Individual-Based Model to Simulate Crowd Dynamics Using Artificial Potential Fields

Mohamed H. Mabrouk

Abstract — **Crowd dynamics has been investigated for several years by many researchers. Computer based models prove to offer the optimum way to simulate the crowd dynamics especially in situations that need simulation of realistic behaviour such as in the emergency situations. In this paper, an individual-based model to simulate the behaviour of human crowd is developed. Using the artificial potential fields, a new approach to link the behaviour of each member in the crowd and the components of the environment is presented. Generalized Morse potential (GMP) is used to describe the interaction forces between each member and the environment on one side and amongst the crowd members on the other side to add realistic flavour to the predicted crowd behaviour along with simple calculations that are required especially for crowds with large number of individuals. Numerical results that match the observations of human crowd behaviour prove the efficiency and applicability of the model .**

***Index Terms*— Pedestrian Dynamics, Agent-Based Modeling, Artificial Potential Fields, Generalized Morse Potential, Lyapunov stability.**

I. INTRODUCTION

The study of human crowd dynamics has recently become of growing interest, as the population grows dramatically in many countries. This fact drags the attention of researchers in the field to the need for developing reliable models to predict the behaviour of crowds especially in critical situations such as evacuation processes [1-5].

Pedestrian motion has been investigated using many approaches. Some used the approaches that count on empirical data and observation to help further investigation in the field [2,6-8], others used the computer based modeling approaches [9-11] as the computer model has the ability to be re-run many times, altering parameters to evaluate different situations. Generally, the individual-based modeling approach, in which crowd members behave individually, becomes more practicable as computer power and storage capacity has recently increased.

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As the majority of the available studies investigated a crowd as a collection of isolated individuals, each having his own speed, direction of motion, and destination point [1, 9, 12, 13].

The computer based models that predict the evacuation of a group of pedestrian from a room demonstrates a noticeable transition from aggregate to individual-level modeling. The term ‘group’ refers in such models to number of pedestrians that happen to exist close to each other rather than individuals who have social ties such as friends or family members. The behaviour according to social ties amongst the crowd members is considered out of scope of this paper and is currently under the investigation as a future work.

Most recently, more investigations are based on the ‘physics’ of crowd motion [14-16]. The physical definition of interaction force is “the negated gradient of the potential” [17]. Different forces are associated with specific potentials such as the Columb potential, van der Waals potential, generalized Morse potential (GMP) and the Lennard-Jones potential. The idea that many physical systems attempt to relax their configuration to a minimum-energy state is employed using the artificial potential field method for describing the interactions amongst members in crowds as will be explained in the following section.

II. THE INDIVIDUAL-BASED MODEL

The artificial potential field method (APF), introduced by [18], has been an important method for describing the self-driven forces of multi-agent systems [19-21]. In APF, both attractive and repulsive fields are formed separately and then added to form a global potential field. Many types of potentials can be used for both the attractive and repulsive potentials. GMP is preferable because its exponentially decaying nature defines a realistic way of constructing the field [22] as well as its simpler computations required especially when dealing with a large number of crowd members. These two main advantages of the GMP also make it preferable for repulsive forces. Although the artificial potential field method has been used to describe the interactions amongst groups of interacting particles by many researchers [22-24], fewer attempts were presented in the field of pedestrian dynamics which either did not present realistic way to define the interactions amongst the crowd members [25-27] or were based on methods that demand high calculations especially for the large crowds [8, 28-30].

To define the model components which determine behaviour of each individual in each time period of the simulation and to achieve a non-colliding individual motion neither with the obstacles nor with other individuals, the potential fields are defined by assigning repulsive potentials (repulsive forces) to the obstacle positions and other crowd members while assigning attractive potentials (attractive forces) to the goal positions.

The model is considered here for the i^{th} individual with mass m_i, position $\mathbf{r}_i$ and velocity $\mathbf{v}_i$. A dissipative friction force with coefficient β_i is added to control the i^{th} individual's speed. The global potential affecting the i^{th} individual is characterised by attractive goal and repulsive obstacle potential fields of strength C_{ig} and C_{io} with ranges l_{ig} and l_{io} respectively and an individual interaction potential function $V_{interaction}(\mathbf{r}_i)$ that includes only the individual's repulsive potential field of strength C_{ri} with range l_{ri}. This means of defining the interaction amongst the individuals define each individual to have no attractive force to any of the other individuals in the environment which matches former approaches to study the pedestrian dynamics [31].

