{z_2}^3-1.5k{z_2}^2-1.5k^2z_2$$
$$-0.5k^3+1.5k-3Rz_2-3kR \qquad (40)$$
$$-1+\mu_2(\theta_2,\lambda_2)-\dot{k}$$

Substituting into equation (38),

$$\dot{V}_2\le-\sigma R^3-k\sigma R^2+\sigma R^2-c_1{z_2}^2$$
$$=-\sigma R^2\left(R+k-1\right)-c_1{z_2}^2$$

V_2 is negative definite for $R+k>1,\ \sigma>0,\ c_1>0$, hence there is local asymptotic stability.

Substituting for ψ into equation (37)

$$\dot{Z}_2=-c_1z_2+\sigma R^2+2k\sigma R^2 \qquad (41)$$

Next we define $z_3=\psi-\gamma_2\left(R,z_2,\mu_1(\theta_1,\lambda_1),\mu_2(\theta_2,\lambda_2)\right)$, with

$$\gamma_2\left(R,z_2,\mu_1(\theta_1,\lambda_1),\mu_2(\theta_2,\lambda_2)\right)=$$
$$\begin{bmatrix}c_1z_2-\sigma R^2-2k\sigma R^2-1.5z_2\\ -0.5{z_2}^3-1.5k{z_2}^2-1.5k^2z_2\\ -0.5k^3+1.5k-3Rz_2-3kR\\ -1+\mu_2(\theta_2,\lambda_2)-\dot{k}\end{bmatrix} \qquad (42)$$

$$\therefore z_3=\begin{bmatrix}\psi-c_1z_2+\sigma R^2+2k\sigma R^2\\ +1.5z_2+0.5{z_2}^3+1.5k{z_2}^2\\ +1.5k^2z_2+0.5k^3-1.5k\\ +3Rz_2+3kR+1\\ -\mu_2(\theta_2,\lambda_2)+\dot{k}\end{bmatrix} \qquad (43)$$

$$z_3=\begin{bmatrix}\psi-c_1z_2+\sigma R^2+2\left(\dfrac{\mu_1R^3}{\sigma}\right)^{\frac{1}{2}}\\ +1.5z_2+0.5{z_2}^3+1.5\left(\dfrac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}{z_2}^2\\ +1.5\left(\dfrac{\mu_1}{\sigma R}\right)z_2+0.5\left(\dfrac{\mu_1}{\sigma R}\right)^{\frac{3}{2}}\\ -1.5\left(\dfrac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}+3Rz_2+3\left(\dfrac{\mu_1R}{\sigma}\right)^{\frac{1}{2}}\\ +1-\mu_2(\theta_2,\lambda_2)+\dot{k}\end{bmatrix} \qquad (44)$$

$$\dot{z}_3=\frac{\partial z_3}{\partial\psi}\dot{\psi}+\frac{\partial z_3}{\partial R}\dot{R}+\frac{\partial z_3}{\partial z_2}\dot{z}_2$$
$$+\frac{\partial z_3}{\partial\mu_1}\dot{\mu}_1+\frac{\partial z_3}{\partial\mu_2}\dot{\mu}_2+\ddot{k} \qquad (45)$$

$$\dot{z}_3=\dot{\psi}+k_0\dot{\mu}_1+k_1\dot{R}+k_2\dot{z}_2-\dot{\mu}_2+\ddot{k} \qquad (46)$$

$$\dot{z}_3=-u+k_0\dot{\mu}_1+k_1\dot{R}+k_2\dot{z}_2-\dot{\mu}_2+\ddot{k} \qquad (47)$$

where,

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$$k_0 = \frac{3\left(\frac{R^3\mu_1^{\;3}}{\sigma}\right)^{\frac{1}{2}}}{\mu_1} + \frac{1.5\left(\frac{R\mu_1}{\sigma}\right)^{\frac{1}{2}}}{\mu_1} - \frac{0.75\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}}{\mu_1} + \frac{1.5z_2}{\sigma R} + \frac{0.75\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}}{\sigma R} + \frac{0.75z_2^{\;2}\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}}}{\mu_1} \tag{48}$$

