

A Performance Based Fire Analysis of Simple Beams Supporting Non-structural Concrete Slabs

E. Ufuah

Abstract— Most building constructions incorporate simple beam-column members designed to carry both axial and flexural loadings. These elements can exhibit different behaviour under fire conditions. Large displacements are often experienced by these elements when subjected to elevated temperatures making the analysis to become more nonlinear. This paper presents results from a performance based thermal and structural analyses of simple beams carrying non-structural concrete slabs. Numerical as well as simple theoretical models are developed and a comparison is made using the experimental results from tests conducted at the Warrington research centre, UK. Maximum coefficients of variation (COV) of about 2% and 6%, for mid-span deflection and temperature distribution respectively, demonstrates that the numerical model and trial results are in good agreement. Moreover, correlation coefficients of 99% or better can be achieved by implementing the theoretical models in predicting the deformation behaviour of the beams.

Index Terms— Beam-column, fire, load level, performance based methodology.

I. INTRODUCTION

The behaviour of steel beams under fire conditions has been extensively investigated both numerically and experimentally [1]-[5]. However, simple theoretical computations are hardly available for the assessment as well as preliminary design of the response parameters such as strength and deformation characteristics. A few have adopted the stochastic method to assess the performance of steel frames in fire while most modellers have employed the traditional techniques. These two approaches have been found to be reasonable in the computation of the required response parameters. The new trend in research seems to rely more on valid numerical predictions in order to circumvent the excessive cost that are inevitably present as contained by experimental techniques. Some of the parameters that can greatly influence the behaviour of steel beams in fire include, but not limited to, boundary conditions, restraint to thermal expansion, fire scenario, section factor and load level [1].

Recently, the trend of fire design has begun to move from the traditional prescriptive approach to a more robust performance based approach. Though, an increase in material ductility with increasing temperature enhances structural integrity, at high enough temperatures the strength and stiffness of structural steels are reduced. The traditional

prescriptive approach to design is based on individual member analysis to assess the strength and behaviour under various loading conditions. This approach, though conservative in most cases, has been successfully implemented in most of the existing buildings. In the case of rolled steel joist I-section, the critical element is recognized to be the bottom flange of the beam. However, the web can be seen as critical for laterally unrestrained sections which can often result in lateral-torsional buckling under compressive loadings. In this paper, a performance based methodology as well as simple theoretical formulations is developed to evaluate the behaviour of simply supported beams carrying non-structural concrete slabs under fire conditions.

II. THERMO-MECHANICAL PROPERTIES OF STEEL AT ELEVATED TEMPERATURES

The materials which are commonly used to fabricate steel beams for onshore structural works include S235 and S275. Nevertheless, the provision for high strength requirement may necessitate the use of S355 steel in this environment. When a steel structure is exposed to fire, the temperature of the element will increase from ambient to a high level, thereby leaving the material to degrade in strength and stiffness. According to Eurocode 3 Part 1-2 [6], various models are recommended to determine the reduction of elastic modulus and yield strength of different steels at elevated temperatures. In addition, the thermal properties of carbon steel at elevated temperatures; namely thermal conductivity and specific heat used in this present study were computed using the correlations in Eurocode 3, Part 1-2 [6]. These properties at high temperatures are illustrated in Fig. 1.

The reduction factors for proportional limit, yield strength and linear elastic range for carbon steel at elevated temperatures is represented in Fig. 2. A typical stress-strain-temperature curve for carbon steel is shown in Fig. 3 whereas the yield values for the various steel sections used in the present study are given in Table 1. The stress-strain model for carbon steel at elevated temperatures is illustrated in Fig. 4. A spreadsheet is usually developed to calculate the stresses and strains in conjunction with the stiffness as demonstrated in Fig. 4. Although, stiffness computation in this regard is somewhat implicit, it should be noted that it is a function of the linear elastic range within the stress strain characterization. It should also be emphasized that the model of Fig. 4, which explicitly excludes the effect of strain hardening, is derived by implementing the correlations defined in Eurocode 3, Part 1-2 [6].

