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About the Simulation of Stochastically Excited Elastic Systems and Their Stability

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Abstract—**An efficient method for the analysis of nonlinear elastic systems under the action of parametric forces in the form of Gaussian random stationary processes is suggested. Spectral densities of input random stationary processes are assumed in the form of rational functions. The method is based on the simulation of stochastic processes and the numerical solution of differential equations, describing the motion of the system. Considering a sample of solutions, statistical characteristics of trajectories can be found. The effect of parameters of input random processes on indicated statistical characteristics is investigated. A special attention in the work is devoted to the investigation of the stability of the unperturbed motion of systems. To analyze the stability of the unperturbed motion the motion due to perturbations of initial conditions is considered. The method of the stability investigation is based on the numerical solution of differential equations, describing the perturbed motion of the system, and the calculation of top Liapunov exponents. The method results in the estimation of the stability with respect to statistical moments of different orders. It is remarked that in some cases the impose of a stochastic noise on the deterministic periodic excitation can render a stabilizing effect on the motion of elastic systems.**

Index Terms—**Elasticity, nonlinear oscillations, stability, top Liapunov exponents, random stationary processes**

I. Introduction.

The behavior of nonlinear mechanical systems, subjected to random loads in the form of random stationary processes, was considered in works [1] - [3], where an extensive enough review of investigations in the indicated direction is contained. As a rule, these investigations were developed at different restrictions. They imposed on the character of stationary processes. (For example, sometimes these processes were assumed as narrow-band processes and in this case the method for the solution was used, which is similar to the method of the harmonic balance. In other cases the level of the nonlinearity was presupposed as small. In such case the method of stochastic averaging was applied [1], [2]. For the estimation of the reliability of nonlinear systems, the Melnikov stochastic process is used in the book [3].
Some questions of the stability of elastic and viscoelastic systems, subjected to random loads in the form of random stationary process, were considered in the work [4]. There was suggested a numerical method of the solution for nonlinear problems with help of the method of canonical expansions of stationary processes. The present paper is devoted to a numerical analysis of nonlinear oscillations of elastic systems under the stochastic excitation in the form of a Gaussian stationary process with a rational spectral density. The analysis is based on the numerical simulation of the input stationary process, on a numerical solution of differential equations, describing the motion of the system, and, in a case of the stability investigation of this motion, on the calculation of the top Liapunov exponent. On the example of a plate, subjected to a random stationary load acting in the middle plane, peculiarities of the application of the proposed method are considered. A particular attention is devoted to "the interaction" of a deterministic periodic and stochastic excitations from the stability point of view of the motion. It is shown that in some cases the impose of a "colored" noise can render a stabilizing effect on the motion of elastic deterministic systems.

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II. Statement of the problem.

The dynamic behavior of an elastic system with regards to finite deflections provided that strains are small, is described by the system of nonlinear partial differential equations. Using different methods, for example, the method of finite elements, the method of finite differences, the Bubnov-Galerkin method etc., this system can be replaced by a system of ordinary differential equations. Then by the expansion of the phase space the indicated system can be substituted by a system of nonlinear differential equations of the first order

$$\dot{\mathbf{z}} = \mathbf{F}(\mathbf{z}, \alpha(t), t), \tag{1}$$

where $\mathbf{z}$ is the vector of unknowns, $\alpha(t) = \alpha^*(t) + \alpha^o(t)$, $\alpha^*(t)$ is a deterministic function, $\alpha^o(t)$ is a random stationary process, t is the time.
The point denotes the derivative with respect to time t.
The random stationary process $\alpha^o(t)$ is presupposed as a Markov process, which is a result of passing of the Gaussian white noise through a linear filter of the m - th order, i.e. the function $\alpha^o(t)$ is the solution of the stochastic differential equation

$$\alpha^{o^{(m)}} + d_1\, \alpha^{o^{(m-1)}} + .. + d_{m-1}\dot{\alpha}^o + d_m\alpha^o = h\xi(t), \tag{2}$$

where d_k, $(k = 1, 2, ..., m)$, h are constants, $\xi(t)$ is the Gaussian white noise.
For the analysis of the behavior of the system the method of statistical simulation is used, which is based on the numerical solution of differential equations (by the Runge-Kutta method) in the combination with a numerical method of obtaining of realizations of random stationary processes.

