Computer-Aided Modelling for a Poly Articulated Robotic Arm with Spherical Joints

Violeta Dumitru, Sorin Dumitru

Abstract: The paper refers to a series of poly articulated robotic arms, with multiple vertebrae, having functions of exploration, inspection, intervention and gripping in operating spaces with reduced restrictions. At first, the paper relates about the construction of the digital model of the arm, the analysis techniques and the construction of the prototype from polymer powders using the 3D technology. We propose an analytic method, with SMB type high performance software, for the assembly of the model, the redundancy, the kinematics and reversed kinematics analysis. Then, it analyses the manoeuvrability of the arm to obtain an imposed target.

Index Terms: poly articulated, robotic arm, modelation

I. INTRODUCTION

The conventional poly articulated mechanic configurations are discrete mechanisms obtained by the connection in cascade of certain rigid bodies articulated by a connection with a single degree of freedom. The elements of the kinematic chain are made of passive components, the activation being performed only at the level of the joint. In contrast with the classical structures, the serpentine type models (hyper redundant) are, theoretically, built from an infinite number of joints, which allows to obtain a kinematic chain described by a continuous curve that confers it a hypothetically infinite mobility. The discrete mechanical systems can obtain an improved manoeuvrability and flexibility by adding several degrees of freedom. Roboson [1] proposed to classify these types of mechanical systems in three categories:

- Discrete, with conventional structure;
- serpentine, snake shape;
- continuous.

The serpentine systems are composed of numerous joints connected by very short rigid connections, being a very mobile structure in the apparent form of a smooth curve, like a snake body. The continuous systems are permanently undulating along or by means of resilient deformation. We must underline the fact that there are fundamental differences between the three classes of mechanical systems, as we can notice from figure 1. For the mechanical systems with discrete structure, each joint is actuated and rigidly connected to the next joint, while the continuous robot (fig. 1.b) can perform the same operation by energizing a single actuating element. This concept also exists for the hyper-redundant manipulators which have an additional number of degrees of freedom, therefore they need the actuation of n joints or it is enough a single actuating element, figure 1.a.

![Fig. 1. The structural differences existent between hyper-redundant and continuous structure; (a) mechanical system with hyper redundant structure; (b) Mechanical system with continuous structure](image_url)

Poly articulated arms with discrete elements are a class of manipulators which perform exploration, inspection, intervention.

![Fig. 2. Geometrical shapes of the pieces used to build up the tentacles](image_url)

In figure 2 there are shown the geometrical shapes of the pieces that can be used to build up the tentacles. There can be identified simple, symmetrical or asymmetrical geometrical shapes. There can be variants with a single type of piece, with two types or three types of pieces and rarely with several constructive types of pieces. A single piece or a group of pieces forms a geometrical module [2], figure 3. The simple structure of these manipulators permits the performance of some light mechanisms with a wide operation range, capable to operate in restricted areas and with a difficult access. Also, the hyper-redundant mechanisms can be used to position some small loads that do not need a precise positioning, like minimum invasive inspections and explorations. The analysis and the
modulation of these systems are really complex and a great number of researchers have tried to offer solutions. The snake like robots have been investigated for a long time for numerous applications that benefit of an increased dexterity and high capacity to avoid obstacles [3], [4]. Due to the subsistent high number of freedom degrees, the efficient resolution for the inverse kinematics was one of the main theoretical problems that have delayed their use. Chirikjian and Burdick [5] solved this problem by using the variation calculation methods. Previous articulated constructions of shake robots were cumbersome for the constructors, as they needed a lot of mechanical joints, so they had mechanical problems, namely clearances. In order to prevent these problems, a few papers by Walker and Gravagne [6] were concentrated on the kinematics and the control of the spine constructions. Different concepts are also presented in [7], [8], [9], which inspired this paper. The authors have presented tools in the shape of a snake using as material super elastic NiTi that has a higher rigidity than the SMAs, but the heating

![Fig. 3 Geometrical modules made up of one or two pieces](image)

problem and the complexity of the multiple degrees of freedom (DOF) is still relevant, therefore creating DOF limited mechanisms and articulated systems actuated by flexible cables. In the inverse kinematic analysis, Han and Rudolph [10] had obtained remarkable results that represented the basis of the inverse kinematic theory for the hyper redundant robots with

II. THEORETICAL AND TECHNOLOGICAL CONSIDERATION

Technical poly articulated hyper redundant models are complex structures that operate in a tridimensional space. Constructively, the poly articulated robotic arm is composed of a high number of joints connected by very short rigid joints (vertebrae), constituting a mobile structure which has an apparent form of a smooth curve similar to a snake body. Each vertebra is designed for a simple integration and to facilitate the performance of the requests. These requirements include:

- more sections for bending along the hyper redundant arm;
- aspect (range 1:60 with a video capacity at the end).