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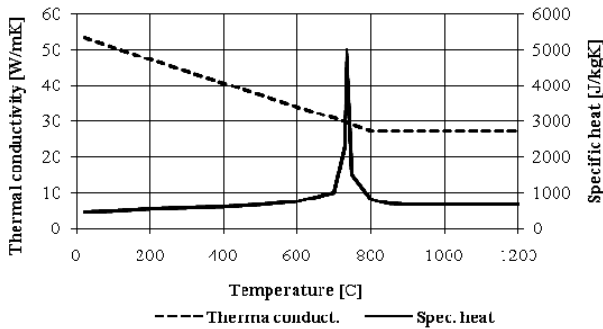


Fig. 1 Thermal properties of carbon steel at elevated temperatures.

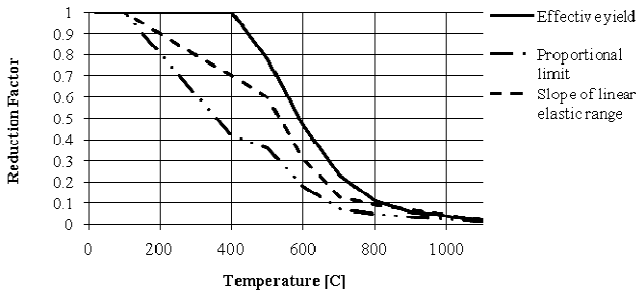


Fig. 2 The reduction factors for yield strength, proportional limit and linear elastic range for carbon steel.

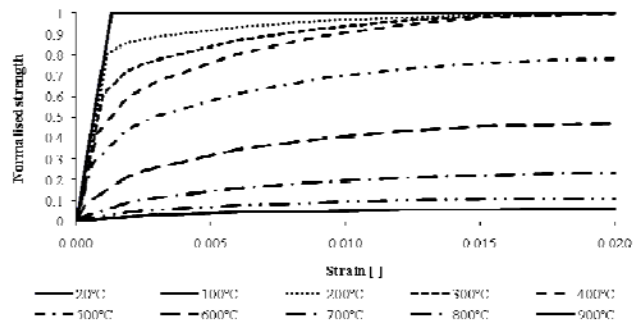


Fig. 3 Stress-strain curves for carbon steel at elevated temperatures

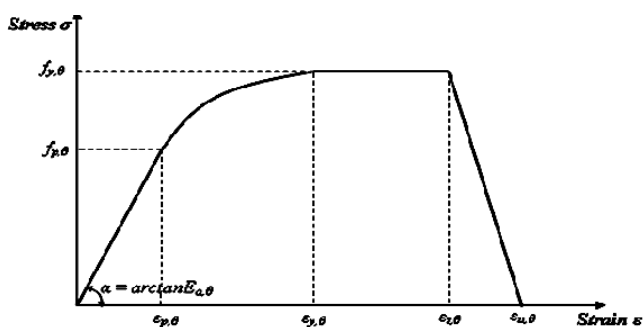


Fig. 4 Stress-strain model for carbon steel at elevated temperatures

III. MODELLING TECHNIQUES

The numerical calculations were carried out using a general-purpose commercial code-ABAQUS [7], which has the capability of predicting the thermal and mechanical responses of structures based on finite element method. The large displacement effects of the non-linear finite elements are considered using an updated Lagrangian formulation. The modelling techniques include heat transfer analysis and

structural response using the standard ISO 834 fire load [8]. This methodology rather considers the fire behaviour by adopting a standard temperature time curve that characterizes the heating regime of a fire.

The multi-linear stress-strain curves at various temperatures describe the non-linear behaviour of the material. The plasticity is calculated by the von Mises yield criteria and the associated flow rule [7]. The results of the heat transfer analysis are implemented in the structural analysis making it possible to account for the effects of thermal strains. Therefore, the simulation assumes a sequentially coupled thermal-stress procedure in which a thermal analysis is followed by a stress analysis. All the beams investigated are 254x146x43UB and subjected to various point loads with a load level of 0.6 [9]. The loads were computed using elastic design in accordance with BS5950 Part 8 [10]. The segmented concrete toppings, nominally 650mm wide and 130mm thick, were characterized as non-structural and their influence on the resultant deformation of the beams was immaterial in all the analyses. However, to avoid stress concentration the total variable load at each loading point was divided equally amongst nine nodes. In addition, a theoretical formulation was developed from a regression analysis for the various grades of steel to calculate the deformation characteristics at elevated temperatures.

