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Design of a Switching PID Controller for a Magnetically Actuated Mass Spring Damper

Shabnam Armaghan, Arefeh Moridi and Ali Khaki Sedigh

***Abstract*— In this paper the use of proportional-integral-derivative (PID) switching controllers is proposed for the control of a magnetically actuated mass-spring-damper system which is composed of two masses M_1 and M_2; each mass is jointed to its own spring. Two different modes occur during the system motion; a PID controller is designed for each mode and a switching logic is applied in order to recognize the system's position to switch to the proper controller. Finally, simulation results are employed to show the performance of the proposed switched PID controller. Also, comparison results with the previously used model predictive controller (MPC) are provided.**

***Index Terms*—Hybrid systems, MPC, PID control, switched systems**

I. Introduction

DYNAMICAL systems introduced by interaction between continuous and discrete dynamics, namely the hybrid systems, are widely used in many industrial plants [1]. Hence, hybrid control has received considerable attention during the past two decade, and the class of switching systems are specifically employed in many industrial applications [2]. A switched system consists of several subsystems and a switching law that selects the proper subsystem. Previous research [3] showed that MPC can be applied to the recently proposed mechanical plant. Design and implementation of MPC can be complex and time consuming. These complications motivated the present research to design a conventional PID controller for the plant.

In this paper, a switching PID controller algorithm is applied to the proposed mechanical plant in [4], which is a difficult control benchmark.

The paper starts by briefly describing the mechanical system in section II. The PID controller and its integrator wind up are considered in section III. In section IV, the switching strategy and the proposed switching scheme are briefly discussed and also simulation results are provided to show the effectiveness of the proposed algorithm. Conclusions are given in section V.

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A. Khaki Sedigh is Professor at the School of Electrical Engineering, K. N. Toosi University of Technology, Shariati Avenue, Tehran, Iran, (corresponding author. phone: +98-21-84062317; e-mail: Sedigh@kntu.ac.ir).

Sh. Armaghan is MSc Student in the School of Electronical Educations of Shiraz University, Zand St, Shiraz, Iran, (email:871017@st.vus.ir).

A. Moridi is MSc Student in the School of Electronical Educations of Shiraz University, Zand St, Shiraz, Iran, (email:871037@st.vus.ir).

II. System Description

The considered mechanical system is composed of two subsystems which are masses M_1 and M_2 connected by two springs with stiffness k_1 and k_2 respectively. The schematics of the system is shown in Figure 1. Mass M_1 is pulled by a force F which actuates the system and is given by [4]:

$$F = \frac{k_a i^2}{(x + k_b)^2} \tag{1}$$

where $x = x_{eq1} - x_1$ is the distance between the active mass and the actuator, k_a, k_b are constant parameters and i is the current that passes through the coil. The main aim is to make the position of mass M_1 (x_1) track as fast as possible an external reference with a small control effort. Note that the position and velocity of mass M_2 are not controllable.

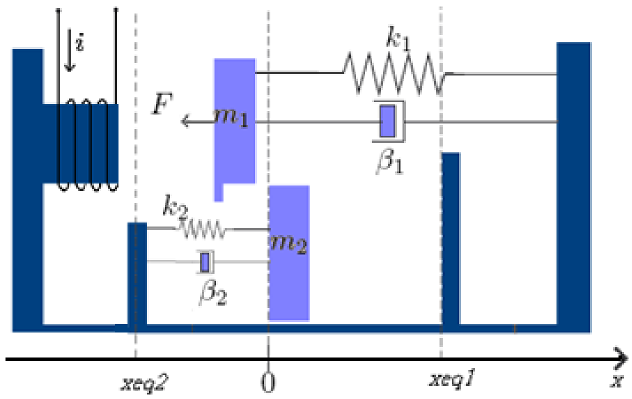

Fig. 1. Mass spring damper system schematics [3].