The equations of motion of the i^{th} individual are as follows:

$$\mathbf{v}_i = \dot{\mathbf{r}}_i \tag{1}$$

$$m_i \dot{\mathbf{v}}_i = -\beta_i \mathbf{v}_i - \nabla_i V_{global}(\mathbf{r}_i) \tag{2}$$

where the global potential corresponding to the i^{th} individual, $V_{global}(\mathbf{r}_i)$, is defined as follows:

$$V_{global} = V_{interaction} + V_{obstacles} + V_{goals} \tag{3}$$

The obstacles potentials is defined as:

$$V_{obstacles}(\mathbf{r}_{io}) = \sum_{z=1}^{N_o} C_{io_z} e^{-|\mathbf{r}_i - \mathbf{r}_{o_z}|/l_{io_z}} \tag{4}$$

$$V_{goals} = -\sum_{k=1}^{N_g} C_{ig_k} e^{-|\mathbf{r}_i - \mathbf{r}_g|/l_{ig}} \tag{5}$$

$$V_{ineraction}(\mathbf{r}_i) = \sum_{j \neq i}^{N_p} C_{r_j} e^{-|\mathbf{r}_i - \mathbf{r}_j|/l_{r_j}} \tag{6}$$

Therefore the global potential is defined as:

$$V_{global}(\mathbf{r}_i) = \sum_{j \neq i}^{N_p} C_{r_j} e^{-|\mathbf{r}_i - \mathbf{r}_j|/l_{r_j}} + \sum_{z=1}^{N_o} C_{io_z} e^{-|\mathbf{r}_i - \mathbf{r}_{o_z}|/l_{io_z}} - \sum_{k=1}^{N_g} C_{ig_k} e^{-|\mathbf{r}_i - \mathbf{r}_g|/l_{ig}} \tag{7}$$

where N_o is the total number of point obstacles that constitute the boundaries (walls), N_p is the number of crowd members, N_g is the total number of goals (destination points), $\mathbf{r}_{gk}$ is the k^{th} goal position and $\mathbf{r}_{oz}$ is the z^{th} obstacle point position. Eq. (1-3) have the advantages of taking the physical terms of velocity and acceleration into account as well as the relatively simple required calculations. This way of defining the model elements gives the model the advantages of adaptation to real situations especially in crisis situation, e.g evacuation situations, in addition to the employment of Morse potential to define the social forces amongst the individuals that gives more realistic flavour to the model. This is the key concept to be developed in this paper.

The main elements of the model include the environment in which the individual navigates, the individual's perception about the environment, and the individual's behaviour. Utilising perception techniques in biological systems can be useful to increase the perception of the individual about the environment [32], which is simplified here by representing the repulsion potential range (l_{io}) affecting the i^{th} individual as a function of an obstacle constant (l_o), which characterises the physical nature of the obstacle, and the individual repulsion potential range (l_{ri}) that characterises the individuals minimum distance to its nearest neighbours.

This means of defining (l_{io}) increase each individual's perception about the obstacles (boundaries and other individuals) in the environment, while the repulsion potential strength affecting the i^{th} individual (C_{io}) can be represented as the obstacle constant (C_o) for simplification. Also for simplification, the attraction potential range of the goal affecting the i^{th} individual (l_{ig}) can be represented as a function of the goal constant (l_g), which characterises the physical nature of the goal, while the attraction potential strength of the goal affecting the i^{th} individual (C_{ig}) can be represented as the goal constant (C_g) such that

$$C_{io} = C_o \tag{8}$$

$$l_{io} = l_o + l_{ri} \tag{9}$$

$$C_{ig} = C_g \tag{10}$$

$$l_{ig} = l_g \tag{11}$$

III. Stability analysis

Now, the approach from [32] is adopted to prove the stability of a system of interacting particles where the system members tend to relax into a minimum energy state [33].

A. Model analysis for obstacle free scenario

Considering the potential field defined in Eq. (3-7), the potential field is now a function of the individual interaction parameters. According to the equations of motion and assuming unit mass, the equation of motion of a single individual i at position $\mathbf{r}_i$ is:

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$$\ddot{\mathbf{r}}_i = -\beta_i \mathbf{v}_i - \nabla_i V_{global}(\mathbf{r}_i) \tag{12}$$

where $V_{global}(\mathbf{r}_i)$ is the global potential that affects the i^{th} individual. From Eq. (3) – Eq. (7) and for an obstacle free system, the global potential function for a single individual at position $\mathbf{r}_i$ attracted by a goal is defined as

$$V_{global} = -C_g e^{-|\mathbf{r}_{ig}|/l_g} \tag{13}$$

Then, the system effective energy ϕ, for N_p members of the system that behave individually, will be defined as follows

$$\phi = \frac{1}{2}\mathbf{v}_i^2 + V_{global} \tag{14}$$

so that its time derivative is

$$\dot{\phi} = \mathbf{v}_i.\dot{\mathbf{v}}_i + \nabla_i V_{global}.\mathbf{v}_i \tag{15}$$

Substituting from Eq. (2) in Eq. (15), it can be seen that

$$\dot{\phi} = -\beta \mathbf{v}_i^2 \tag{16}$$

Since $\beta > 0$, it is clear that $\dot{\phi} < 0$ and therefore the system is *Lyapunov* stable [32-33].