$$k_1 = 3z_2 + 2\sigma R + \frac{3\mu_1}{2\sigma}\left(\frac{R\mu_1}{\sigma}\right)^{\frac{1}{2}} + \frac{3R^2\mu_1^{\;3}}{\sigma}\left(\frac{R^3\mu_1^{\;3}}{\sigma}\right)^{-\frac{1}{2}} - \frac{1.5\mu_1 z_2}{\sigma R^2} + \frac{0.75\mu_1}{\sigma R^2}\left(\frac{\mu_1}{\sigma R}\right)^{-\frac{1}{2}} - \frac{0.75\mu_1}{\sigma R^2}\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}} - \frac{0.75\mu_1 z_2^{\;2}}{\sigma R^2}\left(\frac{\mu_1}{\sigma R}\right)^{-\frac{1}{2}} \tag{49}$$

and

$$k_2 = 3R - c_1 + 3z_2\left(\frac{\mu_1}{\sigma R}\right)^{\frac{1}{2}} + 1.5z_2^{\;2} + 1.5\left(\frac{\mu_1}{\sigma R}\right) + 1.5 \tag{50}$$

Define $V_3(R, z_2, z_3) = \frac{R^2}{2} + \frac{z_2^{\;2}}{2} + \frac{z_3^{\;2}}{2}$ (51)

$$\dot{V}_3 = R\dot{R} + z_2\dot{z}_2 + z_3\dot{z}_3 \tag{52}$$

$$\therefore \dot{V}_3 = \left[\begin{array}{l} R\begin{pmatrix} -\sigma R^2 - k\sigma R + \sigma R \\ -z_2\sigma R - 2k\sigma R z_2 \end{pmatrix} \\ +z_2\left(-c_1 z_2 + \sigma R^2 + 2k\sigma R^2\right) \\ +z_3\begin{pmatrix} -u + k_0\dot{\mu}_1 + k_1\dot{R} \\ +k_2\dot{z}_2 - \dot{\mu}_2 + \ddot{k} \end{pmatrix} \end{array}\right] \tag{53}$$

Using the control law in equation (39),

$$u \le \left[c_2 z_3 + k_0\dot{\mu}_1 + k_1\dot{R} + k_2\dot{z}_2 - \dot{\mu}_2 + \ddot{k}\right] \tag{54}$$

Hence,

$$\therefore \dot{V}_3 \le -c_2 z_3^{\;2} - \sigma R^3 - k\sigma R^2 + \sigma R^2 - c_1 z_2^{\;2}$$

$$\therefore \dot{V}_3 \le -c_2 z_3^{\;2} - c_1 z_2^{\;2} - \sigma R^2\left(R + k - 1\right) \tag{55}$$

V_3 is negative definite for

$R + k > 1,\ \sigma > 0,\ c_1,\ c_2 > 0$,

hence there is local asymptotic stability.
Substituting for *u* into equation (47) produces

$$\dot{z}_3 = -c_2 z_3 \tag{56}$$

The resulting feedback system is:

$$\dot{R} = -\sigma R^2 - k\sigma R + \sigma R - z_2\sigma R - 2k\sigma R z_2 \tag{57}$$

$$\dot{z}_2 = -c_1 z_2 + \sigma R^2 + 2k\sigma R^2 \tag{58}$$

$$\dot{z}_3 = -c_2 z_3 \tag{59}$$

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V. SIMULATION RESULTS I

Simulation results of the unstable system without disturbances are shown in Figs. 1-4

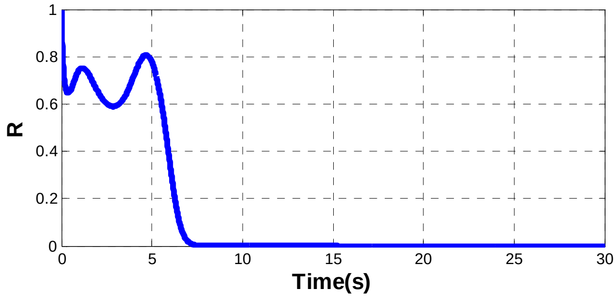

Fig. 1. Trajectory for R (without noise)

