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Tolerance Analysis of Involute Splines

Jacek Kroczak, Marian Dudziak

***Abstract*—The authors present the analysis of the tolerance system of involute splines. They explain the concept of effective variation, actual and effective tooth size. The analysis of the effective variation, which is defined as the deviation allowance, for the different spline tolerance classes is presented. Pitch diameter is assumed as a reference parameter during the analysis.**

***Index Terms*—effective variation, form deviation, involute spline**

I. Introduction

A spline is defined as an equally spaced, multi-toothed connector, which is used to transmit rotary motion from an input to an output and to align two mating elements. Splined connections are applied to couple elements and transfer torque from splined shaft to splined hub or to other rotary component such as gear, pulley, flywheel, or the like (fig. 1). These connections can be found in mechanical transmission systems, e.g. gas turbine engine mainshaft, intermediate shaft of tractor gearbox, and automobile drive shaft.

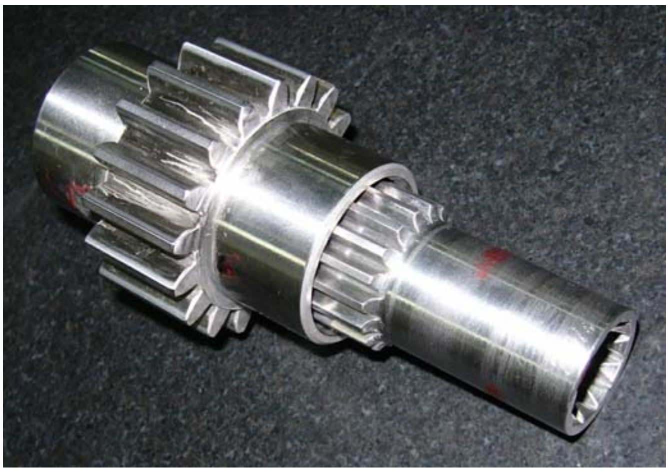

Fig. 1. View of splined connection

Splined connections are made in two types: straight-sided and involute profile. The involute spline is stronger, due to the gradual increase of tooth thickness and less stress concentration factor (about 1.7 times less), than straight-sided spline [6]. Further, the involute splines guarantee self-centering action of hub and shaft which are being under load. There are two basics types of splines: the flexible spline (working) and the fixed splined (nonworking). Flexible spline allows some rocking motion whereas fixed spline allows no relative or rocking motion between two coupling elements.

Involute splines are very similar to gears, but their teeth are shorter in height (a stub tooth). Therefore, an involute spline can be cut and measured by the same machines as for gear teeth. Standard involute splines utilize the nominal pressure angle values of 30^0, 37.5^0 or 45^0. However, in special applications, the pressure angles of 14.5^0, 20^0 or 25^0 are sometimes chosen, which are commonly used in gears.

Involute splines are specified by the international standard [4] or by the national standards, e.g. [1]-[3]. This paper considers the international standard only. According to the ISO standard there are four classes of machining tolerance for splines (4, 5, 6, 7) and six fit classes (H/d, H/e, H/f, H/h, H/js, H/k). Fittings mentioned above concern side fits only. Tooth flanks transmit the torque and simultaneously create the coaxial connection.

II. Actual and Effective Tooth Size

After machining and heat treatment of involute splines, one can not obtain a perfect involute spline. There will always be some form deviations. These form deviations affect the maximum material condition and hence the fit of connection. The form deviations are complex and occur on each flank of space and tooth. The most important spline deviations are: pitch deviation, profile deviation, and lead deviation. Therefore, the tolerance system of involute splines must take into consideration the influence of existing form deviations on the clearance for the different accuracy classes.

In order to consider a proper fit of assembly of two mating parts, there are two different dimensions for tooth thickness and space width. The first one is called the actual (real) tooth thickness (S_{max}, S_{min}) and it is just the circular thickness of tooth. The second one is the effective tooth thickness (S_{vmax}, S_{vmin}) and it is equal to the circular space width on the pitch circle of an imaginary perfect internal spline which would fit the external spline without looseness or interference. The effective tooth thickness includes the actual tooth thickness and the variations of tooth form. The effective dimension controls the clearance, while the actual dimension determines the strength of the tooth [5]. Fig. 2 and 3 illustrate the relationship between these two dimensions.

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Jacek Kroczak is with the Chair of Basics of Machine Design, Poznan University of Technology, 60-965 Poznan, Poland (corresponding author, phone: +48-61-665-2042; fax: +48-61-665-2074; e-mail: jacek.kroczak@put.poznan.pl).
Marian Dudziak is with the Chair of Basics of Machine Design, Poznan University of Technology, 60-965 Poznan, Poland (e-mail: marian.dudziak@put.poznan.pl).