III. Oscillations of the plate under the action of the random load in the middle plane.

For the illustration of the present method let us consider transverse oscillations of a thin rectangular plate hinged along all edges and subjected to an uniformly distributed load applied in the plate plane to two opposite edges (Fig.

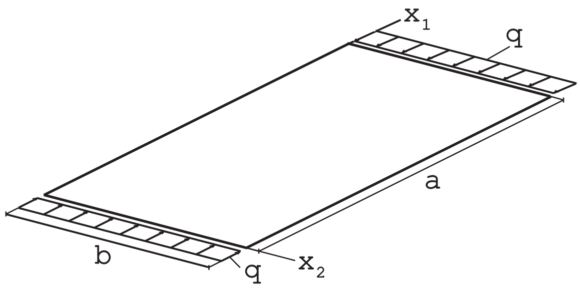

Fig. 1. The rectangular plate under loads applied to its middle plane.

1). It is presupposed that opposite edges of the plate can be removed in direction of axes x_1 and x_2, but the edges remain parallel to one another.

If the material of the plate is isotropic, then the equations of the plate oscillations for the case of finite deflections of von Karman type [5] is written in the following form

$$D\bigtriangledown^4 w - h(\Phi_{,22}w_{,11} - 2\Phi_{,12}w_{,12} + \Phi_{,11}w_{,22}) = -\gamma\ddot{w} - k\dot{w} \quad (3)$$

$$(1/E)\bigtriangledown^4 \Phi = (w_{,12}^2 - w_{,11}w_{,22}), \quad (4)$$

where w is the deflection of the plate, Φ is the function of stresses, acting in the middle surface of the plate, h is the thickness of the plate, γ is the mass per unit area of the plate, k is the damping coefficient, $\bigtriangledown^4$ is the biharmonic operator, E is the Young modulus, μ is the Poisson coefficient.

If the form of the plate is near to square and initial conditions have the form

$$w(t, x_1, x_2)|_{t=o} = f_0 \sin\frac{\pi}{a}x_1 \sin\frac{\pi}{b}x_2,$$

$$\dot{w}(t, x_1, x_2)|_{t=o} = v_0 \sin\frac{\pi}{a}x_1 \sin\frac{\pi}{b}x_2, \quad (5)$$

then the deflection of the plate can be found in the similar shape

$$w(t, x_1, x_2) = f(t)\sin\frac{\pi}{a}x_1 \sin\frac{\pi}{b}x_2. \quad (6)$$

Really, even for initial conditions, given by expressions (5), the solution of equations (3, 4) has a more complicated form. However, since we focus on the qualitative aspect of the problem rather than the quantitative one, we restrict the consideration to the function $w(t, x_1, x_2)$, given by the formula (6). Substituting relation (6) in the right-hand side of Eq. (4) and solving it with respect to the function Φ, we will obtain

$$\Phi = \frac{a^2}{32b^2}E\,f^2(t)\left(\cos\frac{2\pi}{a}x_1 + \frac{b^4}{a^4}\cos\frac{2\pi}{b}x_2\right) - \frac{qx_1^2}{2h}. \quad (7)$$

It is not difficult to prove that the boundary conditions concerning the parallelism of opposite edges are fulfilled [4]. To find the plate deflection amplitude $f(t)$ let us use the Bubnov-Galerkin method and to this end we multiply both sides of Eq. (3) by $\sin(\pi/a)x_1 \sin(\pi/b)x_2$ and integrate the resultant relation over the plate area. Then we obtain the differential equation

$$z'' + 2\varepsilon z' + z - \alpha z + \frac{3}{4}(1-\mu^2)\frac{a^4+b^4}{(a^2+b^2)^2}z^3 = 0. \quad (8)$$

Here $z = f/h, 2\varepsilon = k/(\gamma\omega), \omega$ is the fundamental frequency of plate oscillations

$$\omega^2 = \frac{D}{\gamma}\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}\right)^2, \qquad \alpha = \frac{\pi^2 q}{Db^2}\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}\right)^{-2}.$$

The prime denotes the derivative with respect to dimensionless time $t_1 = \omega t$.