In table I and II are shown the functional requirements and the design specifications for a poly articulated mechanical system.

Constructively, the poly articulated robotic arm consists in two concentric tubes (figure 4). I will refer to these tubes as being two mechanisms, an exterior one and the other an inside one.

![Fig. 4. Structural view of the poly articulated robotic arm](image)

Each mechanism enclosed several adjacent elements similar to some rigid vertebra, interconnected by cables (wires), disposed in series. Each link is made with inverted segments (opposite) single or double curved (spherical surfaces) and foreseen with three semi-circular longitudinal channels. When the inside mechanism is inserted into the exterior mechanism, the three operational ports (channels) can be aligned, forming a three dimensional space used to pass the instruments from the first end (proximal) to the second one (distal) of the robotic arm. The vertebrae are maintained in contact by means of cables (wires); 3 cables for the exterior mechanism, disposed at 120 degree between them and a central cable for the inside mechanism, figure 1.4. By actuating two cables of the exterior mechanism it is possible to rotate the final link with an angle of ± 10° in any direction. By combining the unique configurations of the exterior/ interior links (proximal, intermediary and distal) of the exterior/ interior mechanisms we offer the tentacular robotic arm the ability to make any route defined in the 3D space.

<table>
<thead>
<tr>
<th>Requirements</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>3D complex configuration</td>
<td>Task directed design</td>
</tr>
<tr>
<td>Forward/ backward movement</td>
<td>Forward/ Backward: ± 220 mm</td>
</tr>
<tr>
<td>Adaptability/ Manoeuvrability</td>
<td>Reconfiguration of the components</td>
</tr>
<tr>
<td>Safety during operation</td>
<td>Miniaturizing (reduced sizes/ small loads)</td>
</tr>
<tr>
<td>Aspect report</td>
<td>Increase the field of vision</td>
</tr>
<tr>
<td></td>
<td>Range 1:60</td>
</tr>
</tbody>
</table>

The materials for the vertebrae and cables: thermal plastic polymers–polyamide (durmadi), high density polyethylene, garolite (consolidated fibre). The features of the thermal plastic polymers are shown in table III.
Table III

<table>
<thead>
<tr>
<th>Polymer</th>
<th>Density (g/cm³)</th>
<th>Tₐ, °C</th>
<th>Tᵣ, °C</th>
<th>Isotropic module (GPa)</th>
<th>Crystallization module (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA-6</td>
<td>1.13</td>
<td>50</td>
<td>220</td>
<td>2.8</td>
<td>160</td>
</tr>
<tr>
<td>HDPE</td>
<td>0.95</td>
<td>120</td>
<td>130</td>
<td>0.4 - 2</td>
<td>300</td>
</tr>
</tbody>
</table>

### III. THE DIGITAL MODEL

This paragraph describes the CAD modeling techniques in order to establish the optimal form of the exterior/interior vertebrae (proximal, intermediary, distal) starting from a basic structure represented by the IGES surface and finally reaching to the virtual model of an intermediary external vertebra. This model can serve to the performance of virtual assemblies that will serve to numeric simulations, respectively to its practical performance using the nonconventional technology of Rapid prototyping. The CAD modeling of the vertebrae was performed using conventional techniques available in CAD Solid Work 2010 software. We started by sketching out a shape that would satisfy from the functional point of view the needs of such vertebrae. These needs consist in the possibility of the vertebrae to be fixed by cables (wires). Thus, in the first phase, it was modulated an intermediary exterior vertebra which fulfills to a certain extend the functional requirements imposed to such a construction. For the performance of this vertebra are needed n modeling operations like: jet moulding, the creation of plans, etc. In order to make the assembly, it was considered the optimization of the shape for the intermediary exterior vertebra, so that it should have two major functionalities: the capacity for a precise positioning; the possibility to keep the full contact of the two adjacent entities during the requests. In figure 5 is shown the basic sketch after which we designed the intermediary exterior vertebrae. In order to force the free design of the sketch, between the constituent geometric elements were established, depending on the case, relations of co axiality, symmetry, parallelism, verticality, etc. From the constructive point of view, the prototype of the intermediary exterior vertebra is characterized by the following elements (figure 5): transmission outlet (1), with a 7.5 mm diameter allows the intermediary interior vertebra, with a 6.5 +/- 0.1 mm diameter, to pass; the first cone (2); the second cone (3); the hemisphere (4), with a 14.5 mm diameter, receives the second end of the previous adjacent vertebra; the hemisphere (5), with a 14.5 mm diameter, receives the first end of the posterior adjacent vertebra; transmission outlets (6), three of them with a 1.25 mm diameter providing the guiding of the operating cables (wires); holes (7), with a 1.25 and 2.5 mm provide the releasing of the cables operating (wires); semi-circular channels (8), three of them, identical, equidistant, together with the exterior semi-circular channels provide the guidance of the operating cables (wires). In figure 6 is shown the final model of the intermediary exterior vertebra.

