Triaxial Accelerometer Static Calibration

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Abstract— **A new approach is studied to calibrate a low-cost, microelectromechanical system (MEMS) triaxial accelerometer. The accelerometer is modeled as a linear system with inter-axis misalignment correction. Mathematically, at least nine different equations, equivalent to nine positions of an accelerometer, are needed to solve for nine unknowns (3 scale factors, 3 zero bias, 3 misalignment angles). However, a new approach is studied to minimize the number of positions needed. Three new linear equations corresponding to each axis are formulated. To validate the equations, a triaxial accelerometer was positioned at twelve known orientations and its outputs were measured. All possible combination of positions were attempted iteratively. Singularity is identified using the principle of matrix rank. Results from twelve positions and six positions were almost identical, suggesting that six positions are adequate to solve nine unknowns.**

Index Terms— **triaxial accelerometer, static calibration, scale factors, misalignment, zero bias**

I. INTRODUCTION

MEMS type accelerometers are popular nowadays in lots of motion detection applications [1,2] such as navigation system, portable gait analysis, etc. Static calibration is crucial to determine the coefficient matrix and zero bias of a linear modeled accelerometer before use. Conventionally, rotary table is used to calibrate an accelerometer. However a precision rotary table is usually expensive and the process is time-consuming [3,4]. Some researchers [2,3,6] had promoted in-use calibration that optimizes a cost function of an accelerometer. However, the accuracy of the estimation is greatly dependant on the experiment process, the quasi-static detection determinant and the optimization algorithm. This paper proposes a possible solution to calibrate a triaxial accelerometer that utilizes minimum static positions, using a simple test rig. The scale factors, misalignment angles and zero bias are assumed to be invariant to time. The outputs is assumed to display ignorable Gaussian White noises.

II. THEORY

A. Accelerometer

An inertial measurement unit of five degree of freedom (IMU-5DOF) in the form of a PCB breakout (SparkFun Inc.) as shown in Fig. 1a was used. It consists of a triaxial (X,Y,Z) accelerometer (ADXL335, Analog Device) and a

dual axial (X_R, Y_R) gyroscope (IDG500, InvenSense Inc.). It is light weight (~2g) and small size (20x23mm). The axes assignments follow the recommendation of ADXL335 datasheet. The direction of axes assignments follow the norm that the accelerometer axis points upward against the gravity (Fig. 1b). Accelerometer is ratiometric [8], that means zero bias and sensitivity varies relative to the input voltage change. Investigation of a gyroscope is out of the scope of this paper.
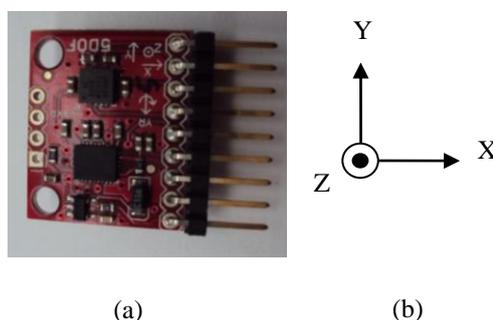

(a)　　　　　　　(b)

Fig. 1: (a) IMU-5DOF, (b) Accelerometer Axis assignment
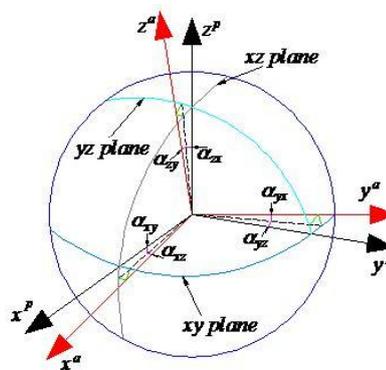
B. Linear Model With Error Correction

Fig. 2: Inter-axis misalignment between an accelerometer coordinate and a platform coordinate. Reproduced from [6,7]

The output precision of an accelerometer suffers from small angle inter-axis misalignments (α_{xy} α_{xz} α_{yx} α_{yz} α_{zx} α_{zy}) as shown in Fig. 2 between the sensor axis and the platform coordinate. Skog [6] showed that the error model can be simplified by assuming x^p coincides with x^a as shown in (1), where s^p and s^a are g-vector in the platform coordinate and the sensor axis respectively. T_a^p is a transformation matrix or named as correction matrix that maps s^a to s^p.

$$s^p = T_a^p s^a, \qquad T_a^p = \begin{bmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ 0 & 1 & -\alpha_{zx} \\ 0 & 0 & 1 \end{bmatrix} \qquad (1)$$

The sensor can be linearly modeled as (2).

$$V_i = K[T_a^p]^{-1} s_i^p + b \qquad (2)$$

where:

