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Large Deflection Analysis of Clamped Circular Plates

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Abstract— **Geometrically nonlinear bending analysis of clamped circular plates under axisymmetrical transverse load is made in this computational study. The thickness of the plate is considered to be uniform and the plate material is assumed to be isotropic and homogeneous. Since both the plate geometry and the loading are axisymmetric a set of nonlinear ordinary differential equations are solved in the paper. The system of nonlinear algebraic equations which is obtained by the finite difference method is solved by the Newton-Raphson method. The boundary conditions at the support and at the center of the plate are satisfied exactly. The accuracy of the results is verified by checking the maximum deflection with the results available in the literature. In case of uniform pressure almost identical central deflection is obtained.**

Index Terms—**Bending, large deflection, nonlinear, plate**

I. Introduction

Nonlinear analysis is one of the most challenging topics in applied mechanics. Since closed form solutions are available only for a limited number of cases, numerical methods have been used extensively [1]-[3].

When the deflections are small in comparison with the plate thickness, the Kirchhoff plate theory is applicable. However; when the deflections are beyond a certain level, the relation between the external load and the deflection is no longer linear [4].

In the current study, geometrically nonlinear bending analysis of clamped circular plates is made numerically. The axisymmetric transverse load (q) is considered to be uniform. The diagrams of deflection and stress resultants are plotted and the results are compared with the Kirchhoff plate theory.

II. Formulation

Geometrically nonlinear shallow spherical shell equations derived by Huang [3] are used in the study. First, due to the radial symmetry of the plate the partial differential shell equations are transformed to ordinary differential equations. Next, the equations are reorganized in terms of three displacement components (w, u, β), and three stress resultants (n_r, q_r, m_r). Finally, the height of the shallow spherical shell is set to zero in the current study and a circular plate of radius a is obtained. Therefore,

$$m_r + r\,m_r' + D\left(\frac{w'}{r} + \nu\beta'\right) - r\,q_r = 0 \tag{1}$$

$$(1-\nu)n_r + r\,n_r' - Et\frac{u}{r} = 0 \tag{2}$$

$$q_r + r\,q_r' + r\beta' n_r + \frac{Et}{r}w'u + \nu w' n_r + r\,q = 0 \tag{3}$$

$$m_r + D\beta' + D\frac{\nu}{r}\beta = 0 \tag{4}$$

$$u' + \frac{1}{2}\beta^2 - \frac{(1-\nu^2)}{Et}n_r + \nu\frac{u}{r} = 0 \tag{5}$$

$$w' - \beta = 0 \tag{6}$$

are obtained where $(\ )' = \dfrac{d(\ )}{dr}$.

Here, w, u, β are the deflection, the horizontal radial displacement and the rotation. The symbols denoted by r, ν, E, t are the radial coordinate, Poisson's ratio, Young's modulus, and the thickness of the plate, respectively. The stress resultants n_r, q_r, m_r are the membrane force, transverse shear, and the bending moment. The uniform external pressure is introduced by q.

The non-dimensional parameters are introduced by

$$w = Wt, \quad u = Ut, \quad r = \xi a, \quad c = \frac{a}{t}, \quad D = \frac{Et^3}{12(1-\nu^2)} \tag{7}$$

$$q = QE, \quad n_r = EtN_r, \quad q_r = EtQ_r, \quad m_r = Et^2M_r \tag{8}$$

where D is the bending rigidity of the plate. Substituting the non-dimensional variables into "(1-6)", we have the ordinary differential operators $L_1, L_2, L_3, L_4, L_5, L_6$ given by

$$L_1 = (1-\nu^2)M_r + (1-\nu^2)\xi\frac{dM_r}{d\xi} + \frac{1}{12c^2\xi}\frac{dW}{d\xi} + \frac{\nu}{12c}\frac{d\beta}{d\xi} - (1-\nu^2)c\,\xi\,Q_r = 0 \tag{9}$$