In order to compute the thermal and structural responses of the model beam, the governing differential equations for heat conduction and structural equilibrium together with the corresponding constitutive material model are required. However, the 3-D heat conduction equation can be generally stated as follows [11]:

$$-\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right) + Q(x, y, z, t) = \rho c \frac{\partial T}{\partial t}(x, y, z, t) \quad (1)$$

where $q_r(r=x,y,z)$ represents the heat flux in the three coordinate directions, T is the current temperature, $Q(x,y,z,t)$ is the rate of internal heat generation, t is the time, ρ is the density and c is the specific heat.

The conductive heat flux defined by the Fourier law can be written in general form as follows:

$$q_r = -k_r \frac{\partial T}{\partial r}, \quad r = \text{coordinate of flow} \quad (2)$$

However, the thermal conductivity values of steel do not show significant spatial variation so that a homogeneous thermal conductivity k can be adopted in the following form:

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) + Q(x, y, z, t) - \rho c \frac{\partial T}{\partial t}(x, y, z, t) = 0 \quad (3)$$

The proper boundary conditions required to solve (3) for the temperature field relate to initial temperature T , internal heat generation $Q=f(q)$ and heat fluxes q_r , as given in (4) and (5).

$$T(x, y, z, t = 0) = T_0(x, y, z) \quad (4)$$

Table 1 Matrix of test specimens implemented in the present study

Test No.	Effective span (m)	Exposed length (m)	Recorded yield (MPa)	Concentrated variable load (N)	Number of loading points	End condition
1	4.5	4.0	Flange =284	44150	2	Simple supports
			Web =306			
2	4.585	4.0	Flange =411	46730	4	Simple supports
			Web =399			
3	4.585	4.0	Flange =250	32540	4	Simple supports
			Web =277			
4	4.465	4.0	Flange =408	33920	4	Simple supports
			Web = 409			

$$k\left(\frac{\partial T}{\partial x} N_x + \frac{\partial T}{\partial y} N_y + \frac{\partial T}{\partial z} N_z\right) + q(x, y, z, t) + h_c(T_s - T_\infty) + \sigma \epsilon VF(T_s^4 - T_f^4) \quad (5)$$

where N_x , N_y and N_z are the direction cosines orthogonal to the boundary surface, h_c is the film coefficient, T_s is the surface temperature assumed uniform through the thickness, T_f is the fire temperature, T_∞ is the temperature of the surrounding medium, ϵ is the surface emissivity of the material, σ is the Stefan Boltzmann constant ($= 5.667 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$) and VF is the configuration factor.

This temperature field was then applied to the mechanical model as a forcing function to calculate the stresses and strains by means of incremental plasticity theory with temperature-dependent mechanical material properties. The transport properties comprising convective heat transfer coefficient and radiative emissivity utilised in the numerical computations of temperature profile are $25 \text{W/m}^2\text{K}$ and 0.7 respectively. The view factor was assumed unity by neglecting the position and shadow effects.

The material model often adopted in the analysis of steel structures is assumed to follow the von Mises yield criterion and the associated flow rule [12]-[14]. The general purpose non-linear finite element analysis code ABAQUS [7] adopts the Newton-Raphson approach to incrementally solve the resulting differential equations. In this state, the mechanical strength and behaviour of a material need to be assessed by implementing the correlations (6) and (7) [14].

$$\sigma_{ij,j} + f_i = 0 \quad (6)$$

$$\{\Delta\sigma\} = \{D^{ep}\}\{B\}\{\Delta U_e\} - \{C^{th}\}\{M\}\{\Delta T_e\} \quad (7)$$

where σ_{ij} is a stress tensor, f_i is a body force, $\Delta\sigma$ is the incremental stress, D^{ep} is the elasto-plastic stiffness, B is strain-displacement relation, U_e is nodal displacement, C^{th} is thermal stiffness, M is temperature shape function and ΔT_e is nodal incremental temperature.