In conclusion, the system is defined by the following differential equations:

$$\ddot{x}_1 = \frac{1}{M_1}\left[-F - k_1\left(x_1 - x_{eq1}\right) - b_1(\dot{x}_1)\right]$$

$$\ddot{x}_2 = \frac{1}{M_2}\left[-k_2\left(x_2 - x_{eq2}\right) - b_2(\dot{x}_2)\right] \tag{2}$$

where k_1 and k_2 are spring stiffness, b_1 and b_2 are friction coefficient of dampers. For modeling the multibody automotive mechatronic actuator, hybrid dynamical systems, are used. Here we consider two operating modes:

- M_1 and M_2 are detached, M_1 moves freely and M_2 is at the stop point (x_2=0). This mode is described by the following equations:

$$\ddot{x}_1 = \frac{1}{M_1}\left[-F - k_1\left(x_1 - x_{eq1}\right) - b_1(\dot{x}_1)\right]$$

$$x_2 = 0, \dot{x}_2 = 0. \quad (3)$$

- M_1 and M_2 move together. The system equations become:

$$\ddot{x}_1 = \frac{1}{M_1 + M_2}\left[-F - k_1\left(x_1 - x_{eq1}\right) - k_2\left(x_2 - x_{eq2}\right) - (b_1 + b_2)\dot{x}_1\right]$$

$$x_2 = x_1, \quad \dot{x}_2 = \dot{x}_1 . \quad (4)$$

We assume that only M_1 position (x_1) is available.

A. State Space of Each Mode

The state space model of the mechanical system for the first mode, when the masses are not in contact, is as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{-k_1}{M_1} & \frac{-b_1}{M_1} \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 & 1 \\ \frac{-1}{M_1} & \frac{k_1 x_{eq1}}{M_1} \end{bmatrix}$$
$$C_1 = [1 \quad 0]$$
$$D_1 = [0 \quad 0] \quad (5)$$

For the second mode, where the two masses are in contact, the state space has the following specific form:

$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{-(k_1 + k_2)}{M_1 + M_2} & \frac{-(b_1 + b_2)}{M_1 + M_2} \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0 & 1 \\ \frac{-1}{M_1 + M_2} & \frac{k_1 x_{eq1} + k_2 x_{eq2}}{M_1 + M_2} \end{bmatrix}$$
$$C_2 = [1 \quad 0]$$
$$D_2 = [0 \quad 0] \quad (6)$$

For the above state space equations, we determine x_1 (position of M_1) and $\dot{x}_1$ (velocity of M_1) as system states. As mentioned before, x_1 is the main output (that is controllable).

III. PID CONTROLLER DESIGN

In this section, the design of PID control is briefly described. The Proportional Integral Derivative (PID) controller is the most commonly used controller in control system engineering [5], and [6]. The PID controller is applied to derive the set point tracking error between the measured output value and a desired set point to zero. A variety of PID tuning strategies are available in the literature. The most common tuning methods used are the Ziegler-Nichols based tuning methods [7].

In this paper, the Ziegler-Nichols method is used to obtain the initial PID parameters and these parameters are further refined to achieve the desired performance. For the present mechanical system (mass spring), two PID controllers are designed for each mode. As the control variables in the designed system reach the actuators limits, the feedback loop is broken and the error will continue to integrate. Integral action in a PID controller is an unstable mode and therefore drifts to very large values.

This is the integrator windup and to overcome it, we require that the error has opposite sign as long as the system recovers. Hence, the anti wind up PID controllers are used [7]. Figure 2 depicts a block diagram of a PID controller with anti-windup.

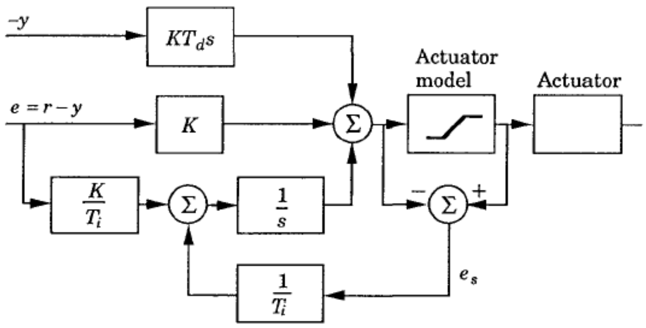

Fig. 2. PID Controller with anti-windup [7].