The proximal exterior vertebra, the distal exterior vertebra, as well as the interior vertebrae (proximal, intermediary, and distal) had been shaped using the same classical technique of modelling like for the intermediary exterior vertebra.

In order to verify the wanted functionality of the designed vertebrae (exterior, interior), it was performed its assembly with 20 exterior vertebrae, respectively 33 interior.

This assembly was obtained using the Solid Works 2010 software, figure 7. In order to make the vertebrae we used one of the rapid prototyping techniques (rapid manufacturing of the prototypes – 3D models), namely the prototyping technique by 3D printing. The 3D prototyping system Z printer 310 produced by Z-Corporation and owned by the company CAD Work Craiova, with the following features: the sizes of the prototype 200 x 250 x 330; resolution (100 microns); speed (24 mm/hour); material (polyamide, polystyrene).

Fig. 5. Basic sketch after which the intermediary exterior vertebra was

Fig. 6. The final model of the intermediary exterior vertebra

Fig. 7. The poly articulated robotic arm assembly designed with Solid Work 2010 software

### IV. THE MOVEABILITY OF THE ROBOTIC ARM
The movement of a robotic arm, even a poly articulated hyper-redundant one or any other type, can be seen from two angles:

- from the kinematic point of view;
- from the dynamic point of view.

The kinematics studies the movement of the robotic arm. In a kinematic analysis is calculated, at a given moment, the position, the speed and the acceleration of each segment of the manipulator and all the derivatives of the position variables without taking into consideration the forces that cause this movement. In order to analyse a mechanic system, the MBS type performant software allow several operations and namely: the analysis of the feasibility (the assembly of the model), redundancy, static balance, the kinematic analysis, the dynamic analysis and the inverse dynamic. These can be performed separately or together in a certain sequence depending on the type of analysis and the degree of freedom (DOF) of the analysed system. The degree of freedom of a mechanical system is given by the number of movements that are not kinematically determined:

\[
\text{DOF} = \text{DOM} - \text{ND}
\]

where, \(\text{DOM}\) (Degrees of Mobility) represents the degree of mobility, namely the number of the generalized coordinates that define all the elements in the mechanism and the DN (Driven Number) is the number of driven/ generated movements (phenome restrictions, time dependent). The movement of a rigid body is described by six generalized coordinates: the coordinates that are origins for the marker attached to the body and the directions of the axis of this marker relatively to the axis of the global reference system (inertial). Accordingly, the degree of liberty is equivalent with the total number of independent coordinates in the system:

\[
\text{DOF} = 6 \times n - r
\]

where \(n\) is the number of mobile bodies in the system and \(r\) is the number of restrictions imposed in the limits of certain admitted tolerances. The kinematics deals with aspects of redundancy, avoiding collisions and avoiding singularities. The redundancy consists in the identification and elimination of the redundant connections (additional) from the over constrained mechanical system. In the shaping phase, the elimination of the redundancies implies the elimination of the duplicates and the replacement of the joints from the initial module with other type of joints of inferior classes, in the conditions in which the initial movement of the mechanical system must be observed. The redundant constrains correspond to the movements that are not produced in the mechanical system, the redundant constrains correspond to the movements that are not kinematically determined.

\[
\text{DOF} = 6 \times n - (r - R).
\]

For direct kinematic we considered a serial kinematic structure with 20 rigid vertebrae, with lengths \(l_1, \ldots, l_i, \ldots, l_{20}\), connected with spherical joints for a poly articulated robotic arm. It was considered the lengths of the elements and the relative movements from the rotational spheres:

\[
\psi_{10}, \psi_{21} \ldots, \psi_{19,20}.
\]

