Proceedings of the World Congress on Engineering 2011 Vol III
WCE 2011, July 6 - 8, 2011, London, U.K.

Aerostatic Spherical Bearing for Mass Properties Machine (MPM)

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***Abstract*-Aerostatic Bearing offers high stiffness and very low friction with zero stiction. When complemented with spherical geometry, the aerostatic bearing provides a frictionless pivot and allows three degrees of rotational freedom. These characteristics find its use in applications such as precision motion simulators, zero gravity systems and measurement devices. High stiffness, precise axis definition, low friction, zero wear and clean operating conditions are the specific advantages of aerostatic bearing which makes it an appropriate candidate for application in precise test equipments for spacecrafts. The bearing for Mass Properties Machine (MPM) is optimized to have low tare mass and inertia. It is also configured to accommodate protruding elements of the payload at the hollow central cavity of the bearing, thus resulting in lowering the Centre of Gravity (CG) vertically during measurements. This paper discusses design approach for an Aerostatic Spherical Bearing (ASB) specially tailored for MPM to measure Mass moments of Inertia and Dynamic Unbalance of spacecraft precisely. The characteristic equations for the pressure distribution, thrust load capacity, mass flow rate and stiffness of the bearing are presented. Numerical solution for a typical model bearing, design considerations for the bearing parts, precautionary measures to ensure crash-proof operation and related fabrication aspects are also discussed.**

***Index terms* - Aerostatic Spherical bearing, Centre of Gravity, Mass moments of Inertia, Spacecrafts, Stiffness Thrust load**

I. INTRODUCTION

AEROSTATIC BEARINGs are externally pressurized using clean air or occasionally with gas like dry Nitrogen introduced between the bearing surfaces through precision holes (orifices), grooves, steps or by using porous compensation techniques. The air discharges through the edges of the bearings to the ambient atmosphere. High stiffness can be obtained in air bearings by appropriate design, precise manufacturing processes and tuned flow controls.

Manuscript received January 25, 2011. Revised March 23, 2011
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The radial stiffness directly indicates the precision of axis definition of the bearing and hence the measurement reference axis of MPM.

Aerostatic Spherical Bearing (ASB) as discussed in this paper is configured to interface with all the associated machine elements of the Mass Properties Machine (MPM) such as Spin Table, DC Brushless Motor, Torsion Flexure, Static and Dynamic Flexures, Auxiliary Journal Air Bearing, Lock Devices etc. Fig.'1' shows the ASB configured for MPM. The MPM is designed to allow selective measurement of any one of the mass properties viz., Centre of Gravity (CG) or Mass Moment of Inertia (MOI) or Dynamic Unbalance while locking /disabling the other measurement modes.

During the measurement of MOI, the ASB is required to allow un-damped free torsional oscillation proportional to the spring rate of the torsional flexure and MOI of the object.

The CG measurement requires indexing the rotor of the ASB about its vertical axis through definite angle and the measurement of Dynamic Unbalance requires precise spinning of the rotor of the ASB along with the object at constant speed using a DC Brushless motor.

The design objectives for the ASB is defined from the functional requirements said above dictated by MPM specifications such as mass of the object to be measured, required positional accuracy, stiffness, and the available supply air conditions at the test facilities. The ASB replaces the conventional design wherein a combined thrust and journal bearing integrated with complex flexural hinges.

