Abstract—Grasping is concerned with characterizing and achieving the conditions that will ensure that a robot gripper holds an object securely, preventing, for example, any motion due to external forces. A system where in a desired object is gripped by the fingers of a robot (or human) and is generally called a grasp. Most of past researches are restricted to tip prehension grasp (object gripped by fingertips only). In the present work, a grasp is used often to mean the grasped object itself, and this is also true with the literature. The Theory of grasping internal forces, formulated in the form of equal and opposite pairs of forces acting along the lines of contact, can be used to selectively orient the net force vector. A grasp situation, which satisfies this condition, is a stable grasp. This Paper deals with the optimization to obtain the most stable grasp for a nominated set of contact points and loading on an object. The equilibrating forces have been calculated on the basis of algorithms developed. The values of friction angles are optimized so as to satisfy the condition of stable grasp. The unit cube considered for these calculations was assumed to be loaded under its own weight (no moment appears due to the weight of the body because of symmetry; there are no external moments either) and had its body diagonal coincident with the z-axis. The body diagonal of the cube is now shifted from the z-axis and the cube is subjected to what is known as quasi-static motion. The study concludes that the stable grasp for a nominated set of contact points and loading condition is obtained at maximum friction angles and minimum contact points.

Index Terms—Algorithm, Equilibrating, forces friction angles, quasi-static motion, Robotic Grasping.

I. INTRODUCTION

Over the last two decades, grasping has showed a somewhat marginal topic to an important field of robotics research. This increasing interest in grasping is partly due to the evolution of industrial automation towards flexible automation. The transition from large batch size to medium and small sizes has led to the replacement of special purpose devices with more general purpose and effectors enabling the manipulation of a broader class of objects. At the same time, more attention has been given to fine manipulation algorithms.

This has pointed out the needs for tools able to increase the robot’s manipulative capacity with fine position and force control. Therefore, as end effector becomes more flexible, control becomes more complex and a better understanding of grasping turns out to be a challenging issue [1].

Regarding the grasping and manipulation by a robot hand, many papers have been published [2]-[6]. Authors have reported various multi-fingered hands for manipulating objects skills fully. They have also done analytical studies of grasping and manipulation by robot hand. Numerous factors are available to determine the effective grasp of an object. Researchers [7]-[10] in robotics have tried to analyze, what constitutes good grasps. Nine analytic measures were shown, for describing a grasp, compliance, connectivity, force closure, from closer, grasp isotropy, internal forces, manipulability, resistance to slipping and stability [11]. Multitude of properties that an articulated grasp must possess in order to able to perform everyday tasks similar to those performed by human hands has been discussed [12]. Even after these studies, mechanics of grasping, manipulation & grasp properties have not yet fully been understood so far [13]. This understanding is important for designing robot hands and for developing grasping and manipulation algorithms.

II. METHODOLOGY

Minimums of three fingers are needed to grasp an object. In carrying out a sequence of motion a particular finger may reach a joint limit or start slipping. It then becomes necessary to relieve that finger with an unused finger. Hence dexterous manipulability requires at least four fingers [14]. Even simple operations in real life, like turning a coke can about its axis of symmetry require phasing in and out of the fingers. A algorithm is developed for Computation of contact force at the contact point, when to phase out a finger.

The algorithm developed will be used to numerically optimize the values of the friction angles (\(\theta_i\)) as well as to satisfy the stable grasp requirement. However, a numerical computation of friction angles can only offer coherence to the theory behind the entire analysis but also reveals a few interesting trends. The unit cube considered for these calculations was assumed to be loaded under its own weight (no moment appears due to the weight of the body because of symmetry; there are no external moments either) and had its body diagonal coincident with the z-axis. The body diagonal of the cube is now shifted from the z-axis and the cube is subjected to what is known as quasi-static motion. That is, the cube is now set in motion and moves about the \(0.707 \hat{i} - 0.707 \hat{j}\) axis from -15° to +15°, in intervals of 1°. The interaction forces and angles are recomputed for every such interval and the trend is plotted.
A. The point contact

The development in this section is algebraic. Whenever possible, a parallel geometric interpretation of the result will be shown. Vectorially the force system can be represented for a three-point contact by

\[
P_1 = F_1 + k_1 u_1 - k_2 u_2 \\
P_2 = F_2 - k_3 u_3 - k_2 u_2 \\
P_3 = F_3 - k_1 u_1 - k_3 u_3
\]