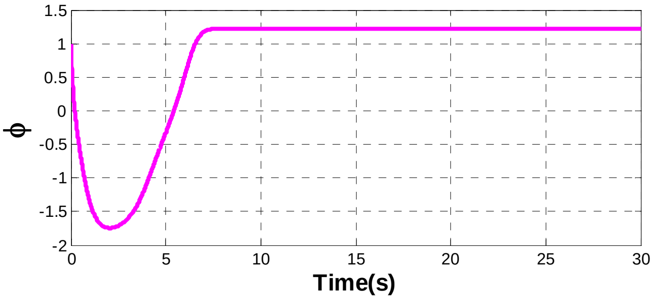

Fig. 2. Unstable trajectory for ϕ (without noise)

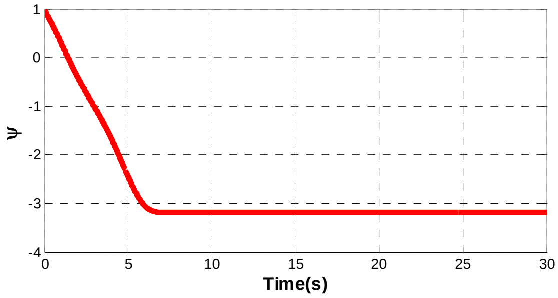

Fig. 3. Unstable trajectory for ψ (without noise)

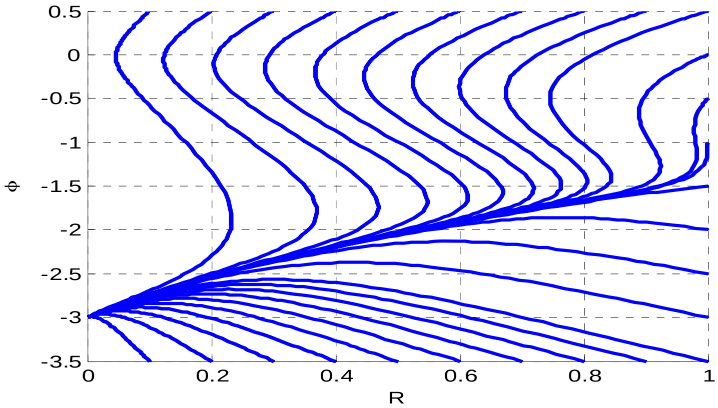

Fig. 4. Phase portrait of ϕ against R (without noise)

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Simulation results of the system when subjected to disturbances are shown in Figs. 5-7.

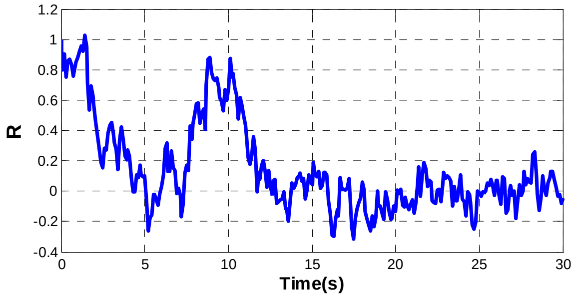

Fig. 5. Plot of R in the presence of noise (disturbances).

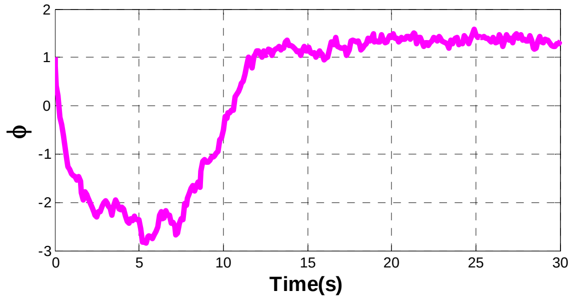

Fig. 6. Trajectory of ϕ in the presence of noise (disturbances).