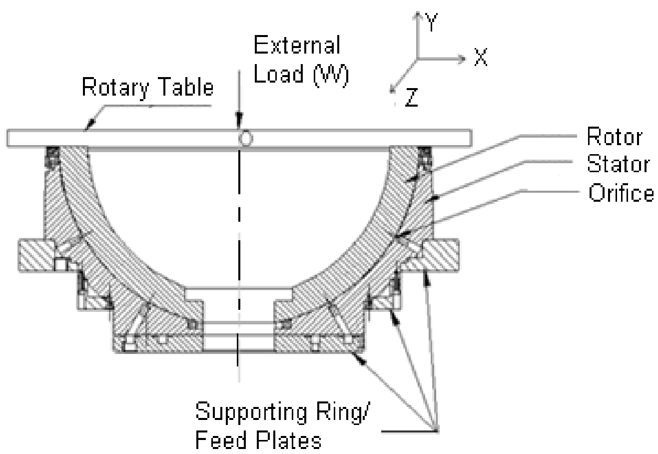

Fig 1:
Configuration of Aerostatic Spherical Bearing for Mass Properties Machine

II. THEORY

Among the many variants and classifications in Thrust air bearing design as discussed in [1],[2], the orifice fed bearing stabilized by the external working force is chosen here, owing to its simplicity of realization and high stiffness characteristics among other types. Fig. '2' shows an orifice fed hemi-spherical bearing. The inner and the outer spherical bowls form the rotor and stator of the bearing respectively. Under ideal conditions, both the stator and rotor hemisphere have the common geometrical centre 'O', thereby their radii R_b and R_r are concentric to each other.

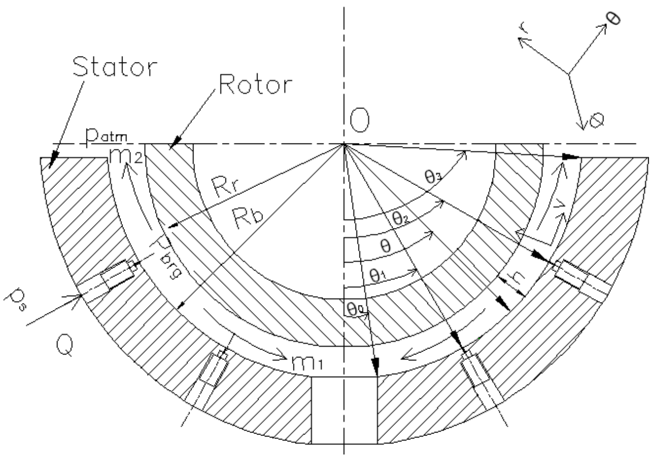

Fig 2: Sectional View of Hemispherical Air Bearing

Referring to Figure B, for large bearings, the multiple rows of orifices are positioned between angle θ_1 and θ_2 circumferentially equi-spaced about vertical axis of bearing. The supply pressure 'p_s' is admitted into the clearance through a set of orifices. The exit pressure at the orifices is termed as bearing pressure 'p_{brg}' which ideally remains constant between θ_1 and θ_2 for identical supply pressure conditions. The θ_0 and θ_3 are inner and outer exit angles where the air exits through annular bearing gaps to ambient atmospheric pressure 'p_{atm}'. The film thickness 'h' and the orifices diameters are usually very small (in the order of 25 to 75 microns) and it is shown exaggerated in Fig. '2' for clarity. The air pressure drops as it flows out of the bearing due to the acceleration of the air as it expands. A smaller clearance will reduce the pressure drop that gives a higher load capacity. It is desirable to achieve an optimum condition at which a maximum stiffness occurs where the change of load when divided by the change of clearance is a maximum.

The objective of deriving the relationships of various parameters such as thrust load capacity, stiffness of the bearing, pressure profile at the bearing gap and mass flow rate is achieved proceeding sequentially from the first principles of momentum equation for a Newtonian fluid having constant density (ρ) and constant viscosity (μ) in spherical coordinates (r, θ, and Φ) (here r varies from '0' to 'h') which is given by Navier Stokes Equation,

$$R_b\,(d^2v_\theta/dr^2) = 1/\mu\,(dp/d\theta) \quad \text{--- (1)}$$

with the following assumptions:

- The flow is laminar
- Fluid inertia is neglected
- No body forces
- No slip exists at the boundaries between the fluid and plates
- Pressure always remains constant in direction normal to the direction of flow etc...