$$L_2 = (1-\nu)N_r + \xi\frac{dN_r}{d\xi} - \frac{1}{c\xi}U = 0 \tag{10}$$

$$L_3 = Q_r + \xi\frac{dQ_r}{d\xi} + \xi\frac{d\beta}{d\xi}N_r + \frac{1}{c^2\xi}\frac{dW}{d\xi}U + \frac{\nu}{c}\frac{dW}{d\xi}N_r + c\,\xi\,Q = 0 \tag{11}$$

$$L_4 = 12(1-\nu^2)c\,M_r + \frac{d\beta}{d\xi} + \frac{\nu}{\xi}\beta = 0 \tag{12}$$

$$L_5 = \frac{1}{c}\frac{dU}{d\xi} + \frac{1}{2}\beta^2 - (1-\nu^2)N_r + \frac{\nu}{c\xi}U = 0 \tag{13}$$

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$$L_6 = \frac{1}{c}\frac{dW}{d\xi} - \beta = 0 \qquad (14)$$

"(9-14)" are converted to algebraic equations via the finite difference method. The points of the finite difference mesh are located along the radial coordinate and six unknowns ($W, U, \beta, N_r, Q_r, M_r$) are defined at each point. Therefore, the boundary conditions at the clamped edge (i.e., at $\xi = 1$; $W = U = \beta = 0$) and at the center of the plate (i.e., at $\xi = 0$; $U = \beta = Q_r = 0$) are satisfied exactly. The system of nonlinear algebraic equations are solved by the Newton-Raphson method.

III. NUMERICAL RESULTS

The parameters considered in the numerical procedure are shown in the following:

$$E = 2\times10^6 \ N/m^2, \quad \nu = 0.3, \quad a = 1\ m, \quad t = 0.10\ m, \quad c = 10, \quad \alpha = Qc^4 \qquad (15)$$

The diagrams of deflection and stress resultants are plotted for several values of α (Figs. 1-4).

TABLE I
NON-DIMENSIONAL CENTRAL DEFLECTION (W_0) FOR UNIFORM PRESSURE

	$\alpha = 1$ $Q = 0.0001$	$\alpha = 3$ $Q = 0.0003$	$\alpha = 10$ $Q = 0.0010$
Present study	0.1678	0.4583	1.0509
Large deflection analysis	0.1680 [1]	0.4588 [1]	1.0512 [1]
Large deflection analysis (approximate solution) (*)	0.1687 [5]	0.4655 [5]	1.0937 [5]
Kirchhoff plate theory (**)	0.1706 [4]	0.5119 [4]	1.7062 [4]

(*) These results were computed by the author(s) via the formula given on page 412 and Table 82 on page 410 in [5].

(**) These results were computed by the author(s) via the formula given in [4].

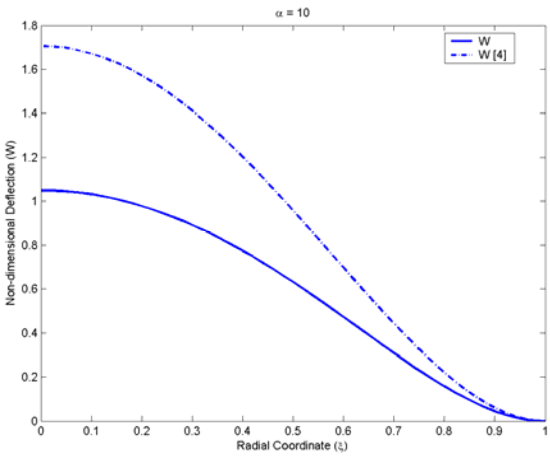

Fig. 1. Non-dimensional deflection (W)