B. Model analysis for problem scenario

According to Eq. (1-7), and for a single individual at position $\mathbf{r}_i$, it can be seen that

$$\ddot{\mathbf{r}}_i = -\beta \mathbf{v}_i - \nabla_i V_{goal}(\mathbf{r}_{ig}) - \nabla_i V_{obstacles}(\mathbf{r}_{io}) \tag{17}$$

where $V_{goal}(\mathbf{r}_{ig})$ is the potential field of a single goal and $V_{obstacles}(\mathbf{r}_{io})$ is the obstacles potential field that affect the i^{th} individual. However, from Eq. (4-7) it can be seen that

$$V_{obstacles}(\mathbf{r}_{io}) = \sum_{z=1}^{N_o} C_{io_z} e^{-\left|\mathbf{r}_i - \mathbf{r}_{o_z}\right|/l_{io_z}} \tag{18}$$

where N_o is the number of obstacles. Then, from Eq. (18) it can be seen that

$$\nabla_i V_{obstacles}(\mathbf{r}_{io}) = -\sum_{z=1}^{N_o} \frac{C_{io_z}}{l_{io_z}} e^{-\left|\mathbf{r}_i - \mathbf{r}_{o_z}\right|/l_{io_z}} \hat{\mathbf{r}}_{io_z} \tag{19}$$

Noting that C_{io} is given very small values compared to C_g to avoid formation of local minimum inside the trap [18], then the term $\nabla_i V_{obstacles}(\mathbf{r}_{io}) \approx 0$, then Eq. (17) will approximate as

$$\ddot{\mathbf{r}}_i + \beta \dot{\mathbf{r}}_i = -\nabla_i V_{goal}(\mathbf{r}_{ig}) \tag{20}$$

which is the equation for a damped oscillator (due to the linear dissipation term) with an external forcing term generated by the goal [34]. This indicates the tendency of the individual to move to the goal position and then come to rest.

IV. Numerical Results

Models to simulate crowd dynamics during emergency situations are considered the most recent amongst the crowd dynamics simulation attempts [2, 4, 12, 35-36]. From the observations recorded and presented in [1], the behaviour of people in panic situations is irrational due to the fact that people get nervous, therefore they try to escape in a speedy random fashion, which is too unmanageable to be predicted and is considered out of the scope of this paper.

However, if people do not panic in emergency situations, their behaviour at bottlenecks will be different. They seem to follow the crowd members, who succeed to pass the neck, with higher speed than that in the normal conditions. Also the number of pushing pedestrians becomes less with time as the crowd members pass the neck which increases the chance to evacuate the room in lower time than that in the normal conditions. All the aforementioned characteristic features are taken into consideration during the design of the model presented in this paper by linking the behaviour of each individual to the environment (destination point, boundaries, and other individuals in the crowd) using Eq. (3) – Eq. (11). Also using the potential fields to define a destination point for every individual makes them all have the same desire to reach their destinations at the same time giving realistic flavour to the predicted behaviour. In order to illustrate the use of the model in a simple way, we will investigate a problem of low population crowd members, from random initial positions and velocities, to evacuate a single-exit room in both normal and emergency situations.

In normal situation the motive of the individuals to leave the room is not strong, therefore their average speeds are low and there is no pushing among them which means that each individual keeps a certain distance from its nearest neighbour in the crowd. Potential fields parameters should be chosen in a way that matches the situation features. A moderate attraction potential goal point outside the room that affects all the individuals, moderate dissipation factor, and low repulsion potential amongst the individuals are chosen. On contrary, when crowd individuals attempt to leave the room under emergency situation, a stronger attraction potential goal point that affects all the members of the crowd, lower dissipation factor, and high repulsion potential amongst the individuals are chosen.