For a two dimensional stress analysis, after measuring the released strains following both thermal and mechanical loadings, the stresses in two orthogonal directions can be calculated by implementing (8) and (9) [15]-[16].

$$\sigma_x = \frac{E}{1-\nu^2} [\epsilon_x + \nu\epsilon_y] \quad (8)$$

$$\sigma_y = \frac{E}{1-\nu^2} [\epsilon_y + \nu\epsilon_x] \quad (9)$$

However, in fire conditions with increase in strains the resultant stresses are reduced and thus the thermo-elastic stresses should be calculated according to (10) and (11) [15].

$$\sigma_x = \frac{E}{1-\nu^2} [\epsilon_x + \nu\epsilon_y - (1+\nu)\alpha_T T] \quad (10)$$

$$\sigma_y = \frac{E}{1-\nu^2} [\epsilon_y + \nu\epsilon_x - (1+\nu)\alpha_T T] \quad (11)$$

where σ_x , σ_y are two orthogonal direct stresses, ϵ_x , ϵ_y are the two orthogonal direct strains, E is elastic modulus, ν is Poisson's ratio, α_T is coefficient of thermal expansion and T is temperature.

IV. RESULTS AND DISCUSSION

The results of heat transfer analysis were used as part of the forcing function in the mechanical computation. These results are shown in Table 2. Moreover, the results of stress analysis are given in Table 3 and illustrated in Figs. 5-8. Table 4 shows the theoretical correlations from the results of a regression analysis conducted. A comparative assessment between the experimental results and the model predictions gave maximum coefficient of variation (COV) of 0.021 for mid-span deflection. However, a COV of 0.06 was also computed between experimental and numerical results for temperature distribution.

Obviously, the heterogeneity of temperature distribution across the beam section has a significant influence on the strength and behaviour of the beam. However, it was also observed that the thickness of the section plays an important role in dictating the magnitude of temperature reached without considering thermal equilibrium. Coupled with the numerical computation, simple theoretical calculations were conducted based on the method of least squares as illustrated in the appendix. Test beam is designated TB whereas COV represents coefficient of variation.

Table 2 Comparison of fire test results with finite element predictions: Temperature

Beam No.	Test results				Finite element (FE) results				Ratio of test to FE results			
	Test period [min]	UFT [°C]	WT [°C]	LFT [°C]	FRP [min]	UFT [°C]	WT [°C]	LFT [°C]	FRP	UFT	WT	LFT
TB1	35.5	535	691	682	35.5	521.33	735.77	727.48	1	1.03	0.94	0.94
TB2	23	423	660	647	23	396.92	658.80	633.43	1	1.07	1	1.02
TB3	22	423	634	623	22	394.12	645.22	624.90	1	1.07	0.98	0.997
TB4	30	519	701	692	30	498.54	704.01	692.46	1	1.04	0.996	0.999
Mean		475	671.5	661		452.73	685.95	669.57	1	1.05	0.98	0.99
COV		0.127	0.045	0.048		0.147	0.035	0.073	0	0.02	0.028	0.06

Table 3 Comparison of fire test results with finite element predictions: Mid-span deflection

Beam No.	Test results		Finite element (FE) results		Ratio of test to FE results	
	Test period [min]	Mid-span deflection [mm]	FRP [min]	Mid-span deflection [mm]	FRP	Mid-span deflection
TB1	35.5	148	29.11	147.626	1.22	1
TB2	23	144	21.78	146.434	1.06	0.98
TB3	22	140	21.44	146.955	1.03	0.95
TB4	30	152	28.11	156.618	1.07	0.97
Mean		146		149.418	1.095	0.975
COV		0.035		0.032	0.078	0.021

Table 4 Regression analysis results for mid-span deflection δ of the studied beams

Beam No.	Steel grade	Theoretical correlations for mid-span deflection
TB1	S275	$\delta = 0.0043t^3 - 0.133t^2 + 4.06t - 7; \quad t \geq 2 \text{ min}$
TB2	S355	$\delta = 0.00326t^3 - 0.957t^2 + 12.2t - 28; \quad t \geq 3 \text{ min}$
TB3	S275	$\delta = 0.0407t^3 - 1.13t^2 + 12.66t - 27; \quad t \geq 3 \text{ min}$
TB4	S355	$\delta = 0.01t^3 - 0.326t^2 + 6.4t - 16; \quad t \geq 3 \text{ min}$