If $a = b$ and $\mu = 0.3$, then Eq. (8) acquires the form

$$z'' + 2\varepsilon z' + z - \alpha z + 0.34125z^3 = 0. \quad (9)$$

Introducing new variables $z_1 = z, \quad z_2 = z'$, we replace the differential Eq. (9) by the system of first-order differential equations

$$z_1' = z_2, \quad z_2' = -2\varepsilon z_2 - (1-\alpha)z_1 - 0.34125z_1^3. \quad (10)$$

The solution of these equations should satisfy the initial conditions

$$z_1(0) = f_0/h, \quad z_2(0) = v_0/h.$$

Let us express the function $\alpha(t)$ in the form of the sum

$$\alpha(t) = \alpha_0 + \alpha_1 sin\omega t + \alpha^o(t),$$

where α_0, α_1 are deterministic constants, ω is a frequency of the deterministic periodic part of the load, $\alpha^o(t)$ is a stationary random process with zero mathematical expectation $\langle\alpha^o(t)\rangle = 0$ and the correlation function

$$K(\tau) = \sigma^2 \exp(-\delta|\tau|)\left[cos\,\theta\tau + (\delta/\theta)sin\,\theta\tau\right], \quad (11)$$

where $\tau = t_1 - t_2$, σ^2 is the dispersion of the process, δ, θ are parameters, characterizing the scale of the correlation and the frequency of the implicit periodicity respectively.

Here and further angle brackets denote the operation of the mathematical expectation.

The spectral density $S(\omega)$ in this case has the form

$$S(\omega) = \frac{2\sigma^2\delta}{\pi}\,\frac{\delta^2+\theta^2}{(\omega^2-\theta^2-\delta^2)^2 + 4\delta^2\omega^2}.$$

The considered random process is differentiable and Eq. (2) is written in the following way [5, 6]

$$\ddot{\alpha}^o - a_2\dot{\alpha}^o - a_1\alpha^o = b_2\sigma\xi(t), \quad (12)$$

where $a_1 = -(\delta^2+\theta^2)$, $a_2 = -2\delta$, $b_2 = \sqrt{2(\delta^2+\theta^2)}$, $\xi(t)$ is the Gaussian white noise, simulated by the expression

$$\xi(t) = \sqrt{2\delta/\Delta}\,\epsilon_\Delta(t).$$

Here

$$\epsilon_\Delta(t) = \epsilon_i, \quad t \in [i\Delta,\,(i+1)\Delta],$$

ϵ_i is a sequence of normally distributed uncorrelated numbers with zero mean value and $\langle\epsilon^2\rangle = 1; \quad \Delta = \Delta t$ is the step of time.

Further let us consider some results, obtained at $\varepsilon = 0.1$, $\delta = 0.5$, $\theta = \omega = 1.4$, $\Delta t = 0.1$ by means of the numerical solution of equations (12) by the fourth-order Runge-Kutta method. The number of increments n and initial conditions in all cases were assumed the same, namely, $n = 10^4$, $z_1(0) = 1.0$, $z_2(0) = 0$.

Fig. 2 shows the most typical trajectories of the plate motion on the phase plane $z_1 \sim z_2$. For each of these trajectories the limits of the variation of values z_1 and z_2 are indicated. These results can be explained as follows.