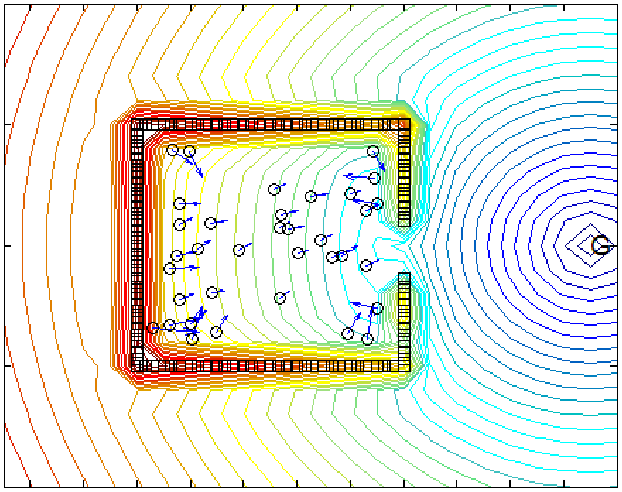

Fig.1.a. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, $t = 5$

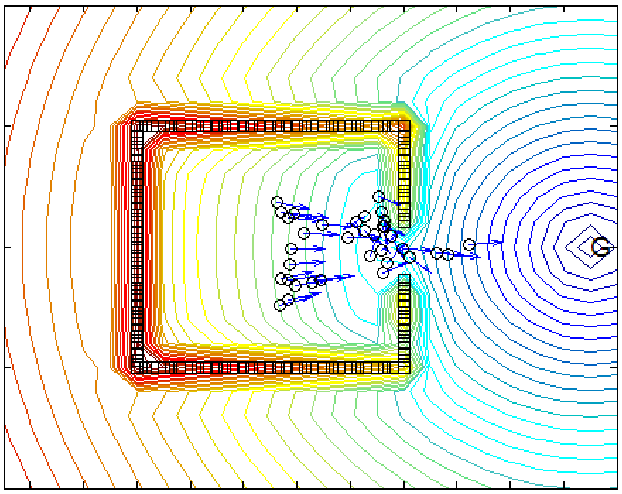

Fig.1.b. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, $t = 32$

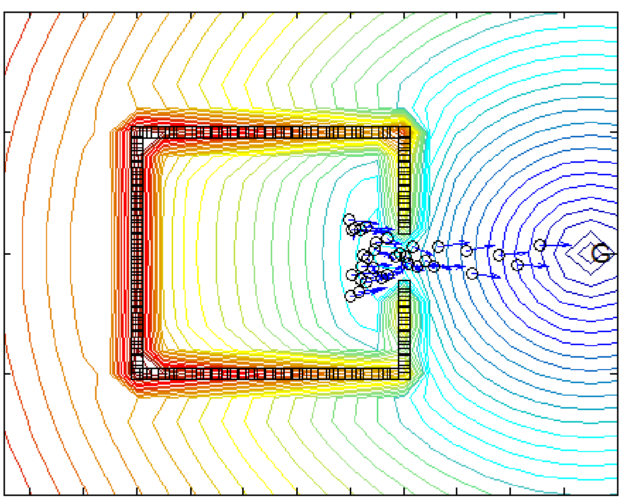

Fig.1.c. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, $t = 50$

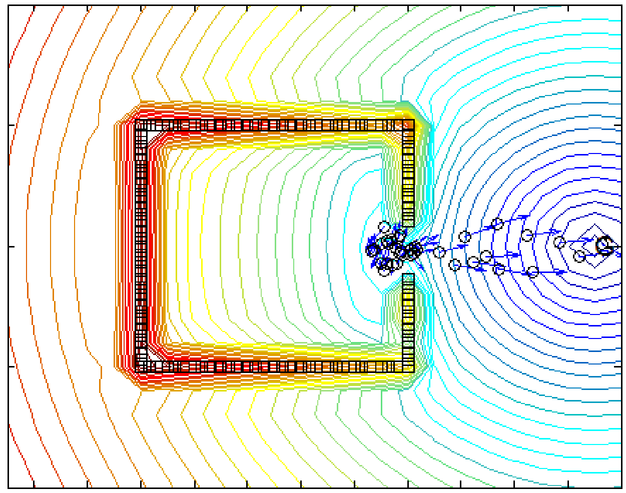

Fig.1.d. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, $t = 65$

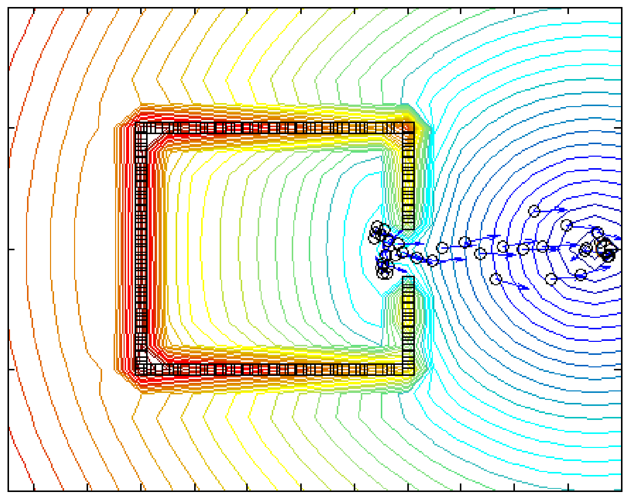

Fig.1.e. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, $t = 93$

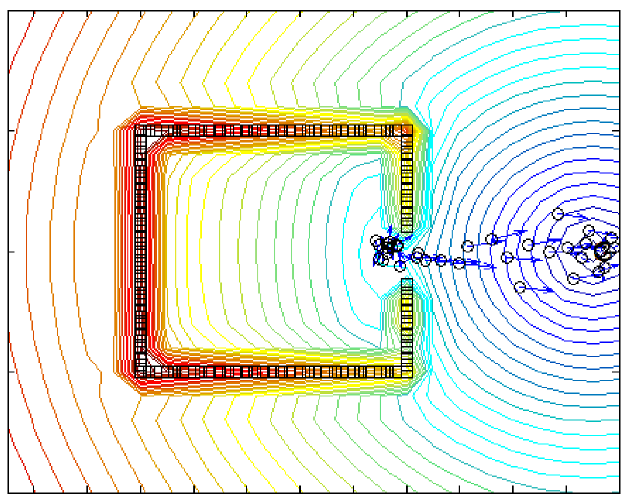