The flow velocity 'v_θ' is obtained by double integration of the above equation (1) and by applying the boundary conditions,

$dv_\theta/dr = 0$ @ $r=h/2$ and $v_\theta = 0$ @ $r = 0$ and $r = h$

$$v_\theta = 1/2\mu\{(dp/d\theta)\,(r^2-hr)\}(1/R_b) \quad \text{--- (2)}$$

and maximum velocity can be obtained at $r = h/2$.

$$v_{\theta\ max} = -\,(1/8\mu\,(dp/d\theta)\,h^2)\,(1/R_b) \quad \text{--- (3)}$$

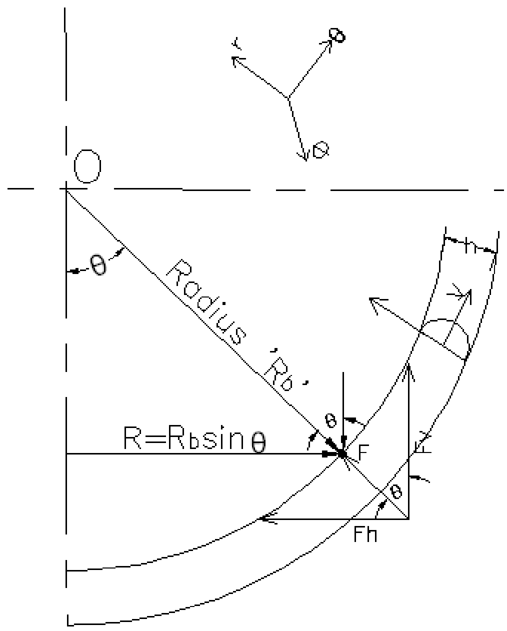

Fig 3: Geometry of Spherical Bearing

III. MASS FLOW RATE

The differential mass flow rate is given by

$$dm = dA\; v_\theta\; \rho \quad \text{---(4)}$$

where, v_θ is the flow velocity given by equation (2)
ρ is the density of the fluid
$dA = (2\pi(dR))*h$ and $R = R_b\,\mathrm{Sin}\theta$
(Refer Fig C)
Substituting, we get

$$dm = (2\pi\, d(R_b\,\mathrm{Sin}\theta)*h)\;*(1/2\mu\{(dp/d\theta)\,(r^2-hr)\}(1/R_b))\;\rho \quad \text{--- (5)}$$

Integrating eq. (5) for the limits r=0 to r=h, we get,

$$m = ((\pi\rho h^3)/6\mu)*(\mathrm{Sin}\theta)\;dp/d\theta \quad \text{--- (6)}$$

For an ideal gas (at isothermal condition) $\rho = (p/RT)$. Substituting the above in eq. (6) and rearranging, we get,

$$pdp = (m^*(6\mu RT)/(\pi h^3)) \text{ Cosec}\theta \, d\theta \quad \text{--- (7)}$$

Integrating Equation (7) between the limits θ_0 & θ_1 for inward mass flow and between the limits θ_2 & θ_3 for outward mass flow, we get

$$m_1 = \left|(p^2_{brg} - p^2_{atm})^* \pi h^3)/ (12\mu \mathbf{R} T^* \ln (\tan (\theta_1/2) / \tan (\theta_0/2)\right| \quad \text{--- (8a)}$$

$$m_2 = \left|(p^2_{brg} - p^2_{atm})^* \pi h^3)/ (12\mu \mathbf{R} T^* \ln (\tan (\theta_3/2) / \tan (\theta_2/2)\right| \quad \text{--- (8b)}$$

Adding Eq. (8a) and (8b) for total mass flow rate, we have

$$m_{tot} = \frac{((p^2_{brg} - p^2_{atm})^* \pi h^3)}{12\mu \mathbf{R} T} * \left(\left(\ln\left\{\frac{\tan(\theta_1/2)}{\tan(\theta_0/2)}\right\}\right)^{-1} + \left(\ln\left\{\frac{\tan(\theta_3/2)}{\tan(\theta_2/2)}\right\}\right)^{-1} \right) \quad \text{--- (8c)}$$

The total mass flow through the bearing as given m_{tot} (equations 8c) is equal to the flow through the orifices when steady state is attained. The expression for flow through the orifices is given as:

$$m_o = C_d A \rho_0 \left[(\gamma/\gamma\text{-}1)^* 2\mathbf{R} T \left\{ (p_{brg}/p_s)^{(2/\gamma)} - (p_{brg}/p_s)^{(\gamma+1)/\gamma} \right\}\right]^{1/2} \quad \text{--- (8d)}$$

Equation 8c is equated to equation 8d for sizing of the orifices. However, the later paragraph will express condition for maximum stiffness to choose a choked flow which defines the diameter and number of orifices.