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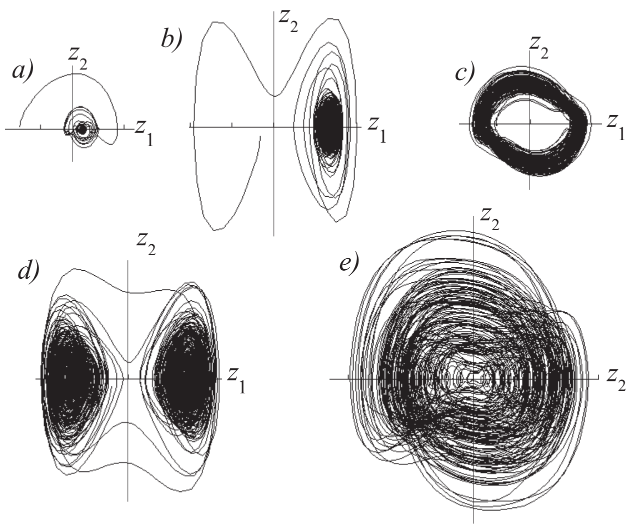

Fig. 2. Trajectories of the plate motion in the phase plane.

If the load is deterministic and constant with time, then the quantity z, corresponding to an equilibrium state of the plate, can be found from the cubic equation

$$(1-\alpha_0)z + 0.34125z^3 = 0. \tag{13}$$

If $\alpha_0 < 1$, then this equation has only one root $z = 0$, which corresponds to the unbent equilibrium configuration of the plate, and this state is known to be stable.
If $\alpha_0 > 1$, then Eq. (13) has three roots

$$z_{(1)} = 0, z_{(2),(3)} = \pm\sqrt{(\alpha_0 - 1)/0.34125}.$$

It can be shown, that the solution $z_{(1)}$ is unstable and nontrivial solutions $z_{(2)}$ and $z_{(3)}$ are stable. The quantity $\alpha_0 = 1$ is critical for an elastic plate.
The results for the dynamic (deterministic or stochastic) statement of the problem are similar. Indeed, if parameters α_0, α_1, σ are sufficiently small, then, obviously, oscillations perform in the neighborhood of the trivial equilibrium state. This statement is confirmed by plots in Fig. 2,a, obtained at $\alpha_0 = 0$; $\alpha_1 = 1.0$; $\sigma^2 = 0.09$ and they damp with time.
But if the same parameters α_0, α_1, σ are sufficiently large, then the plate motion becomes much more versatile. The plate can oscillate in a neighborhood of a certain equilibrium (Fig. 2, b at $\alpha_0 = 1.0$; $\alpha_1 = 0$; $\sigma^2 = 0.04$) or jump between two equilibria (Fig. 2, c at $\alpha_0 = 0.5$; $\alpha_1 = 1.0$; $\sigma^2 = 0.01$) and (Fig. 2, d at $\alpha_0 = 0.5$; $\alpha_1 = 0.5$; $\sigma^2 = 0.25$); moreover, in some cases the plate motion is chaotic (Fig. 2, e at $\alpha_0 = 1.0$; $\alpha_1 = 1.0$; $\sigma^2 = 0.25$). It should be said that in such cases with increasing of time the solution of nonlinear equations becomes steady-state and moreover different initial conditions may lead to different steady-state solutions.
The variation of the deflection z_1 with time (Fig. 4), which correspond to the same plate (Fig. 2, e), is presented in Fig. 3.

In Fig. 4 the histogram of the value z_1 is shown, corresponding to the same input data and obtained at $n = 10^5$ ($\langle z_1 \rangle$ and σ_1 are the mean value and the mean square scattering of z_1).

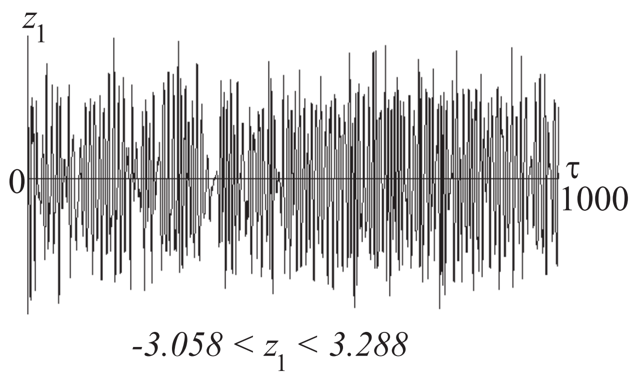

Fig. 3. Realizations of the generalized displacement $z_1 = f(t)$.