V_i Output vector $[v_x\ v_y\ v_z]^T$ at i-sample in volts

K Diagonal matrix of scale factors. $Diag([k_x\ k_y\ k_z])$ in volts/g

s_i^p g-force vector in platform coordinate $[g_x\ g_y\ g_z]^T$ at i-sample

T_a^p inter-axis misalignment correction matrix

b Zero bias vector of each axis $[b_x\ b_y\ b_z]^T$ in volts

From (2), nine setup parameters as listed in (3) are of interest and must be solved.

$$\begin{bmatrix} k_x & k_y & k_z & \alpha_{yz} & \alpha_{zy} & \alpha_{zx} & b_x & b_y & b_z \end{bmatrix} \qquad (3)$$

Mathematically, (2) needs at least nine different samples to solve if there exists a solution. However, the author attempts to solve the problem with less samples. Equation (2) is expanded into (4).

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_i = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix} \begin{bmatrix} 1 & -\alpha_{yz} & \alpha_{zy} \\ 0 & 1 & -\alpha_{zx} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix}_i + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix}$$

$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_i = \begin{bmatrix} b_x + k_x g_{xi} + \alpha_{yz} k_x g_{yi} - k_x g_{zi}(\alpha_{zy} - \alpha_{yz}\alpha_{zx}) \\ b_y + k_y g_{yi} + \alpha_{zx} k_y g_{zi} \\ b_z + k_z g_{zi} \end{bmatrix} \qquad (4)$$

It is obvious that Z-axis is the simplest and only two unknowns $(b_z\ k_z)$, followed by Y-axis with three unknowns $(b_y\ k_y\ \alpha_{zx})$ and lastly X-axis with four unknowns $(b_x\ k_x\ \alpha_{yz}\ \alpha_{zy})$. Hence, using i-number of samples corresponding to the number of unknowns in each axis, (4) can be broken into three linear equations in Z, Y and X axis accordingly as listed in (5), (6) and (7).

$$\begin{bmatrix} v_{z1} \\ v_{z2} \end{bmatrix} = \begin{bmatrix} 1 & g_{z1} \\ 1 & g_{z2} \end{bmatrix} \begin{bmatrix} b_z \\ k_z \end{bmatrix} \qquad (5)$$

$$\begin{bmatrix} v_{y1} \\ v_{y2} \\ v_{y3} \end{bmatrix} = \begin{bmatrix} 1 & g_{y1} & g_{z1} \\ 1 & g_{y2} & g_{z2} \\ 1 & g_{y3} & g_{z3} \end{bmatrix} \begin{bmatrix} b_y \\ k_y \\ k_{yzx} \end{bmatrix} \qquad (6)$$

where:

$$\alpha_{zx} = \frac{k_{yzx}}{k_y}$$

$$\begin{bmatrix} v_{x1} \\ v_{x2} \\ v_{x3} \\ v_{x4} \end{bmatrix} = \begin{bmatrix} 1 & g_{x1} & g_{y1} & -g_{z1} \\ 1 & g_{x2} & g_{y2} & -g_{z2} \\ 1 & g_{x3} & g_{y3} & -g_{z3} \\ 1 & g_{x4} & g_{y4} & -g_{z4} \end{bmatrix} \begin{bmatrix} b_x \\ k_x \\ k_{xyz} \\ k_{xzy} \end{bmatrix} \qquad (7)$$

where:

$$\alpha_{yz} = \frac{k_{xyz}}{k_x}$$

$$\alpha_{zy} = \frac{k_{xzy}}{k_x} + \alpha_{yz}\alpha_{zx}$$

Equation (4), (5) and (6) are arranged in the form of a linear algebra, $y = Ax$. For known outputs and coefficient matrix, solving for x is simply by multiplying the inverse of the coefficient matrix with its outputs, $x = A^{-1}y$, if there exits an solution. A solution of a linear equation is non-singular and unique if $rank(A) = rank(A|y) = number\ of\ variables$.