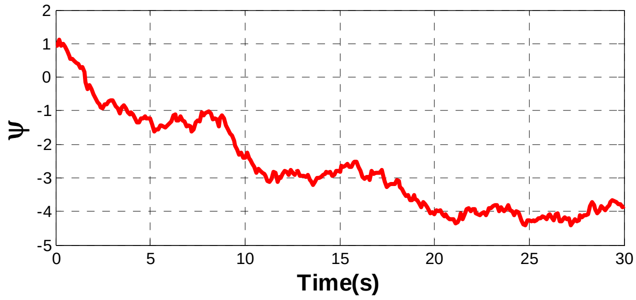

Fig. 7. Trajectory of ψ in the presence of noise (disturbances).

Simulation results of the stabilized system (without noise) are shown in Figs. 8-10.

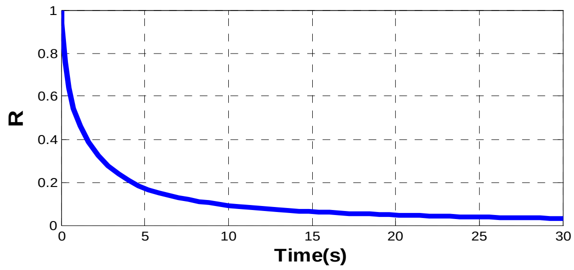

Fig. 8. Stabilized trajectory of R without noise (disturbance)

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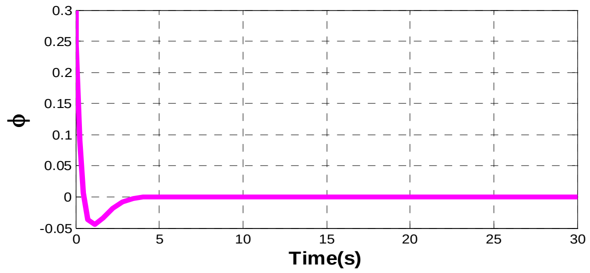

Fig. 9. Stabilized trajectory of ϕ without noise (disturbance)

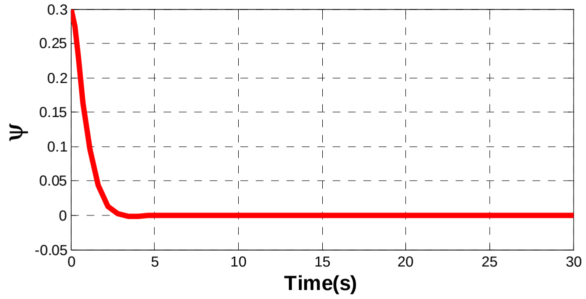

Fig.10. Stabilized trajectory of ψ without noise (disturbance)

Simulation results of the stabilized system which was subjected to disturbances are shown in Figs. 11-13.

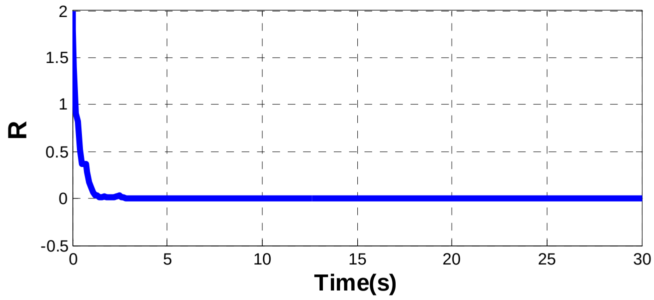

Fig.11. Stable trajectory of R with noise (disturbance) using the robust controller

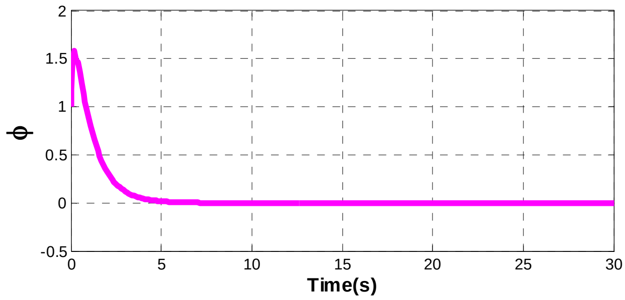

Fig.12. Stable trajectory of ϕ with noise (disturbance) using the robust controller