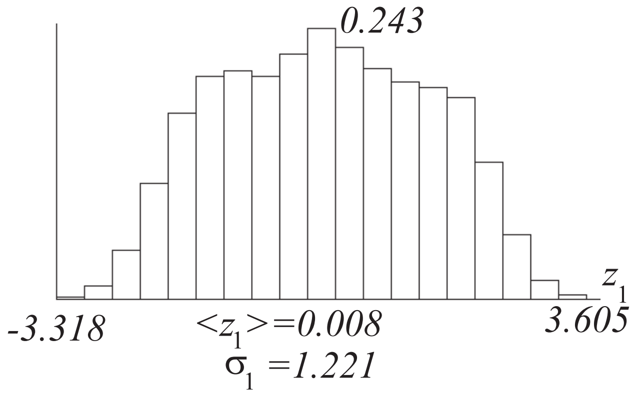

Fig. 4. The histogram of the value z_1 .

IV. Stability of nonperturbed motion of the system.

For the analysis of the stability of nonperturbed motion of the system, describing by Eq. (1), let us consider the perturbed motion, caused by perturbations of initial conditions. In a case of the perturbed motion the solution of equations (1) has the form

$$\mathbf{y} = \mathbf{z} + \delta\mathbf{z},$$

i. e.

$$\dot{\mathbf{y}} = \mathbf{F}(\mathbf{y}, \alpha(t), t), \tag{14}$$

where $\mathbf{z} = \{z_1, z_2, ..., z_n\}^T$ is the vector of unknowns, corresponding to unperturbed motion of the system, the vector $\mathbf{y} = \{y_1, y_2, ..., y_n\}^T$, corresponding to the perturbed motion and the vector of perturbations $\delta\mathbf{z} = \{\delta z_1, \delta z_2, ..., \delta z_n\}^T$.
Let us expand the right-hand side of Eq. (14) in the Taylor series in the neighborhood of the solution $\mathbf{z}(t)$

$$\mathbf{F}(\mathbf{z} + \delta\mathbf{z}, \alpha(t), t) = \mathbf{F}(\mathbf{z}, \alpha(t), t) + \mathbf{F}'(\mathbf{z}, \alpha(t), t)\delta\mathbf{z} + ...$$

Restricting in this expansion to two first terms and taking into account Eq. (1), we obtain a linearized equation

$$\delta\dot{\mathbf{z}} = \mathbf{F}'(\mathbf{z}, \alpha(t), t)\delta\mathbf{z}. \tag{15}$$

The solution of the equation should satisfy to initial conditions $\delta\mathbf{z}(0) = \delta\mathbf{z}_0, \delta\mathbf{z}_0 = \{\delta z_{01}, \delta z_{02}, ..., \delta z_{0n}\}^T$.
For the estimation of the stability of the system the method of top Liapunov exponent is used, which is calculated for each couple of realizations - $\alpha(t)$ and of the corresponding realization of the process $\mathbf{z}(t)$.
Now, many different definitions of the stability of stochastic

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systems are well known. Further, the stability with respect to statistical moments is considered.
(i) The solution $\delta\mathbf{z}(t) \equiv 0$ is called p - stable, if for any $\varepsilon > 0$, such $\Delta > 0$ can be found, that at $t \geq 0$ and $|\delta z_i(0)| < \Delta$, $(i = 1, 2, \ldots n)$, where n is the number of differential equations of the first order in the system of equations (15)

$$|\langle \delta z_i^p(t) \rangle| < \varepsilon.$$

(ii) The solution $\delta\mathbf{z}(t) \equiv 0$ is called asymptotically p - stable, if it p - stable and, in addition, for a small enough $|\delta z_i(0)|$, $(i = 1, 2, \ldots n)$

$$\lim_{t\to\infty} |\langle \delta z_i^p(t) \rangle| = 0.$$

At $p = 1$ the stability in the mean takes place (with respect to the mathematical expectation), and at $p = 2$ - stability in the mean square.
The growth of the vector $\delta\mathbf{z}(t)$ can be estimated with help of the top Liapunov exponent λ, which is defined by the expression