IV. SWITCHING CONTROL LOGIC

Switching control systems are widely studied and used by control and systems engineers [8]. A switched system can be described by a family of subsystems and a rule that coordinates the switching between the subsystems, as shown in fig. 3.

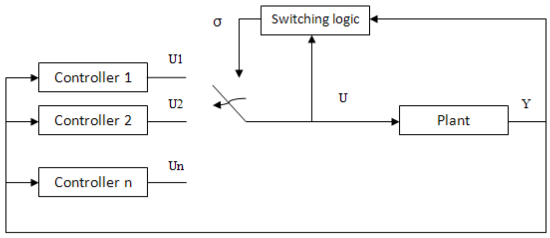

Fig. 3. Block Diagram of a Switching System

In general, a switched system is defined by the following equation:

$$\dot{x} = f_\sigma(x), x \in R^n \quad (7)$$

where $\boldsymbol{\sigma}$ is a piecewise constant signal that is called the switching signal [8]. Several switching methods in control systems such as state-dependent versus time-dependent switching, autonomous (uncontrolled) versus controlled switching, chattering, and slow switching, etc, have already been introduced [1] .

Here, the distance between M_1 and M_2 (x_1- x_2) is applied as the switching logic. Whereas this difference is positive, M_1 has reached M_2 (mode 2). The proposed switching scheme is a class of state-dependent switching.

V. SIMULATION RESULTS AND COMPARISON STUDIES

In this section, simulation studies were carried on two mechanics systems.

Case (a)

Referring to section II consider the mass-spring system consists of two masses M_1=1 Kg and M_2=5 Kg. The masses are impressed by springs with stiffness K_1=1 N/m, K_2=0.1 N/m. The dampers property is defined with friction coefficient in β_1=0.3 NS/m and β_2=0.8 NS/m. Furthermore,

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the values of x_{eq1}, x_{eq2} defined in section II are 10, -10 respectively. The system behavior is simulated for a square wave during 100s. The results are reported in fig. 4.

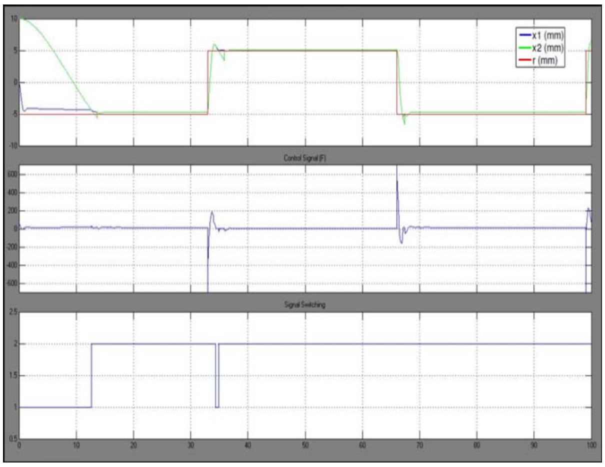

Fig. 4. Switching PID control system: (1) position of masses, (2) control signal (F) and (3) switching signal.

As we can see, the position of $M_1(x_1)$, meets our aim which is tracking the given reference. Switching signal in figure 4 shows which PID controller is active during the simulation. The first PID is designed for mode 1 (M_1 moves freely), the second one is designed for the other mode. In the meantime, switching is occurred exactly where the masses move together. Also we can see that the amplitude of the control signal is increased unlimitedly. Problem occurs due to the integrator windup phenomenon. The practical solution to overcome this problem is using Anti-windup procedure as was discussed in section III. Fig. 5 illustrates what happens when a controller with anti-windup is applied to the system simulated in fig. 4. We can find out that the controller output is limited by using this method.