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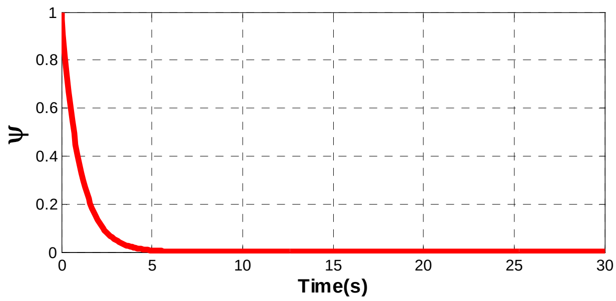

Fig.13. Stable trajectory of ψ with noise (disturbance) using the robust controller

VI. OBSERVER DESIGN (INTEGRATOR BACKSTEPPING WITH AN UNMEASURED STATE)

In observer design the following question is addressed:
Is it possible to estimate, on the basis of external information provided by passed input and output signals, the magnitude of an internal state at time, t?

A. Problem formulation for the observer

Recall the compressor equations (1-3):

$$\dot{R} = -\sigma R^2 - \sigma R(2\phi + \phi^2)$$

$$\dot{\phi} = -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - 3R\phi - 3R$$

$$\dot{\psi} = -u$$

With regard to the Jet compressor model, let us treat the pressure rise ψ as state not measurable. This state could be estimated as $\hat{\psi}$ and the state estimation error $\tilde{\psi}$ which converges exponentially to zero is estimated from:

$$\tilde{\psi} = \psi - \hat{\psi}$$

$$\psi = \hat{\psi} + \tilde{\psi} \qquad (60)$$

With the assumption of no stall (R= 0), the model is rewritten as:

$$\dot{\phi} = -\psi - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 \qquad (61)$$

$$\dot{\psi} = -u \qquad (62)$$

Hence with (65), we have:

$$\dot{\phi} = -\left(\hat{\psi} + \tilde{\psi}\right) - \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 \qquad (63)$$

$$\dot{\hat{\psi}} = -u \qquad (64)$$

$$\dot{\tilde{\psi}} = -\tilde{\psi} \qquad (65)$$

B. Observer design

In designing the observer to estimate the state ψ , $\dot{\hat{\psi}}$ is added as an observer.

Let the error variable, $z = \hat{\psi} - \alpha_1(\phi)$ (66)

$\alpha_1(\phi) = \frac{3}{2}\phi^2$ which is chosen to avoid cancellation of the useful nonlinearity, $-\frac{1}{2}\phi^3$

Due to the presence of $\tilde{\psi}$, we introduce a nonlinear damping term $-s(\phi)\phi$

Select $-s(\phi) = -d_1\phi^2$ (67)

$$\alpha_1(\phi) = \frac{3}{2}\phi^2 - d_1\phi^3 \qquad (68)$$

$$\therefore\ z = \hat{\psi} - \frac{3}{2}\phi^2 + d_1\phi^3 \qquad (69)$$

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or

$$\hat{\psi} = z + \frac{3}{2}\phi^2 - d_1\phi^3 \tag{70}$$

Substituting into (68),

$$\dot{\phi} = -z - \frac{3}{2}\phi^2 + \frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - d_1\phi^3 - \tilde{\psi}$$
$$= -z - \frac{1}{2}\phi^3 - d_1\phi^3 - \tilde{\psi} \tag{71}$$

Select Lyapunov function

$$V(\phi) = \frac{1}{2}\phi^2 \tag{72}$$

$$\therefore \dot{V} = \phi\dot{\phi} = \phi\left(-z - \frac{1}{2}\phi^3 - d_1\phi^3 - \tilde{\psi}\right)$$
$$= -\phi z - \frac{1}{2}\phi^4 - d_1\phi^4 - \phi\tilde{\psi} \tag{73}$$