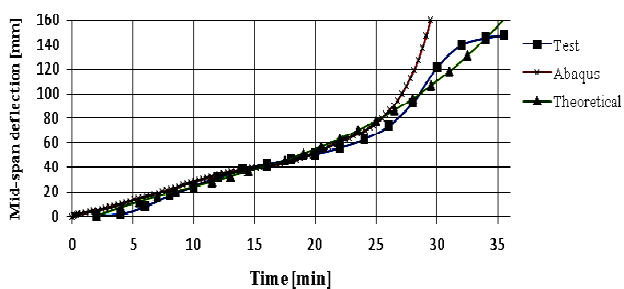


Fig. 5 Mid-span deflection for test beam 1 (TB 1)

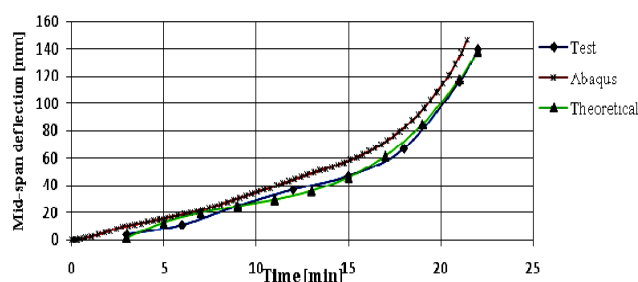


Fig. 7 Mid-span deflection for test beam 3 (TB 3)

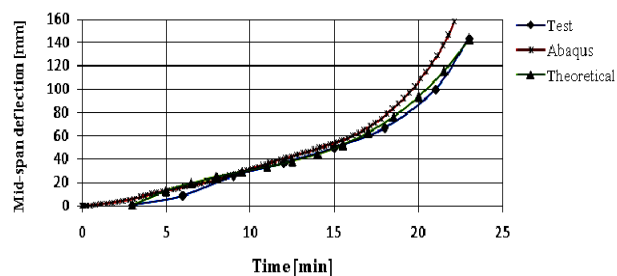


Fig. 6 Mid-span deflection for test beam 2 (TB 2)

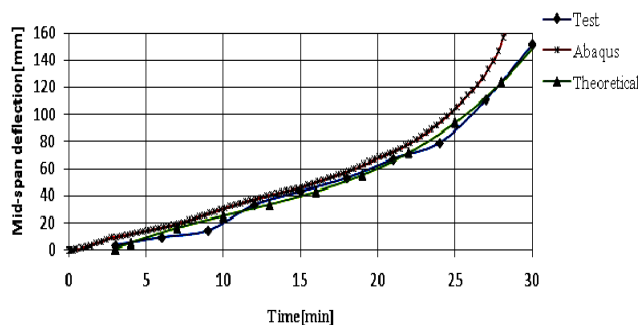


Fig. 8 Mid-span deflection for test beam 4 (TB 4)

V. CONCLUSION

The strength of steel beam-columns subjected to lateral pressure under heat loads was investigated by a series of numerical and theoretical computations. The results demonstrate that the finite element models as well as the theoretical formulations are in good agreement with the test data and may, thus be safely used for preliminary fire design.

APPENDIX

The mid-span deflection δ can be modelled in the form:

$$\delta = f(D, \lambda_i)$$

$$\lambda_i = \text{scalar constants}$$

The residuals can be denoted as follows.

$$r_i \sum_{i=1}^n (\delta / D) = f(\delta, \lambda)$$

$$r_i = \delta_i - f(\delta_i, \lambda)$$

The sum of the least squares of the residuals is given by

$$S = \sum_{i=1}^n r_i^2$$

The regression of the mid-span deflection on time (equivalent to temperature rise) can be achieved by minimizing the sum of the least squares of the residuals.

$$\Delta S = 2 \sum_{i=1}^n \sum_{j=1}^m r_i \frac{\partial r_i}{\partial \lambda_j} \Delta \lambda_j = 0$$

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