III. EXPERIMENT

A simple test rig (Fig. 3) consisting of an adjustable platform, a cube mounted with a triaxial accelerometer and a V-block, was built to perform these twelve positions. The author utilizes twelve static known positions proposed by Hung [5], to validate (4), (5) and (6). These positions are listed in Fig. 4. Before experiments, the platform was adjusted horizontally using an electronic spirit level (resolution, 0.1^0). Theoretically, it can be any arbitrary known angles but for uniformity, all non-orthogonal positions which are in $\pm45^0$ with respect to the platform, i.e. positions (1,3,5,9,10,12), are experimented, by placing the cube inside a V-block. Orthogonal positions (2,4,6,7,8,11) were performed without a V-block. The sensor outputs were sampled for approximately 10 seconds per position, at 200Hz sampling rate. G-vectors in the platform coordinate in all known positions are listed in Table 1. Mean sensor outputs with maximum standard deviation of $4.7 \times 10^{-5}V$ were observed, suggesting ignorable noises at twelve positions. Mean sensor outputs at twelve positions are illustrated in Fig. 5.
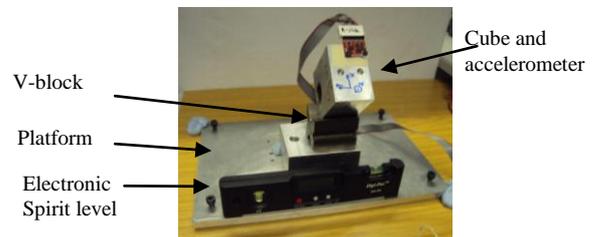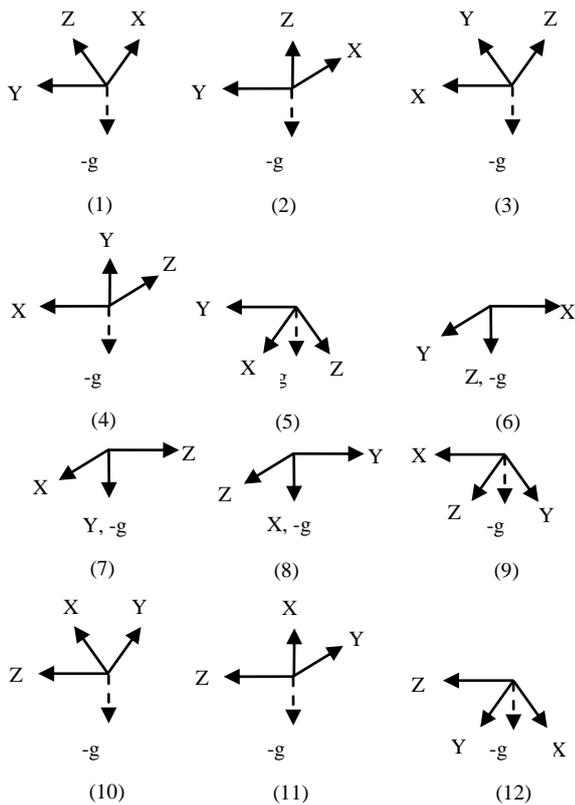

Fig. 3: Experiment Setup

-g indicates the gravity vector

**Fig. 4: Twelve positions in platform coordinate.
Reproduced from [5]**

Table 1: G-vector in platform coordinate at twelve positions ((g_x, g_y, g_z in unit of g)

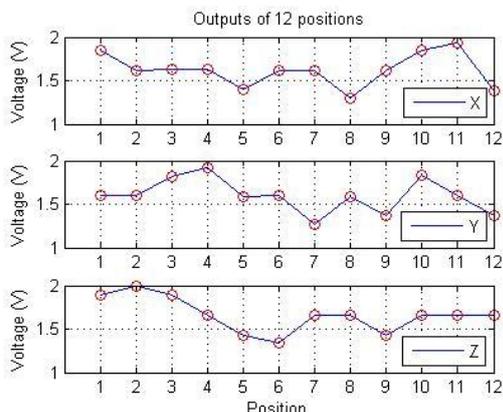| Pos.	g_x	g_y	g_z
1	0.7071	0.0000	0.7071
2	0.0000	0.0000	1.0000
3	0.0000	0.7071	0.7071
4	0.0000	1.0000	0.0000
5	-0.7071	0.0000	-0.7071
6	0.0000	0.0000	-1.0000
7	0.0000	-1.0000	0.0000
8	-1.0000	0.0000	0.0000
9	0.0000	-0.7071	-0.7071
10	0.7071	0.7071	0.0000
11	1.0000	0.0000	0.0000
12	-0.7071	-0.7071	0.0000

Fig. 5: Outputs at twelve positions

IV. RESULTS AND ANALYSIS

A. Analysis Method

Two set of positions were analyzed for comparison purposes, i.e. Set1 (8), all twelve positions and Set2 (9), only the orthogonal positions.

$$Set1 = [1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12] \qquad (8)$$

$$Set2 = [2\ \ 4\ \ 6\ 7\ 8\ 11] \qquad (9)$$

According to the (5), (6), (7), the number of combinations or iterations for solutions, can be calculated using (10) and they are listed in Table 2. For example, in twelve-known positions, two known positions are needed to determine two unknowns as referring to (5). The combination of two out of twelve positions is $\binom{12}{2}$, equivalent to 66 combinations.