Completing the square,

$$\dot{V} = -\phi z - \frac{1}{2}\phi^4 \tag{74}$$
$$-d_1\left(\phi^2 + \frac{\tilde{\psi}}{2d_1\phi}\right)^2 + \left(\frac{\tilde{\psi}}{2d_1\phi}\right)^2$$

$$\dot{V} \le -\phi z - \frac{1}{2}\phi^4 + \left(\frac{\tilde{\psi}}{2d_1\phi}\right)^2 \tag{75}$$

Since $\tilde{\psi}^2$ is the error of an exponentially converging observer, we augment the function $V(\phi)$ with a quadratic term in $\tilde{\psi}$

$$V_1(\phi, \tilde{\psi}) = V(\phi) + \frac{\tilde{\psi}^2}{2d_1\phi^2}$$

$$\dot{V}_1 = \dot{V} + \frac{\tilde{\psi}\dot{\tilde{\psi}}}{d_1\phi^2}$$

$$\dot{V}_1 = \dot{V} - \frac{\tilde{\psi}^2}{d_1\phi^2} \tag{77}$$

$$\dot{V}_1 \le -\phi z - \frac{1}{2}\phi^4 + \frac{\tilde{\psi}^2}{4d_1\phi^2} - \frac{\tilde{\psi}^2}{d_1\phi^2} \tag{78}$$

$$= -\phi z - \frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} \tag{79}$$

Hence, there would be global asymptotic stability

for z = 0

Recall, $z = \hat{\psi} - \alpha_1(\phi)$ (80)

$$\therefore \dot{z} = \dot{\hat{\psi}} - \dot{\alpha}_1(\phi)$$

$$= -u - \frac{\partial\alpha_1(\phi)}{\partial\phi}.\dot{\phi}$$
$$= -u - \frac{\partial\alpha_1(\phi)}{\partial\phi}.\left(\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \hat{\psi}\right)$$
$$-\frac{\partial\alpha_1(\phi)}{\partial\phi}.\tilde{\psi} \tag{81}$$

Let $V_2(\phi, z, \tilde{\psi}) = V_1(\phi, \tilde{\psi}) + \frac{1}{2}z^2 + \frac{\tilde{\psi}^2}{2d_2}$

$$= \dot{V}_1(\phi, \tilde{\psi}) + z\dot{z} - \frac{\tilde{\psi}^2}{d_2} \tag{82}$$

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$$\dot{V}_2 \le -\phi z - \frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} + z\begin{bmatrix} -u - \frac{\partial\alpha_1(\phi)}{\partial\phi}\left(\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \hat{\psi}\right) \\ -\frac{\partial\alpha_1(\phi)}{\partial\phi}.\tilde{\psi} \end{bmatrix} - \frac{\tilde{\psi}^2}{d_2}$$

$$= -\frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} + z\begin{bmatrix} -\phi - u - \\ \frac{\partial\alpha_1(\phi)}{\partial\phi}.\left(\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \hat{\psi}\right) \end{bmatrix} - z\frac{\partial\alpha_1(\phi)}{\partial\phi}.\tilde{\psi} - \frac{\tilde{\psi}^2}{d_2} \qquad (83)$$

The choice of control,

$$u = cz - \phi - \frac{\partial\alpha_1(\phi)}{\partial\phi}.\left(\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \hat{\psi}\right) - d_2 z\left(\frac{\partial\alpha_1(\phi)}{\partial\phi}\right)^2 \qquad (84)$$

yields

$$\dot{V}_2 \le -\frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} + z\begin{bmatrix} cz + \phi + \frac{\partial\alpha_1(\phi)}{\partial\phi}\left(\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \hat{\psi}\right) \\ +d_2 z\left(\frac{\partial\alpha_1(\phi)}{\partial\phi}\right)^2 \frac{\partial\alpha_1(\phi)}{\partial\phi}\left(\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \hat{\psi}\right) \end{bmatrix} - z\frac{\partial\alpha_1(\phi)}{\partial\phi}.\tilde{\psi} - \frac{\tilde{\psi}^2}{d_2} \qquad (85)$$