Fig.1.f. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, $t = 101$

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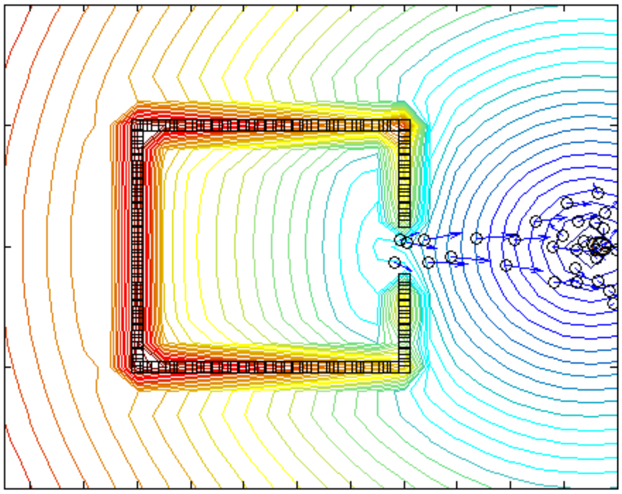

Fig.1.g. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, t = 150

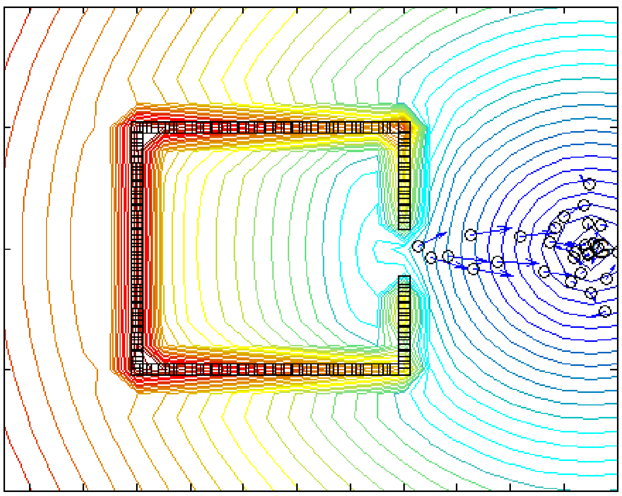

Fig.1.h. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, t = 165

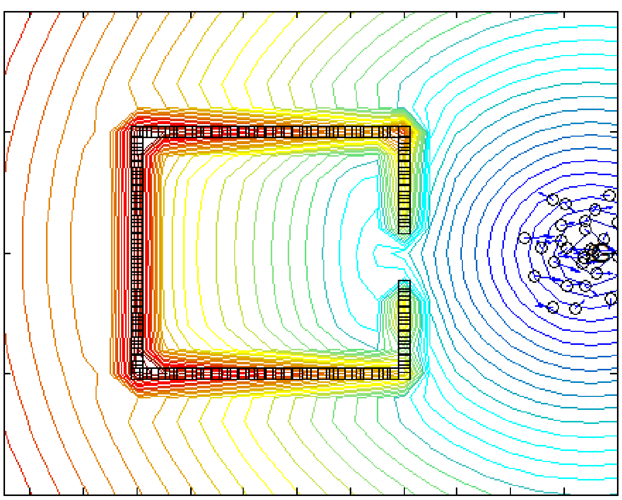

Fig.1.i. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions, t = 200

Fig.1. Simulation of crowd members that attempt to leave a trap to a destination point (G) in normal conditions.