For the condition when the inward flow is equal to the outward mass flow rate, that is, $m_1 = m_2$, we have,

$$\tan(\theta_0/2) * \tan(\theta_3/2) = \tan(\theta_1/2) * \tan(\theta_2/2) \quad \text{--- (9)}$$

The above equation is useful in determining the position of the orifices on the sphere if the inward and outward exit angles are known.

IV. Pressure Inside the Spherical Bearing at Any Position:

Using equation 8a and 8b & substituting pressure 'p' at any angle 'θ' in place of p_{brg} and rearranging, we arrive at the below equations.

$$p = \left(p^2_{brg} \left[1 - \left\{ 1 - (p^2_{atm}/p^2_{brg}) \left\{ \frac{\{\ln(\tan(\theta_1/2)/\tan(\theta/2))\}}{\{\ln(\tan(\theta_1/2)/\tan(\theta_0/2))\}} \right\} \right\} \right] \right)^{1/2} \quad \text{--- (10a)}$$

$$p = \left(p^2_{brg} \left[1 - \left\{ 1 - (p^2_{atm}/p^2_{brg}) \left\{ \frac{\{\ln(\tan(\theta_2/2)/\tan(\theta/2))\}}{\{\ln(\tan(\theta_2/2)/\tan(\theta_3/2))\}} \right\} \right\} \right] \right)^{1/2} \quad \text{---- (10b)}$$

Equation 10a and 10b are the expressions for pressure at any point at position θ between θ_0 & θ_1 and θ_2 & θ_3 respectively. The pressure (p_{brg}) remains constant between θ_1 & θ_2.

V. Thrust Load Capacity of the Spherical Bearing

In simple terms, the pressure profile given by above statements is integrated over the projected area of the bearing to express the thrust resistance. Referring to Fig. '3',

R_b = Radius of the hemisphere
θ= subtended angle at arbitrary position
If 'dF' is the force at any point on the sphere, then
$dF_h = dF \sin\theta$ is the radial load and
$dF_v = dF \cos\theta$ is the thrust load

$$\text{Net Bearing Force } F = A\int p \, dA \quad \text{--- (11)}$$
$$\text{Net Thrust force } (Fv) = F \cos\theta = A\int p \, dA \cos\theta \quad \text{-- (12)}$$

'p' is pressure at any point in the bearing and 'dA' is the elemental area.

As discussed in Para. 'IV', pressure varies between θ_0 and θ_1 (p_{atm} to p_{brg}), remains constant from θ_1 & θ_2 (p_{brg}) and varies between θ_2 and θ_3 (p_{brg} to p_{atm}).

Solving for dA and substituting pressure at any position on the bearing & solving, we get

$$(F_{v1})_{\theta 0 \,\&\, \theta 1} = {}_{\theta 0}\int^{\theta 1} p \,(\pi R_b^2) \sin 2\theta \, d\theta \quad \text{--- (13)}$$
$$(F_{v2})_{\theta 1 \,\&\, \theta 2} = p_{brg} (\pi R_b^2) \; {}_{\theta 1}\int^{\theta 2} \sin 2\theta \, d\theta \quad \text{--- (14)}$$
$$(F_{v3})_{\theta 2 \,\&\, \theta 3} = {}_{\theta 2}\int^{\theta 3} p \,(\pi R_b^2) \sin 2\theta \, d\theta \quad \text{--- (15)}$$

Thus, the total bearing thrust force is given by the summation of equations 13, 14 and 15.

$$F_v = F_{v1} + F_{v2} + F_{v3} \quad \text{--- (16)}$$

The pressure terms for Eq. (13) and (15) are obtained from eq. (10a) and (10b) respectively.

$$\text{Thus, } F_v = f(p_{brg}) \quad \text{--- (16a)}$$

The above equation can be solved in MATHCAD or MATLAB software using integral functions. The plot of equation 16a will be near linear and its constant slope gives the approximate indicating factor for thrust capacity for the given geometry of the bearing.