It is calculated the position and the direction of the final effort, namely the position of the characteristic point \(M = O_{20}\) and the direction of the coordinate system attached to the effectors \((O_{20}X_{20}Y_{20}Z_{20})\), with respect to the reference coordinate system \(O_XX_1Y_1Z_1\). The redundancy consists in the identification and elimination of the operations from the kinematic joints through homogenouus operators [11]. In this respect, to the basic element is associated the Cartesian coordinate system \(O_XX_1Y_1Z_1\) (system 1), \(O_XX_2Y_2Z_2\) (system 2), \(O_XX_2Y_2Z_2\) (system 20). The coordinate systems 1, 2, 3... 20 result after the modulation of relative movements of kinematic joints through homogenouus operators. The relative movement (rotation around the z axis) from the rotation joint, formed between the basic element \(O\) and element 1, is modelled by the rotation operator:

\[
A_{1,0} = R_z(\phi_{01})
\]

(4.1)

This characterizes the position and the orientation of the coordinate system 1 with respect to reference system O0X0Y0Z0. The inverse operator \(A_{1,0}^{-1} = A_{0,1}\) characterizes the position and the direction of the reference coordinate system with respect to the coordinate system 1. The operator’s matrices \(A_{1,0}^{-1}\) and \(A_{0,1}\) are:

\[
\begin{bmatrix} A_{0,1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{\phi_{10}} & -S_{\phi_{10}} & 0 \\ 0 & S_{\phi_{10}} & C_{\phi_{10}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} A_{0,1}^{-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C_{\phi_{10}} & S_{\phi_{10}} & 0 \\ 0 & -S_{\phi_{10}} & C_{\phi_{10}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

where, \(C_{\phi_{10}} = \cos \phi_{10}, \ S_{\phi_{10}} = \sin \phi_{10}\)

The relative movement from the rotation joint formed between the 1 and 2 element is modelled by the rotation operator: \(A_{2,1} = R_z(\phi_{12})\) which characterizes the position and the direction of the coordinate system 2 with respect to the reference coordinate system 1. The position and the direction of coordinate system 20, with respect to the reference coordinate system, are given by the product operator:

\[
A_{0,20} = A_{0,1}A_{1,2} \ldots A_{i,20} A_{20,1} A_{1,0}
\]

(4.2)

The product operators \(A_{k,0}\) (\(k=0, 1, 2, 3... 15\)) characterizes the position and the direction of the coordinate system \(k\). These operators’ matrices are:

\[
\begin{bmatrix} A_{k,0} \end{bmatrix} = \begin{bmatrix} f_{i,j}^{20} & 0 & \ldots & 0 \\ f_{i,j}^{21} & f_{i,j}^{20} & \ldots & f_{i,j}^{20} \\ \ldots & \ldots & \ldots & \ldots \\ f_{i,j}^{18} & f_{i,j}^{18} & \ldots & f_{i,j}^{18} \end{bmatrix}
\]

(4.3)

where, \(f_{i,j}^{20} ; i = 2, 3, 4, \ldots, 18; \; j = 1, 2, 3, \ldots, 18; \; k = 0, 1, 2, \ldots, 18\); From the analysis of the operator \(A_{k,0}\) result the following conclusions the origin of the coordinate system 20 is positioned with respect to the reference system through the coordinates:

\[
x = (\phi_{01}, \phi_{21}, d_{21}) = f_{i,j}^{20},
\]

\[
y = (\phi_{02}, \phi_{22}, d_{22}) = f_{i,j}^{20},
\]

\[
z = (\phi_{03}, \phi_{23}, d_{23}) = f_{i,j}^{20}
\]

(4.4)

In the inverse kinematic analysis of the poly articulated spatial robotic arm with 20 elements is pursued the determination of the relative movements from the kinematic
joints, necessary to realise a certain position and direction of the effectors solidary with the coordinate system \( A_{19,20} \). Knowing the numeric values of the elements of the operator's matrix \( A_{1617} \) (noted \( A_{01} \)), given under the form:

\[
[A_{01}] = \begin{bmatrix}
1 & 0 & \ldots & b_{21} \\
b_{21} & b_{22} & \ldots & b_{33} \\
\vdots & \vdots & \ddots & \vdots \\
b_{161} & b_{162} & \ldots & b_{1818}
\end{bmatrix}
\] (4.5)