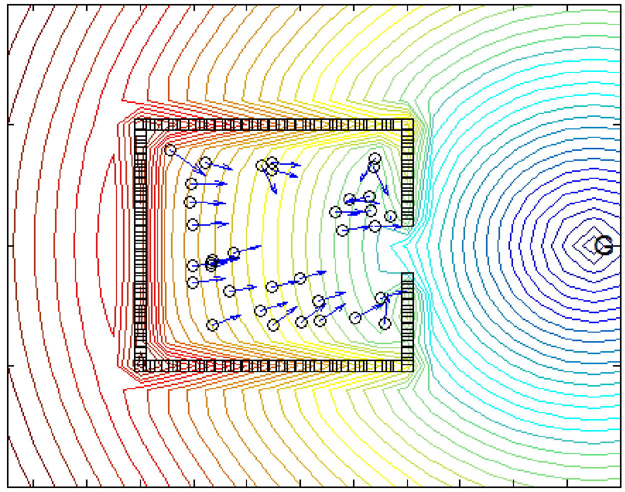

Fig.2.a. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, t = 5

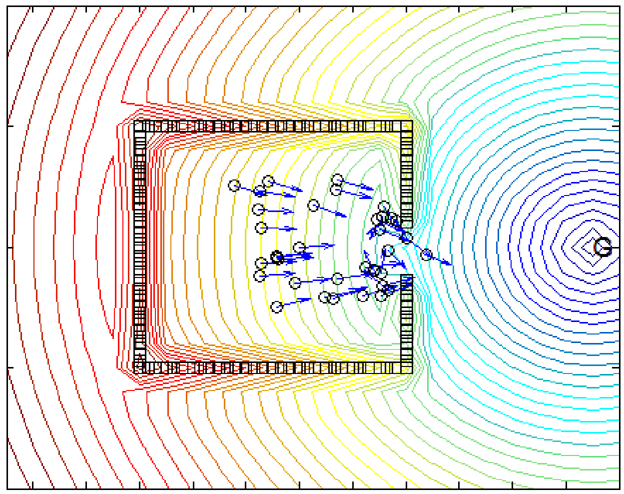

Fig.2.b. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, t = 15

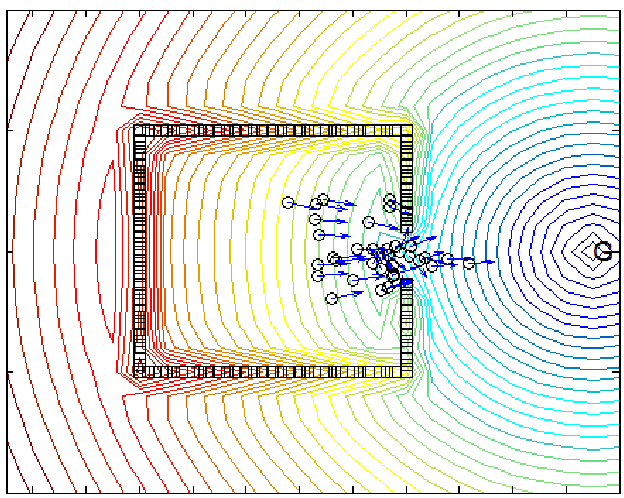

Fig.2.c. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, t = 25

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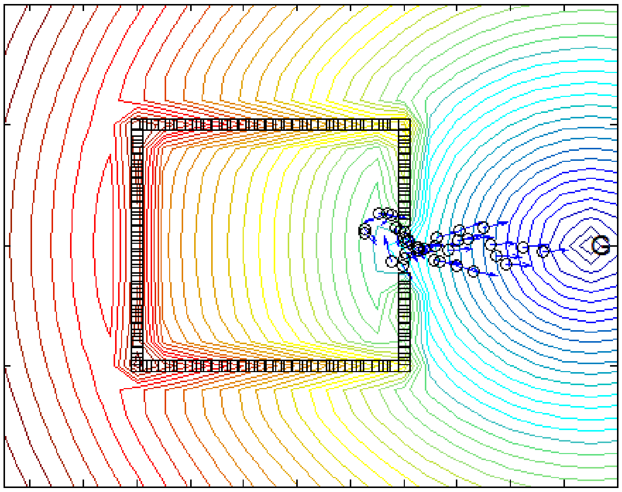