Completing the square yields:

$$= -\frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} - cz^2 - d_2 z^2\left(\frac{\partial\alpha_1(\phi)}{\partial\phi}\right)^2 - z\frac{\partial\alpha_1(\phi)}{\partial\phi}\tilde{\psi} - \frac{\tilde{\psi}^2}{d_2}$$

$$= -\frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} - cz^2 - d_2\left(z\frac{\partial\alpha_1(\phi)}{\partial\phi} + \frac{\tilde{\psi}}{2d_2}\right)^2 - \frac{\tilde{\psi}^2}{d_2} + \frac{\tilde{\psi}^2}{4d_2}$$

$$= -\frac{1}{2}\phi^4 - \frac{3\tilde{\psi}^2}{4d_1\phi^2} - cz^2 - d_2\left(z\frac{\partial\alpha_1(\phi)}{\partial\phi} + \frac{\tilde{\psi}}{2d_2}\right)^2 - \frac{3\tilde{\psi}^2}{4d_1} \qquad (86)$$

$$\dot{V}_2 \le -\frac{1}{2}\phi^4 - cz^2 - \frac{3}{4}\left(\frac{1}{d_1\phi^2} + \frac{1}{d_2}\right)\tilde{\psi}^2 \qquad (87)$$

This also shows a global asymptotic stability with V_2 negative definite
Substituting (84) into (81) yields:

$$\dot{z} = -cz + \phi + d_2 z\left(\frac{\partial\alpha_1(\phi)}{\partial\phi}\right)^2 - \frac{\partial\alpha_1(\phi)}{\partial\phi}.\tilde{\psi} \qquad (88)$$

Hence the resulting closed loop system is

$$\dot{\phi} = -z - \frac{1}{2}\phi^3 - d_1\phi^3 - \tilde{\psi} \qquad (89)$$

$$\dot{z} = -cz + \phi + d_2 z\left(\frac{\partial\alpha_1(\phi)}{\partial\phi}\right)^2 - \frac{\partial\alpha_1(\phi)}{\partial\phi}.\tilde{\psi} \qquad (90)$$

$$\dot{\tilde{\psi}} = -\tilde{\psi} \qquad (91)$$

where $\frac{\partial\alpha_1(\phi)}{\partial\phi} = 3\phi - 3d_1\phi^2$

Hence we have the resulting feedback system as:

$$\dot{\phi} = -z - \frac{1}{2}\phi^3 - d_1\phi^3 - \tilde{\psi} \qquad (92)$$

$$\dot{z} = -cz + \phi + d_2 z\left(3\phi - 3d_1\phi^2\right)^2 - \left(3\phi - 3d_1\phi^2\right).\tilde{\psi} \qquad (93)$$

$$\dot{\tilde{\psi}} = -\tilde{\psi} \qquad (94)$$

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VII. SIMULATION RESULTS II

Using the MATLAB Simulink® software to simulate the developed Observer system, the following results shown in Figs. 14-17, were obtained:

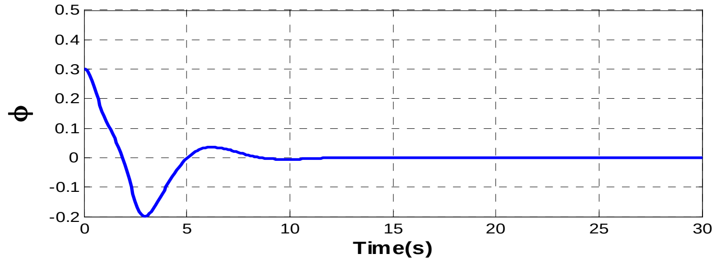

Fig. 14.Time history of ϕ (stabilized)