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \ln \frac{\|\delta\mathbf{z}(t)\|}{\|\delta\mathbf{z}(0)\|},$$

where $\|\delta\mathbf{z}(t)\|$, $\|\delta\mathbf{z}(0)\|$ are norms of the vector $\delta\mathbf{z}(t)$ in the Euclidean space at instants t and $t = 0$.
The value λ can be found numerically with help of the method, proposed in the paper [7]. With this purpose let us divide a large enough time interval $[0, t]$ for m equal intervals $\Delta t = t_{j+1} - t_j$, $(j = 1, 2, \ldots m)$.
Let us assume, that the system (15) is deterministic and at $t = t_j$ the norm of the vector $\|\delta\mathbf{z}(t_j)\|$ is equal to unit. Using this vector as the vector of initial conditions, let us obtain the solution of the system (15) for the instant t_{j+1} with the norm $\|\delta\mathbf{z}(t_{j+1}\| = d_{j+1}$. Continuing the solution of the system (15) with new initial conditions $\delta z_{i0}(t_{j+1})/d_{j+1}$, we will find the sequence of values d_j and then the top Liapunov exponent can be found as the limit

$$\lambda = \lim_{m\to\infty} \frac{1}{m\Delta t} \sum_{i=1}^{m} \ln d_j. \tag{16}$$

Because the system of equations for statistical moments of functions $\delta z_i(t)$ in a case of "colored" noises $\alpha^o(t)$ can't be obtained in a closed form, let us use for their find by the method of statistical simulation [4], [8].
The estimation of statistical moments $< \delta z_i^p >$ for instants t_j can be obtained as a result of statistical averaging of values $\langle \delta z_i^p \rangle$, found from equations (15) for sufficiently large number of realizations q

$$\langle \tilde{\delta} z_i^p(t_j) \rangle = \frac{1}{q} \sum_{m=1}^{q} [\delta z_i^p(t_j)]^{(m)} \tag{17}$$

where $[\delta z_i^p(t_j)]^{(m)}$ is the quantity $\delta z_i^p(t_j)$, corresponding to *m*-th realization of the solution of Eq. (15).
Let us presuppose, that the norm of the vector

$$\langle \delta\mathbf{z}^p(t_k) \rangle = \{\langle \delta z_1^p(t_k) \rangle, \langle \delta z_2^p(t_k) \rangle, \ldots, \langle \delta z_n^p(t_k) \rangle\}^T$$

in the Euclidean space for the instant t_k is equal to unit. The norm of the vector $\langle \delta\mathbf{z}(t)^p \rangle$ becomes equal to $\tilde{d}_{k+1}$ at the instant $t_{k+1} = t_k + \Delta t$. Further the system of equations (15) is solved for each realization of the matrix $\mathbf{F}'(\mathbf{z}, \alpha(t), t)$ with initial conditions

$$\delta z_{i0}(t_{k+1}) = \delta z_i(t_{k+1}) / \left(\tilde{d}_{k+1}\right)^{1/p}.$$

Repeating the stated procedure we obtain the sequence of values $\tilde{d}_k$, with help of which the estimation of the top Liapunov exponent can be found

$$\tilde{\Lambda} = \lim_{l\to\infty} \frac{1}{l\Delta t} \sum_{k=1}^{l} \ln \tilde{d}_k.$$