Where the vector \( P_i \) are the net forces and the vectors \( F_i \) are the equilibrating forces at point of contact \( \text{‘}i\text{’} \). The \( u_i \) are the unit direction vectors along which the equilibrating forces are to be applied and \( k_i \) is the associated scalar factor. Correspondingly the friction angle \( \theta \), at a point is obtained from the relationship

\[
\cos \theta = \frac{n_i \cdot P_j}{|P_j|}
\]

Where \( n_i \) is the unit normal to the surface at contact point \( i \).

The limiting friction problem can be formulated as that of finding a set of internal forces such that the angle \( \theta \) does not exceed the maximum allowable friction angle. A grasp situation, which satisfies this condition, is a stable grasp. The development below produces a set of equations, solution of which yields the most stable grasp possible using a given set of contact points. This achieved by maximizing the minimum of the three friction angles for a set of three contact points. The grasp plane is the plane containing the three points of contact.

Consider

\[
\cos \theta = \frac{n_i(F_1 k_1 + k_2 u_2)}{|F_1 + k_4 u_1 - k_2 u_2|} \quad (3)
\]

For the following section, we shall assume that the inward drawn contact normal and the equilibrating forces are on the same side of the grasp plane for all the contact locations. If this condition is violated, zero friction angles cannot be achieved by manipulating the interaction force field.

B. A Unique Minimum value of Friction Angle \((\theta)\)

As stated above that the object we chose to work on was a unit cube. The notations used in equation system (1) can be modified slightly to include subscripts which clarify the directions of the vectors involved. We let the subscript ‘ij’ denote that a vector is directed from point \( i \) to point \( j \). When used with the \( k \) terms (which are scalars) it denotes the direction in which the interaction forces pertinent to that point are present. We then have,

\[
P_1 = F_1 + k_{12} u_{12} + k_{13} u_{13} \\
P_2 = F_2 + k_{23} u_{23} + k_{21} u_{21} \\
P_3 = F_3 + k_{31} u_{31} + k_{32} u_{32}
\]

The interaction forces in this form still constitute a null solution as \( u_{ij} = -u_{ji} \). Again, in equation \( k_{ij} = k_{ji} \).

In the previous section it has been shown analytically that there is a unique value of \( k_{12} \) and \( k_{13} \) for which \( \cos \theta \) equals unity. Using equation (2) and (3) a MATLAB code was written (listed in Appendix-A) and a mesh surface was plotted (figure 2) for a hundred values of \( k_{12} \) and \( k_{13} \). The plot shows the existence of those values.

The unit cube considered for these calculations had its body diagonal coincident with the \( z \) axis. The mathematical manipulations involved in achieving such an orientation require the use of rotation matrices. The rotation matrix expression, which achieves the necessary coordinate transformations, about the axis 0.707 \( \hat{i} \) – 0.707 \( \hat{j} \) is given by,

\[
R_{\phi} = \begin{bmatrix}
    r_x & r_y & r_z \\
    r_y & r_z & r_x \\
    r_z & r_x & r_y
\end{bmatrix} = \begin{bmatrix}
    \cos \phi & -\sin \phi & 0 \\
    \sin \phi & \cos \phi & 0 \\
    0 & 0 & 1
\end{bmatrix} \quad (5)
\]

Where \( \phi \) is the angle that the cube has to be rotated by to align its body diagonal with the \( z \)-axis. The term \( \forall \phi = \cos \phi = 1 - \cos \phi \). \( C \) and \( S \) are the cosine and sine of \( \phi \) respectively. In terms of the object frame, the cube has its \( z \) axis coincident with one of the edges of the cube. From considerations of geometry, the angle \( \phi \) can thus be found to be \( \cos^{-1}(0.5773) = 54.73^\circ \). The terms \( r_x, r_y, \) and \( r_z \) are the direction cosines of the axis of rotation. In this case, they are respectively 0.707, 0.707, and 0.

C. Cube Motion: The Special Case of Complete Symmetry

One of the ways to grasp a cube with 4-fingers is in a way such that 4 fingers are on four adjacent faces of the cube subject to the condition that no three faces are intersection. However, this type of a grip limits the size of the cube which can be handled. So, a cliff event grip was chosen – one with three fingers, each touching one of the intersecting faces at the corner. It is obvious that with such an arrangement, we will have to hold the cube vertically else it will fall down.