Are determined the values of the parameters \( \varphi_{010}, \varphi_{21}, \ldots, \varphi_{1920} \) so that the poly articulated system to realize the position and the direction imposed. In order to solve the problem of inverse kinematic we start from the relation:

\[
A_{01}A_{12} \ldots \ldots A_{1718} = A_{20} 
\] (4.6)

where, the operator's matrix \( A_{20} \) is given under the form (4.5). Forwards, we follow the successive separation of the unknown values \( \varphi_{010}, \varphi_{21}, \ldots, \varphi_{1920} \) in the right member of the relation (4.41) by successive multiplication of this relation on the left with inverse operators \( A_{01}^{-1}, A_{12}^{-1}, \ldots, A_{1920}^{-1} \). Thus, we obtain the following relations:

\[
A_{k20} = A_{k20}, \; k = 1, 2, 3, \ldots, 19
\] (4.7)

Then, the operator's matrixes \( A_{k20} \) are:

\[
[A_{k20}] = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
0 & k_{21}^{A_{k20}} & \ldots & f_{218}^{A_{k20}} \\
\vdots & \vdots & \ddots & \vdots \\
0 & k_{161}^{A_{k20}} & k_{162}^{A_{k20}} & \ldots & k_{1818}^{A_{k20}}
\end{bmatrix}
\] (4.8)

where, the elements: \( k_{ij}^{A_{k20}} \) \( (i = 2, 3, \ldots, 18); (j = 1, 2, \ldots, 18); (k = 1, 2, \ldots, 19) \) are expressed depending on the relative movements within the joints and elements \( b_{ij} \) \( (i = 1, 2, \ldots, 19) \); \( (j = 1, 2, \ldots, 15) \); \( (k = 1, 2, \ldots, 20) \) of the matrix \( A_{020} \). From the matrix equations (4.7) result the following equations:

\[
k_{20} = b_{20}^{A_{k20}}
\] (4.9)

Forwards, the expressions of the matrixes are systematized:

\[
A_{k17} \; s i \; A_{k17}, \; k = 1, 2, \ldots, 20
\]

and are written the equations out of which are determined the 20th relative movements from the kinematic joints, are characteristic to the analysed kinematic chain, \( (\varphi_{010}, \varphi_{21}, \ldots, \varphi_{1920}) \):

\[
A_{66} = A_{66} \Leftrightarrow A_{01}A_{12}A_{23}A_{34}A_{45}A_{56} = A_{06} \\
A_{16} = A_{16} \Leftrightarrow A_{01}A_{12}A_{23}A_{34}A_{45}A_{56} = A_{16} \Leftrightarrow A_{01} \\
A_{56} = A_{56} \Leftrightarrow A_{23}A_{34}A_{45}A_{56} = A_{12}^{-1}A_{01} \\
\]

The dynamic model of a mechanical structure is analytically represented by a system of differential equations that define the connections occurring between the generalized coordinates or their derivatives and the forces that actuate upon each segment of the manipulator. The models used to determine these differential equations are numerous, Lagrange – Euler, Newton –Euler, D’Alambert’s generalized principle, being only a few of the classical procedures used to calculate the dynamic model for the poly articulated mechanisms [12]. Poly articulated movement mechanism is made in accordance with the governing equations:

\[
H(q)\ddot{q} + c(q, \dot{q}) + G(q) = Q
\] (4.11)

where: \( q \) – the position vector of the joint; \( \dot{q} \) - the speed vector; \( \ddot{q} \) - the acceleration vector; \( H \) – the inertia matrix; \( C \) – describes the coriolis and centrifugal effects; \( G \) – gravitational loading; \( Q \) – the generalized force/torque vector associated with \( q \). The Lagrange form can be written using the Denavit-Hartenberg parameters and is very efficient from the conventional point of view.