Fig.2.d. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, $t = 40$

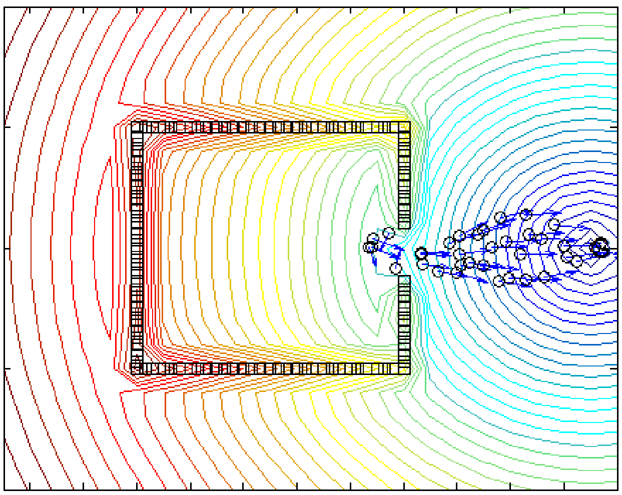

Fig.2.e. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, $t = 54$

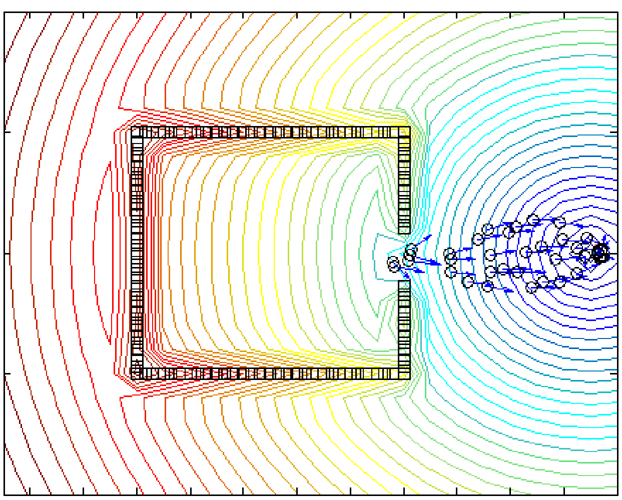

Fig.2.f. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, $t = 61$

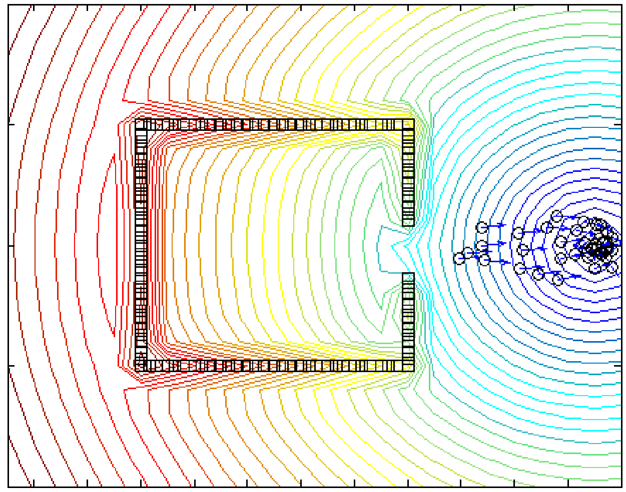

Fig.2.g. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, $t = 76$

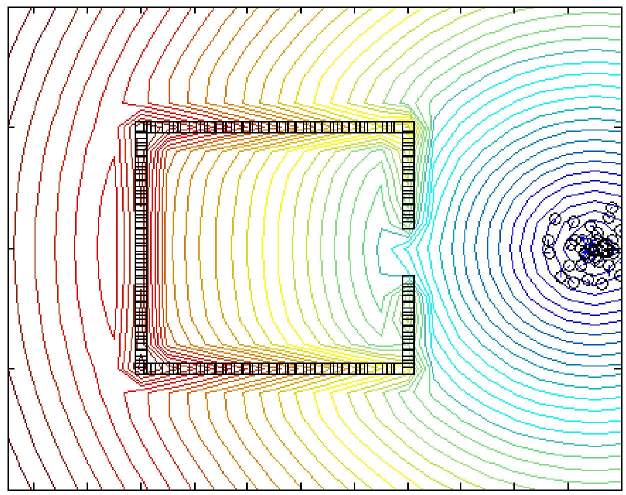

Fig.2.h. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation, $t = 90$

Fig.2. Simulation of crowd members that attempt to leave a trap to a destination point (G) in emergency situation.

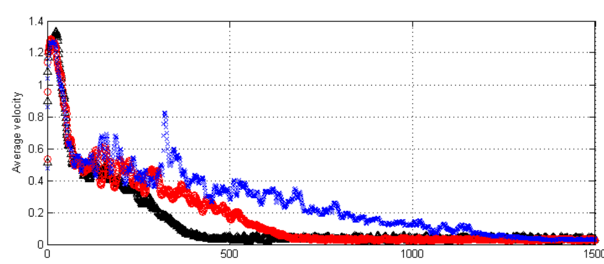

Fig. 3. Individuals' average velocity with time during normal conditions for (200 – 100 – 50) members crowds denoted by (blue 'x'– green'o' – black'Δ') respectively.

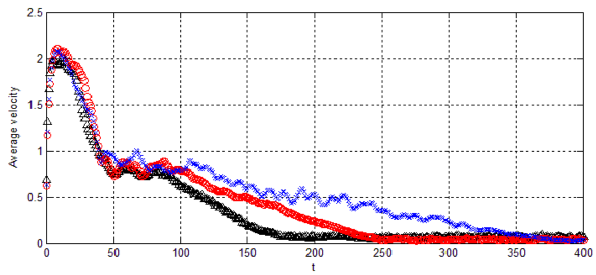

Fig. 4. Individuals' average velocity with time during emergency conditions for (200 – 100 – 50) members crowds denoted by (blue 'x'– green'o' – black'Δ') respectively.