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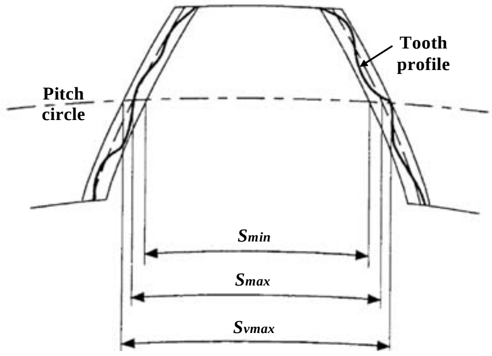

Fig. 2. Actual and effective tooth thickness

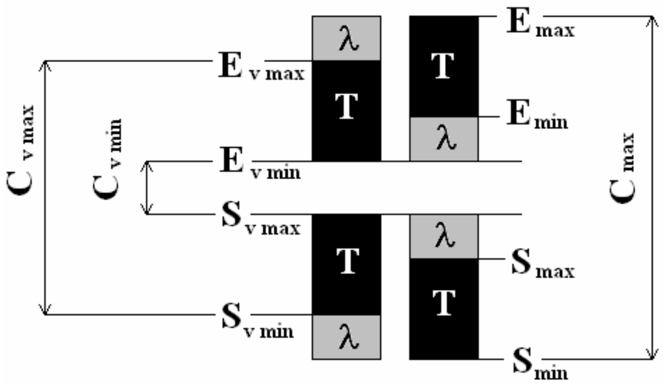

Fig. 3. Tolerance zones for the tooth thickness and the space width: T – machining tolerance, λ – deviation allowance, (T+λ) – total tolerance, $S(E)_{max(min)}$ – maximum (minimum) actual tooth thickness (space width), $S(E)_{vmax(vmin)}$ – maximum (minimum) effective tooth thickness (space width), $c_{vmin(vmax)}$ – minimum (maximum) effective clearance, c_{max} – maximum clearance

The difference between the effective space width and the effective tooth thickness is equal to the effective clearance and defines the fit of the mating parts. The assembly of two splines is controlled by the minimum effective clearance with the maximum effective tooth thickness and the minimum effective space width, and by the maximum clearance with the minimum actual tooth thickness and the maximum actual space width. Hence, the effective size is used to determine if two parts will fit together at assembly.

III. Effective Variation and Form Deviation

The effective variation is defined as the deviation allowance λ. This deviation is the accumulation of the total pitch deviation F_p, total profile deviation F_α, and total lead deviation F_β and has an effect on the effective fit of an involute spline. The deviation allowance is calculated as follows:

$$\lambda = 0.6\sqrt{F_p^2 + F_\alpha^2 + F_\beta^2} \qquad (1)$$

Three types of form deviations are added together statistically and 60 percent of this total value is defined as deviation allowance. The effect of individual spline deviations on the fit is less than their total, because areas of more than minimum clearance can be resized without changing the fit [4].

Formulas for allowable total form deviations are given in table I. These relationships are restricted to the following assumption: the length of engagement is equal to one half of the pitch diameter. Adjustments may be required for other lengths of engagement than assumed one.

On the basis of the above mentioned relationships, the analysis of the deviation allowance for the different spline tolerance classes is conducted. Pitch diameter is assumed as the reference parameter and is defined as follows:

$$d_p = m \cdot z \qquad (2)$$

Pitch diameter in the function of the deviation allowance is analyzed. This analysis can be done for two conditions: *m=const.* or *z=const*. The results in a form of the diagrams are given in fig. 4 and 5. For comparison, the pitch diameter in the function of the machining tolerance is also analyzed (fig. 6 and 7).

IV. Conclusions

The tolerance system of involute splines allows for the effect of form deviations. The effective size includes the actual size and errors due to index, lead, and profile variations. The actual size can be determined by measuring over or between pins. In order to measure the effective values, analytical gear measurement equipment must be used.

TABLE I
FORMULAS FOR ALLOWABLE TOTAL PITCH DEVIATION , TOTAL PROFILE DEVIATION, AND TOTAL LEAD DEVIATION

Spline tolerance class IT	Total pitch deviation F_p (μm)	Total profile deviation F_α (μm)	Total lead deviation F_β (μm)
4	$2.5 \cdot \sqrt{m \cdot z \cdot \pi / 2} + 6.3$	$1.6 \cdot (m + 0.0125 \cdot m \cdot z) + 10$	$0.8 \cdot \sqrt{L} + 4$
5	$3.55 \cdot \sqrt{m \cdot z \cdot \pi / 2} + 9$	$2.5 \cdot (m + 0.0125 \cdot m \cdot z) + 16$	$\sqrt{L} + 5$
6	$5 \cdot \sqrt{m \cdot z \cdot \pi / 2} + 12.5$	$4 \cdot (m + 0.0125 \cdot m \cdot z) + 25$	$1.25 \cdot \sqrt{L} + 6.3$
7	$7.1 \cdot \sqrt{m \cdot z \cdot \pi / 2} + 18$	$6.3 \cdot (m + 0.0125 \cdot m \cdot z) + 40$	$2 \cdot \sqrt{L} + 10$
where: *m* – module, *z* – number of teeth, *L* – the spline length (mm)			

On the basis of the presented diagrams (fig. 4-7), the following conclusions can be drawn:

- the higher the pitch diameter, the higher the machining tolerance and the deviation allowance
- the number of teeth has a low influence on the deviation allowance and the machining tolerance compared with the module
- the module and the number of teeth determinate the value of the deviation allowance.