Let us illustrate general arguments about the investigation of the stability of geometric nonlinear systems on the example of the plate, which was considered earlier.
Eq. (8), linearized with respect to perturbations δz, has the following form

$$\delta z'' + 2\varepsilon \delta z' + 1.02375\, z^2 \delta z = 0. \tag{18}$$

Introducing new variables $z_5 = \delta z$, $\quad z_6 = \delta z'$, let us write the equation (18) as a system of first-order differential equations

$$z_5' = z_6, \quad z_6' = -2\varepsilon z_6 - (1-\alpha) z_5 - 0.34125\, 3 z_1^2 z_5, \tag{19}$$

If the plate subjected to a deterministic periodic load, then at some values of input parameters the motion of the plate can be chaotic (unstable) [9]. It is confirmed by the trajectory of the motion of the elastic plate on the phase plane $z_1 \sim z_2$, shown in Fig. 5,a, which is obtained at $\varepsilon = 0,1$; $\alpha_0 = 0.5$; ; $\alpha_1 = 2.0$; $\omega = 1,4$, $\Delta t = 0.1$ and the number of increments $n = 10^4$.

The value of the top Liapunov exponent at $t = 6.10^4$ is equal to $\lambda = 0.165$ (Fig. 6,a), which confirms the chaos (the instability) of the plate.
If we will impose on the deterministic load a random noise in the form of Gaussian stationary process $\alpha^o(t)$ with characteristics $\sigma^2 = 0.01$; $\delta = 0.5$; $\theta = 1.4$, then the plate becomes stable with respect to statistical moments of the first (Fig. 6, b) $\lambda_1 = -0.082$ and of the second order (Fig. 6,c) $\lambda_2 = -0.164$. These estimates of the top Liapunov

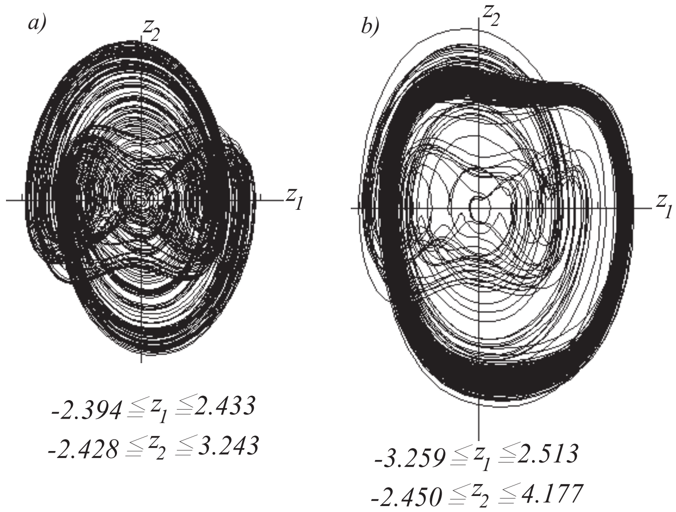

Fig. 5. Trajectories of the elastic plate motion in the phase plane at deterministic (a) and stochastic (b) treatments of the problem.

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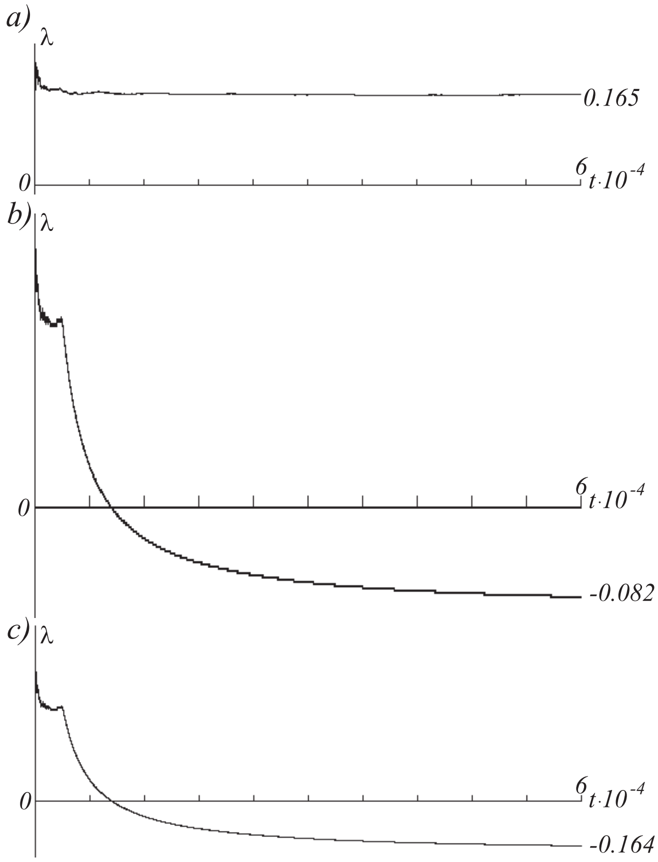