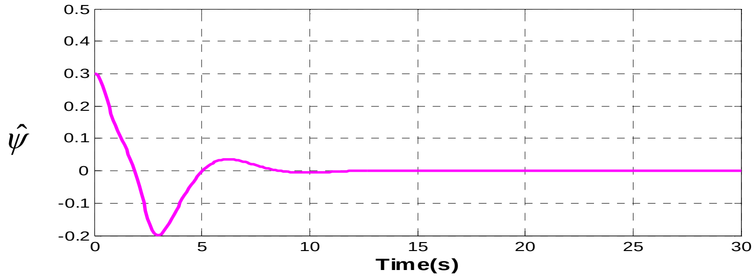

Fig. 15.Time history of the estimate state $\hat{\psi}$

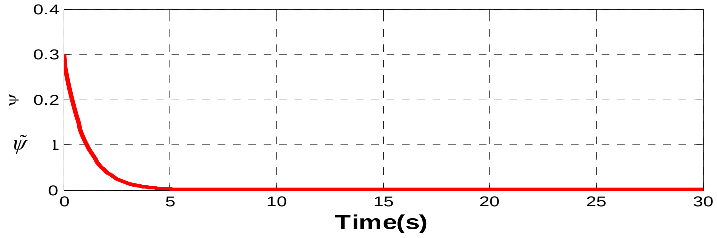

Fig. 16. Time history of the tracking error $\tilde{\psi}$

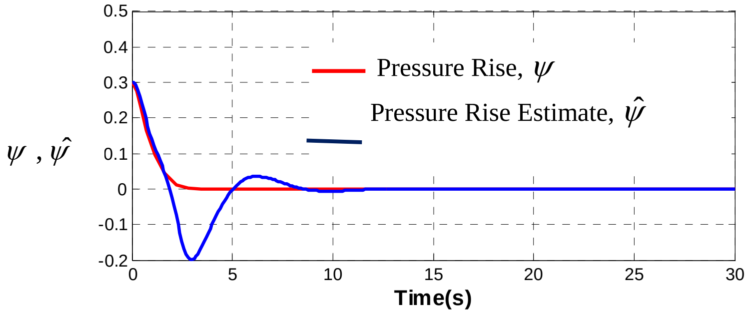

Fig. 17. Comparing ψ and $\hat{\psi}$

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VIII. DISCUSSION OF RESULTS

All the controllers developed for the jet engine compressor, were simulated using the Matlab Simulink® software. In developing the solutions, we used $\sigma = c_1 = c_2 = d_1 = d_2 = 1$. Figs. 1-3 show the trajectories for the unstable signals produced by the Moore-Grietzer three state model.

Fig. 4 shows the phase portrait of R and ϕ in the Moore-Grietzer model without noise. The phase portrait in Fig. 4, again shows that the Moore-Grietzer model for compressor is unstable.

In Figs. 5-7, it is seen that the disturbance introduced to the system by the white noise caused a significant increase (intensity) in the instability of the system.

Figs. 8-10 represent stable trajectory obtained without noise in the system. Implementing the robust controller developed on the unstable system subjected to noise signals, the results show clearly that the resulting system is stabilized as shown in Figs. 11-13.

The results for the observer developed for the system, are shown in Figs.14-17. From the simulation studies, it is seen that using the developed observer on the compressor, results in stability; for the situation where a state that cannot be measured exist, in addition to the absence stall during operation. In this case, ψ is considered as the state that cannot be measured. The tracking error shown in Fig. 16 shows the error in the state estimation dropped from 0.3 to 0 in 5s, and remained at zero as time progressed from 5s. From Fig. 17, it is seen that ψ is sufficiently estimated by $\hat{\psi}$ within the first 2 s then $\hat{\psi}$ peaks low to a value, thereafter there is accurate state estimation from 8 s onwards. Thus, the observer provides accurate estimates of the unmeasured state ψ .

IX. CONCLUSION

We have developed in this work, a robust controller to stabilize the jet engine compressor system in the presence of noise (disturbances). Simulation studies showed that the developed controller was robust to handle the stabilization of compressor system in the presence of noise (disturbances) using integrator backstepping coupled with saturators. The saturators introduced, significantly reduced the effect of noise. Also, the observer developed, accurately estimated the unmeasured state of the compressor.

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