The cube when oriented with the body diagonal coincident with the \( z \) axis presents a case of special interest. Let the angle of rotation (for the quasi static motion mentioned above) be represented by \( \alpha \). When the body diagonal is coincident with the \( z \) axis, \( \alpha \) equals zero. The cube in this condition is symmetrical in all respects. For simplicity of calculation, center-points of the cube faces are chosen as the points of contact. This further lends to the symmetry of the case. For all the calculations the cube is assumed to be loaded under its own weight. The weight vector for all cases is assumed to be directed along the \(-z\) axis. The weight vector will pass through the center of gravity. Since the reference for all the calculations takes the centroid as the origin, for any \( \alpha \) the weight vector will create a moment about the centroid, the lever arm being the
that the three contact points make is then equilateral. Further the grasp plane is perpendicular to the z axis. Consider one contact point. Since all the $k_{ij}$ are equal the analysis is valid for all the three points. The resultant interaction force vector will have magnitude $2k \cos 30^\circ = k\sqrt{3}$. Let the weight of the cube be $w$. For reasons discussed above, the equilibrating forces at each contact point are $w/3$. The minimax problem states that it is desirable to minimize the friction angle. For an angle of zero, the applied contact force $P$ will be in the direction of the surface normal. The surface normal itself is inclined to the $z$ axis (and thus the weight vector) by an angle $\phi = 57.4^\circ$ approximately. Hence the cosine component of $P$ will balance the fraction of weight seen by it and will thus equal the equilibrating force at that point. The sine component of $P$ (which is parallel to the grasp plane) will equal the interaction force as calculated above. We thus have the following relationships:

$$F \sin \phi = k\sqrt{3} \quad \text{------(6)}$$
$$F \cos \phi = w/3 \quad \text{------(7)}$$

where $k_{ij}$ is replaced by $k$ since all $k_{ij}$ are equal. Dividing the two equations we get,

$$k\sqrt{3} = \frac{w \tan \phi}{3} \quad \text{or,}$$
$$k = \frac{w \tan \phi}{3\sqrt{3}} \quad \text{------(8)}$$

For the case $w = 5$, we get $k = 1.3608$.

The algorithm for calculating the two force fields and optimizing the friction angle was implemented on MATLAB. The optimization routine used was called ‘fminu’ and was an unconstrained nonlinear optimization library routine. The subroutine required an initial value of the variable to be optimized, as one of the arguments. The value for $k$ from equation 8 is an apt value for usage in this unbounded case. It further provides a neat check for the results of the algorithm.

IV. RESULTS

Results obtained by running MATLAB Program and further plotted in figure 2. For the quasi-static motion described earlier, $\alpha$ is allowed to take values form $-15^\circ$ to $+15^\circ$. The equilibrating forces and interaction forces, as well as the optimization for $\theta_i$ are recomputed for each interval of $1^\circ$. The results thus obtained were plotted as a graph of the minimized maximum $\theta_i$ along with the values of all $k_{ij}$ against $\alpha$.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Values</th>
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<tbody>
<tr>
<td>$x$</td>
<td>$k_{12}$</td>
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<tr>
<td>$y$</td>
<td>$k_{13}$</td>
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<tr>
<td>$z$</td>
<td>$\theta_i$</td>
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V. CONCLUSIONS

The findings show that the values of $\theta_i$ closely follow each other without any specific tolerance/constraints. The closeness of the values of $\theta_i$ suggests that achieving equality is a distinct possibility. The advantage that accrues from having a case where $\theta_1 = \theta_2 = \theta_3$ is twofold. Firstly, it allows for eliminating two variables from the set of $k_{ij}$. As a result, an indeterminate system of six unknowns in three equations now becomes a deterministic system that calculates various contact forces. Secondly, the system presents a polynomial solution for those constraints whose efficient evaluation methods are known. Optimum stable grasp for a nominated set of contact points and loading condition is obtained at maximum friction angles and minimum contact points. A numerical computation of the friction angles offers solidity to the theory behind the entire analysis [15]. The study may further be extended to include constraints.

REFERENCES