The Lagrange equations result from the equilibrium of the energy equation and it has the following form:

\[
\tau_i = \frac{d}{dt}\left(\sum_{i=1}^{n} \delta \frac{\partial}{\partial \dot{q}_i} T_i - \sum_{i=1}^{n} \delta \frac{\partial}{\partial q_i} U_i + \sum_{i=1}^{n} \delta \frac{\partial}{\partial \dot{q}_i} W_i \right)
\]

(4.12)

where: \( L=K-P \), \( K \) – the kinetic energy of the system; \( P \) – the potential energy of the system; \( \tau_i \) – the torque that operates on the \( i \) joint; \( \dot{q}_i \) – the position of the joint. The kinetic energy \( K_i \) of a mechanism with \( n \) bodies is the sum of the kinetic energies \( K_i \) of individual bodies:

\[
K_i = \sum_{j=1}^{n} K_j
\]

(4.13)

The kinetic energy \( K_i \) of an individual connection \( i \) is obtained by integrating the kinetic energy of the differential mass along the arm:

\[
K_i = \int d_{ki}
\]

(4.14)

where: \( K_i \) is the kinetic energy of the differential mass of the element fixed in the \( i \) joint in the \( r_i \) point. This kinetic energy is dependent of its mass and speed, \( d_{ki} \), respectively \( v_i \). The \( v_i \) speed of the \( r_i \) point is obtained through the differentiation of the global position of the vector \( r_i^0 \):

\[
v_i^0 = \frac{d}{dt}\left(A_i^0 \times r_i^0\right) = \frac{d}{dt}\left(A_i^0 \times A_i^1 \times \ldots \times A_i^n r_i^0\right)
\]

(4.15)

Replacing the time derivate with the variables of the \( q_i \) joint, the speed of the \( r_i \) point can be described as follows:

\[
v_i^0 = \left(\sum_{j=1}^{n} \delta A_i^0 \dot{q} \right) r_i = \left(\sum_{j=1}^{n} u_{ij} \dot{q}_j \right) r_i
\]

(4.16)
where: the matrix \( \mathbf{u}_q = \frac{\partial \mathbf{q}}{\partial \mathbf{q}} \) defines the speed of a fixed point within a joint \( i \) in the global coordinate system as a function of the variables for the \( q \) joint.

The kinetic energy of an \( i \) joint can be expressed as follows:

\[
K_i = \frac{1}{2} \text{Trace} \left( \sum_{p=1}^{N} \sum_{r=1}^{N} \mathbf{u}_{pr} \mathbf{J}_i \mathbf{u}_i^T \mathbf{q}_p \mathbf{q}_r \right)
\]

(4.17)

where \( \mathbf{J}_i \) is the pseudo-inertial matrix for the \( i \) arm, and is calculated using the relation (4.18):

\[
\mathbf{J}_i = \int \mathbf{r}_r \mathbf{r}_d d_m = \begin{bmatrix}
\int x^2 d_m & \int x y d_m & \int x z d_m & \int x d_m \\
\int y x d_m & \int y^2 d_m & \int y z d_m & \int y d_m \\
\int z x d_m & \int z y d_m & \int z^2 d_m & \int z d_m \\
\int d_x d_m & \int d_y d_m & \int d_z d_m & \int d_m
\end{bmatrix}
\]

V. THE ANALYSIS OF THE OPERATING SPACE (MOVEMENT)

The operating space is the space where we can find the final effector in operation, for the performance of the functions, according to the destination and the established features. Generally, it is a volume, particularly a surface. In the considered case, the movement range of the distal exterior vertebrae (instruments) is considered as a volume and namely a cone with the vertex in the origin (figure 8). The designed joints can be tilted up to ±10 degrees. The angle was established by the major dimensions of the exterior vertebrae, respectively the interior ones. To determine the shape and the size of the operating space, we will create a software application that permits to introduce the dimensions of the elements and the length of the fixed element, and to simulate the movement and the determination of the kinematic parameters of the simplified model of a robotic arm, there will be developed a model using the Simulink-Sim Mechanics software from the Matlab programme.

![Operating space](image)

Fig. 8. The operating space (movement) of the poly articulated robotic arm

VI. CONCLUSION

The poly articulated technical models are complex structures, with multiple vertebrae, that operate in the three dimensional space. They perform functions of exploration, inspection, intervention and gripping in operating spaces with reduced restrictions. It is described the constructive solution for the virtual model. The virtual prototype is conceived as a parameterized model, easy adaptable for various interventions, depending on the sizes of the discrete elements. The mathematic modelation of the poly articulated robotic arm takes into consideration both the kinematic approach and the dynamic one. The direct kinematics is used to test the models of inversed kinematics. The inversed kinematics is used for the complete simulation for the movement of the robotic arm. In this respect, we consider for each segment the rotation angles around the arises of the vertebrae. The resulted elements using the technology of 3D Rapid prototyping are within the standards imposed by the shape of the size, the structure and the performance time (from the CAD models to physical models made of plastic material).

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