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!} \qquad (10)$$

Table 2: Combinations for Set1 and Set2

Twelve-known Positions (n)			
	X	Y	Z
unknowns (k)	4	3	2
Combination	495	220	66

Six-known Positions (n)			
	X	Y	Z
unknowns (k)	4	3	2
Combination	15	20	15

Each combination will produce a solution if the linear equation is non-singular and unique. A Matlab m-file was programmed to generate the results iteratively.

B. Results

Means and standard deviations (μ_{12}, s_{12}) of non-singular solutions using twelve positions are listed in Table 3. Low variations in both scale factors and zero bias (max. at 0.0069), suggest that the solutions are consistent for all non-singular solutions. Although the ratios of mean and standard deviation of misalignment angles appear to be larger than the ratios of mean and standard deviation of scale factor and zero bias respectively, but misalignment angles are still kept within small angle range with maximum mean at 0.0101 radian. This agrees with the initial assumptions during the formation of the sensor linear model with error correction. Not all combinations will generate a solution. Table 4 lists the solution check for the combinations using twelve positions. Non-singular solutions occupy a larger portion of the possible combinations. No combination is leaved unchecked.

Table 3: Setup Parameters using 12 positions

	k_x	k_y	k_z
μ_{12}	0.3223	0.3253	0.3271
s_{12}	0.0014	0.0069	0.0026
	α_{yz}	α_{zy}	α_{zx}
μ_{12}	0.0101	-0.0037	-0.0017
s_{12}	0.0049	0.0047	0.0205
	b_x	b_y	b_z
μ_{12}	1.6199	1.5951	1.6627
s_{12}	0.0011	0.0049	0.0017

Table 4: Solution Check for 12 positions

	Singular	Non-Singular	Total
X	63	432	495
Y	44	176	220
Z	17	49	66

On the other hand, means and standard deviations (μ_6, s_6) of non-singular solutions using six orthogonal positions are listed in Table 5. Low variations in scale factors and zero biases suggest that the solutions are consistent for all non-singular solution. Misalignment angles are kept within small angle range. In Table 6, non-singular solutions occupy a larger portion of the combinations.

Comparing to twelve positions, the number of combinations in six positions are greatly reduced. Careful observations into combinations which produce singularity during computation indicate no solution if identical axis within a combination is in horizontal plane. E.g. no solution for Y-axis in position (2,6,8) since Y-axis in each position is in horizontal plane.

Table 5: Setup Parameters using 6 positions

	k_x	k_y	k_z
μ_6	0.3222	0.3246	0.3274
s_6	0.0006	0.0028	0.0027
	α_{yz}	α_{zy}	α_{zx}
μ_6	0.0095	-0.0046	-0.0053
s_6	0.0032	0.0026	0.0091
	b_x	b_y	b_z
μ_6	1.6198	1.5954	1.6626
s_6	0.0006	0.0028	0.0023

Table 6: Solution Check for 6 positions

	Singular	Non-Singular	Total
X	3	12	15
Y	8	12	20
Z	6	9	15

For comparison purposes, a percentage difference as defined in (8) is formulated. In Table 7, low difference percentages in scale factors and zero bias suggest that six positions are adequate to calculate the setup parameters. Although one of the misalignment angle displayed more than 200% discrepancy but in general all misalignment angles are kept within small angle range.

$$Diff = \frac{\mu_{12} - \mu_6}{\mu_{12}}\%$$

(8)

Table 7: Comparison of 12 and 6 positions

	k_x	k_y	k_z
μ_{12}	0.3223	0.3253	0.3271
μ_6	0.3222	0.3246	0.3274
Diff	0.02%	0.21%	-0.07%
	α_{yz}	α_{zy}	α_{zx}
μ_{12}	0.0101	-0.0037	-0.0017
μ_6	0.0095	-0.0046	-0.0053
Diff	5.51%	-23.26%	-213.41%
	b_x	b_y	b_z
μ_{12}	1.6199	1.5951	1.6627
μ_6	1.6198	1.5954	1.6626
Diff	0.00%	-0.02%	0.00%

V. CONCLUSION

Static calibration is crucial to determine the setup parameters that model a linear equation of a triaxial accelerometer. A triaxial accelerometer linear model with correction based on previous works is further expanded and reorganized. Three new linear models in X, Y and Z-axis are reformed in the linear algebra format. Non-singular solution could be determined using the principle of matrix rank. An experiment was performed to validate this new method. Twelve different known positions of a triaxial accelerometer are experimented. Collected output data are tested for a set of twelve positions and another set of six orthogonal positions. The results suggest that using six orthogonal positions could solve for nine setup parameters.

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