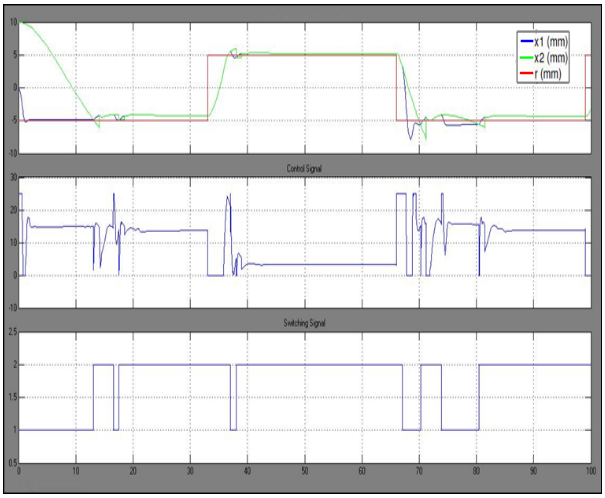

Fig. 5. Switching PID control system by using anti windup

Case (b)

Consider the system defined in case (a). We have just changed the parameters as follows:

M_1=0.08 Kg; M_2=0.07 Kg.

K_1=1.5 . 10^5 N/m; K_2=1.5 . 10^5 N/m.

β_1=15 NS/m; β_2=15 NS/m.

x_{eq1}= 4 mm; x_{eq2}= -0.5 mm.

In this case, we can compare our results with the MPC designed in [3]. The results are shown in Fig. 6, 7. A comparison of the closed loop responses obtained by the MPC and the proposed switching PID controller proposed in this paper reveals that the closed loop performances are fairly similar. However, the simplicity and ease of implementation of the PID controller are the main merits of the proposed design. Also, note that the simple switching logic used makes the PID control of this rather complex mechanical plant practical.

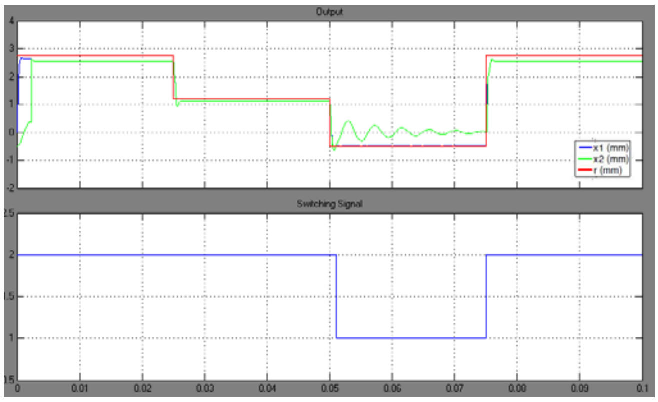

Fig. 6. Switching PID control system: (1) position of masses and (2) switching signal.

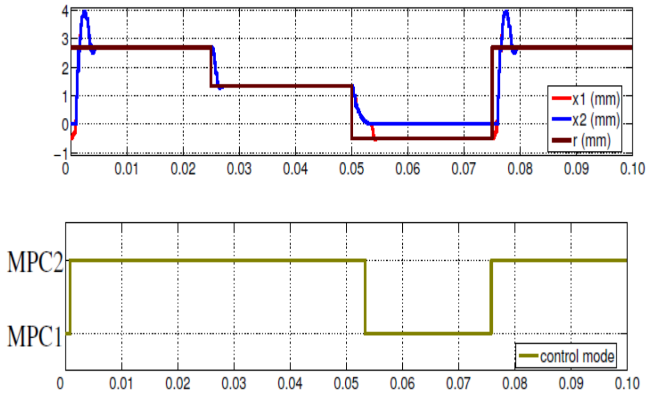

Fig. 7. Switching MPC control system: (1) position of masses and (2) switching signal [3].

VI. CONCLUSION

In this paper, an electromagnetically actuated mass spring damper which is common in automotive systems is considered. By applying switching PID controllers, the position of M_1 tracks a given set point reference. The proposed switching control scheme is in the class of state-dependent switching strategies. The simulation results show that with the conventional PID and advanced MPC controllers the switching logic is almost the same. The closed loop performances are comparable. However, the simplicity of design and implementation are the main virtues of the PID scheme.

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