Fig. 6. Changes in the estimate of the top Liapunov exponent for the elastic plate at deterministic (a) and stochastic (b, c) treatments of the problem.

exponents are obtained as a result of averaging of results at 20 realizations and at $t = 6.10^4$. One of realizations of trajectories on the phase plane in this case, obtained at the number of steps in time $n = 10^4$, is shown in Fig. 5, b.
If we increase the time interval to 10^5, then the same values assume quantities $\lambda_1 = -0.088$ and $\lambda_2 = -0.180$. At last, if the number of realizations is assumed equal to 40, then at $t = 10^5$ we will find $\lambda_1 = -0.087$ and $\lambda_2 = -0.177$.
Let us remark, that the stabilizing effect of the unstable system was discovered for the first time at the investigation of the stability of the first-order differential equation [10], [11]

$$\dot{X} = (b + \sigma\xi)X, \tag{20}$$

where b, σ are constants, ξ is a Gaussian white noise in Ito's sense .
It was shown, that the trivial solution of the equation (20) stable at $b < \sigma^2/2$, what was estimated as a contradicted to physical intuition [11]. On this base the conclusion was made that the white noise in Ito's sense is "physically unrealizable".
If $\xi(t)$ is a white noise in Stratonovich's sense, then the effect of the stabilization disappears [11] and it was remarked, that the unstable deterministic system $\dot{x} = bx(b > 0 - const)$ can't be stabilized by "physically realizable" perturbation of its parameter.
However, in the same work Khasminskii R.Z. [11] showed, that in systems of differential equations of the more higher order the white noise in Stratonovich's sense can render too a stabilizing effect on an unstable deterministic system. Particularly, the trivial solution of the equation of the 2-th order

$$\ddot{z} + (k + \sigma\xi)\dot{z} + \omega^2 z = 0,$$

where k, ω^2 are constant, at definite quantities of the parameter σ can be stable, although the same solution of the deterministic equation (at the same magnitudes of values k, ω^2)

$$\ddot{z} + k\dot{z} + \omega^2 z = 0$$

is unstable.
It should be said, that white noises in Ito's and in Stratonovich's sense are mathematical idealizations, which can't be realized physically (a physical process with an unlimited power doesn't exist). From this point of view the smooth in the mean square process, used in the present paper, can be consider as a physically realizable process.
By Arnold L. with co-workers [12] it were shown that deterministic differential equations can be stabilized (in sense of the almost sure stability) by stochastic wide-band stationary processes. The result, obtained for the plate in the present paper, demonstrates that a "physically realizable" ("colored") noise can render a stabilizing effect (in sense of the stability with respect to statistical moments) at the analysis of the stability of elastic systems, the motion of which is described by nonlinear differential equations.

V. Conclusion

In the present paper an effective method of the investigation of nonlinear oscillations and of the stability of elastic systems at stochastic excitations is proposed. Loads are assumed in the form of Gaussian random stationary processes with rational spectral density ("colored" noises). The method is based on the numerical simulation of random processes, the numerical solution of differential equations, describing the motion of the considered system, and on the calculation of the top Liapunov exponent. It is remarked that in some cases the addition of a stochastic noise on the deterministic periodic excitation can render a stabilizing effect on the motion of elastic plates.

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