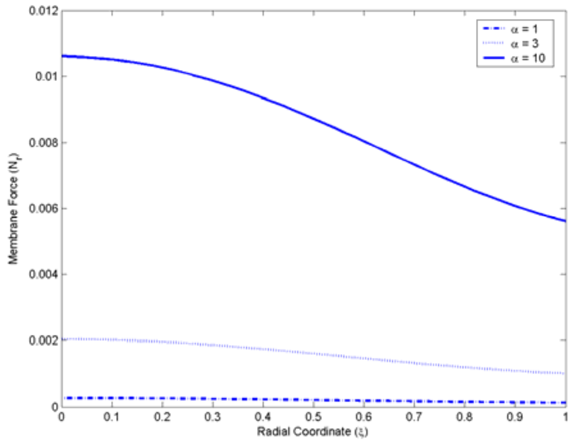

Fig. 2. Non-dimensional stress resultant (N_r)

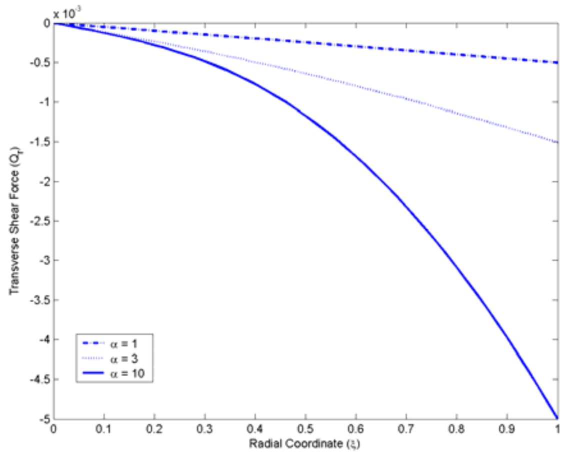

Fig. 3. Non-dimensional stress resultant (Q_r)

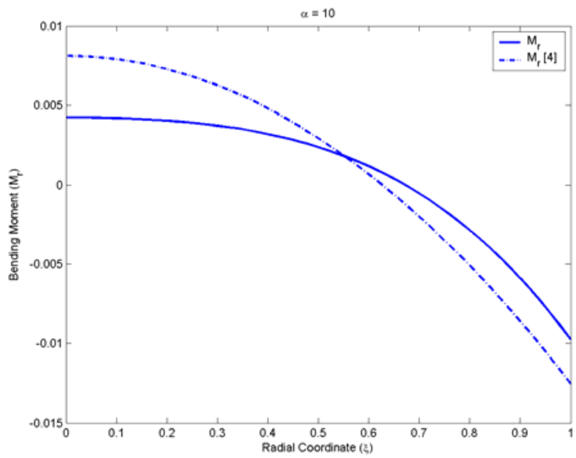

Fig. 4. Non-dimensional stress resultant (M_r)

IV. CONCLUSION

In comparison with the solutions reported by Ye [1] almost identical results are obtained (Table I). The results reveal that Kirchhoff theory yields greater $|M_r|$ than the large deflection theory does (Fig. 4). This statement is also valid for the deflection (Fig. 1). The numerical procedure employed in this computational paper works efficiently and produces results with acceptable accuracy.

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REFERENCES

[1] J. Ye, "Large deflection analysis of axismmetric circular plates with variable thickness by the boundary element method," *Applied Mathematical Modelling*, vol. 15, no. 6, pp. 325-328, Jun. 1991.
[2] Y. Z. Chen and K. Y. Lee, "Pseudo-linearization procedure of nonlinear ordinary differential equations for large deflection problem of circular plates," *Thin-Walled Structures*, vol. 41, no. 4, pp. 375-388, Apr. 2003.
[3] N. C. Huang, "Unsymmetrical buckling of thin shallow spherical shells," *Journal of Applied Mechanics*, vol. 31, no. 3, pp. 447-457, Sep. 1964.
[4] R. Szilard, *Theory and Analysis of Plates*, Englewood Cliffs: Prentice-Hall Inc., 1974, pp. 622.
[5] S. Timoshenko and S. Woinowsky-Krieger, *Theories of Plates and Shells*, New York: McGraw-Hill Book Company, 1959, pp. 410-412.

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