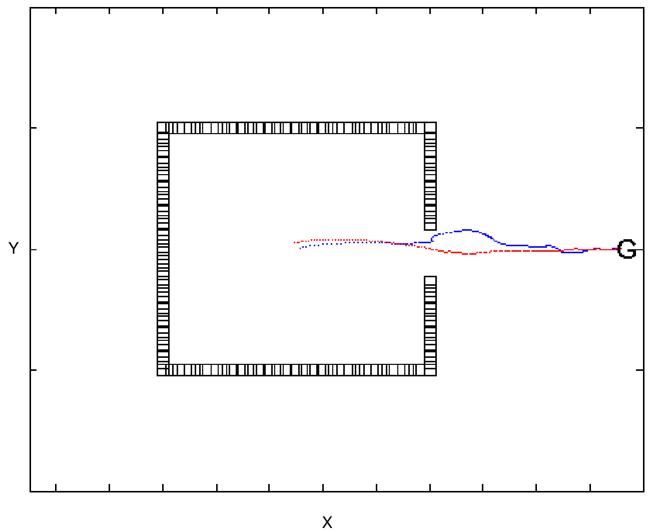

Fig.5. Path of center of a 300-members crowd until reaching a destination point (red line during normal conditions – blue line during emergency situation).

V. Discussion and Summary

The simulation results shown in Fig. 1 demonstrate the behaviour of crowd members that attempt to leave the room in a normal situation through a neck exit using random initial positions and velocities. It can be seen that some individuals pass the neck according to their close initial positions to the neck as shown in Fig. 1(a-b), hence the rest of the individuals seem to follow them and aggregate around the neck as shown in Fig. 1(c-f). The aggregated members leave the room in a line which can be clearly seen in Fig. 1(e-f). The process continues until each individual reaches the destination point while keeping a distance from its nearest neighbour in the crowd as shown in Fig. 1(g-i). The path of crowd centre $\mathbf{r}_c$ is calculated as ($\mathbf{r}_c = \frac{1}{N_p}\sum_{i=1}^{N_p}\mathbf{r}_i$) [37]. Fig. 5 shows the path of the crowd in normal condition (shown in red line) as straight line which emphasis that there are no much fluctuations (pushings) amongst the crowd members.

The simulation results shown in Fig. 2 demonstrate the behaviour of crowd members that attempt to leave the room in emergency situation through a neck exit using random initial positions and velocities. It can be seen that some individuals pass the neck according to their close initial positions to the neck as shown in Fig. 2 (a-b), hence the rest of the individuals haste towards the exit and aggregate around the neck pushing those individuals who are nearest to the exit which makes them squeezed out of the room as shown in Fig. 2(c-d). The number of individuals left in the room decreased with time which decreases the pushings amongst the individuals and the evacuation process becomes faster as can be seen in Fig. 2 (e-f). The evacuation process continues until each individual reaches the destination point while keeping a distance from its nearest neighbour in the crowd as shown in Fig. 2 (g-h). The notches in the path of crowd centre shown in Fig. 5 (shown in blue line for emergency situation), emphasis that there are more fluctuations (pushings) amongst the crowd members during the evacuation process in emergency situation than that in the normal conditions. It is important to note that the crowd behaviour shown in Fig. 1, Fig. 2 does not fulfil the conditions defined by [38] for swarming behaviour. In both results, each individual is repelled from other individuals, which represents the concept of dealing with the problem on an individual basis.

The individuals' average velocity of crowd members $\mathbf{v}_c$ is calculated as ($\mathbf{v}_c = \frac{1}{N_p}\sum_{i=1}^{N_p}\mathbf{v}_i$) [37]. It is shown in Fig. 3 and Fig. 4 for the normal conditions and emergency situation respectively. It can be seen that the average velocity in case of emergency situation is generally higher than that in the normal situation. Also for both cases, the average velocity is almost the same for crowds with higher population at the start of the evacuation process then as the process continues with time, individuals' average velocity is higher for higher crowd populations.

This is due to the fact that during the aggregation of the individuals around the neck in the early regime of the process, there is no enough space for the individuals to have higher speeds, then crowds of different populations almost has same average velocities. Then with time, the number of individuals left in the room decreased which increases the space around the individuals. This makes the pushings amongst the individuals (consequently the average velocity) higher for the crowds of higher populations.

Then crowds member in both cases tend to come to rest when reaching the destination point. This match the stability analysis of the model which proves that the crowd members tend to relax into a minimum energy state achieving real features of crowd members, who desire not to move too far in a short amount of time and to reach their destinations while avoiding other members in the crowd. This matches the observed data that attempted to be simulated by other models [2, 8, 36] yet with lower calculations required.

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VI. Conclusion

This paper presents a computer based model to predict human crowd dynamics. The model uses an efficient way to describe the motion, taking the physical terms of velocity and acceleration into account, employing generalized Morse potential to define the interaction forces amongst the individuals, and linking the behaviour of each individual to the environment components (boundaries, destination point). This way of defining the model elements gives the model the advantages of adaptation to real situations especially in crisis situations as well as the relatively simple required calculations. The stability analysis proves that, using this model, each pedestrian tends to relax to a minimum energy state which achieves the desire of individuals not to move too far in a short amount of time and to reach their destinations while avoiding other individuals. The numerical results which match the real observed data in similar situations show the ability of the model to predict the human crowd behaviour in both normal and emergency situations which means that it can be used effectively to evaluate the evacuation process with the possibility of changing many parameters to adapt the different emergency situations scenarios.

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