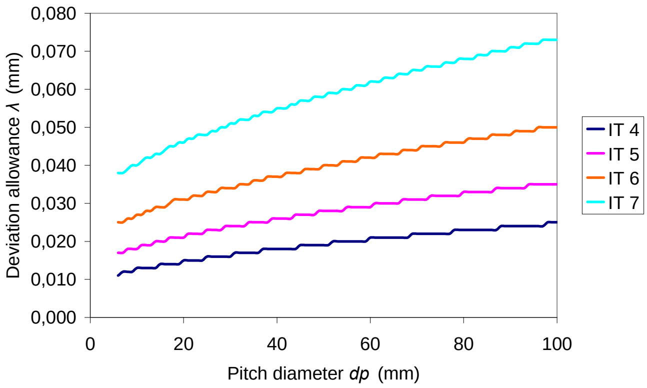

Fig. 4. Diagram of the relationship between the pitch diameter and the deviation allowance for $m = 1$

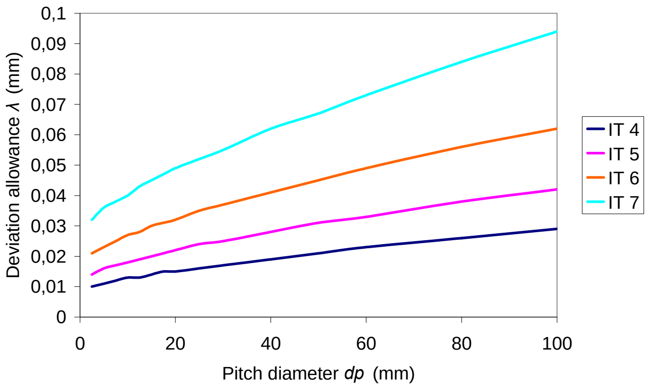

Fig. 5. Diagram of the relationship between the pitch diameter and the deviation allowance for $z = 10$

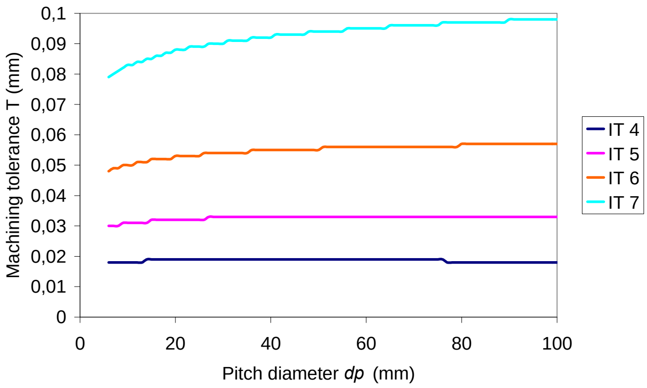

Fig. 6. Diagram of the relationship between the pitch diameter and the machining tolerance for $m = 1$

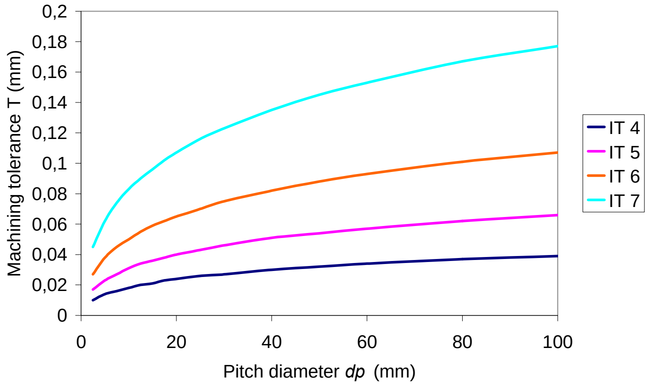

Fig. 7. Diagram of the relationship between the pitch diameter and the machining tolerance for $z = 10$

REFERENCES

[1] ANSI B92.2.2M-1980 *Involute Splines, Metric Module.*

[2] DIN 5480 *Zahnwellen-Verbindungen mit Evolventenflanken*.

[3] GOST 6033-80 *основные нормы взаимозаменяемости. Соединения шлицевые эвольвентные с углом профиля 30 градусов. Размеры, допуски и измеряемые величины. (English translation: Basic norms of interchangeability. Involute splined joints with 30^o profile angle. Dimensions, tolerances and measurable sizes.)*

[4] ISO 4156 *Straight cylindrical involute splines – Metric module, side fit*.

[5] Cedoz R.W., Chaplin M.R., *Design Guide for Involute Splines*. SAE, Warrendale, PA, 1994.

[6] Klebanov B., Barlam D., Nystrom F., *Machine Element: Life&Design*. CRC Press, Boca Raton, FL, 2008.

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