VI. Stiffness of the Spherical Bearing

When the bearing attains steady state conditions, the bearing thrust force 'Fv' equals the external load 'W'. Stiffness of the bearing is the change of load W divided by the change of film thickness (h)

Mathematically, $k = dFv/dh$

Since the expression for Fv does not contain the film thickness 'h', expression for stiffness (k) is derived as follows:

$$\text{Stiffness (k)} = \left(\frac{dF_v}{dp_{brg}}\right)\left(\frac{dp_{brg}}{dS}\right)\left(\frac{dS}{dh}\right) \quad \text{--- (17)}$$

From Eq. (13), (14) & (15), it is seen that Fv is a function of p_{brg} .Hence, the differentiation of Fv w.r.t. p_{brg} is solved directly.

To express, p_{brg} in terms of 'S', we equate the summation of inner and outer mass flow rates (Eq. 8c) and flow through orifices (Eq. 8d) and then solving, we get

$$p_{brg} = \left[p^2_{atm} + S\,(p_s) \left\{ (p_{brg}/p_s)^{(2/\gamma)} - (p_{brg}/p_s)^{(\gamma+1)/\gamma} \right\} \right]^{1/2} \quad \text{-- (18)}$$

$$\text{where } S = \frac{C_d A\, 2(\gamma/\gamma-1)\,(12\mu \mathbf{R} T)}{\left[\ln\left\{\dfrac{\tan(\theta_1/2)}{\tan(\theta_0/2)}\right\}\right]^{-1} + \left[\ln\left\{\dfrac{\tan(\theta_3/2)}{\tan(\theta_2/2)}\right\}\right]^{-1}} * \frac{1}{(\pi h^3)} \quad \text{--- (19)}$$

Thus, differentiation of p_{brg} w.r.t. term 'S' is possible.

Further, from Equation (19), it is seen that the term 'S' is a function of film thickness 'h'. Thus the differentiation of 'S' w.r.t film thickness 'h' can also be done. Thus the equation of stiffness is solved by differentiating the parameters as expressed in eq. (17).

Applying the maxima conditions for stiffness equation (17), and its supplementary equations given above, the conditions for maximum stiffness is derived. Owing to uncertainties, assumptions and other variants assumed to be constant in the equation, it is said that the bearing experimentally tuned to a pressure ratio p_{brg}/p_s in the range 0.5 to o.75 exhibit higher stiffness, though p_{brg}/p_s = 0.528 yields the theoretical highest stiffness. As the bearing pressure is sensitive to the external load, a manual or auto tuning of supply pressure is suggested for maintaining the optimum pressure ratio depending on the application.

VII. Case Study

- Radius of the Bearing = 0. 210 m
- Supply Pressure = 6 bar (6×10^5 N/m^2)
- Bearing Pressure = 3.168 bar (3.168×10^5 N/m^2)
- Viscosity of air (μ) = 1.8×10^{-5} N-sec/m^2
- Velocity of sound = 330 m/sec
- Inlet angle (θ_0) = 14°
- Outlet angle (θ_3) = 85°
- Position of orifice (θ_1) = 22°
- Position of orifice (θ_2) = 60°

θ_1 & θ_2 are determined such that the velocity at which that air flows out does not exceed the supersonic velocity of sound (330 m/sec).

VIII. Theoretical Results

Fig. '4' gives the plot of pressure profile inside the bearing, i.e. (p_{brg}) at any angular position. As discussed, the bearing pressure is constant between angular positions θ_1 & θ_2 and it varies from atmospheric to the bearing pressure between θ_0 to θ_1 and between θ_2 to θ_3. Fig. '5' gives the Thrust load capacity of the bearing under various bearing pressure conditions.

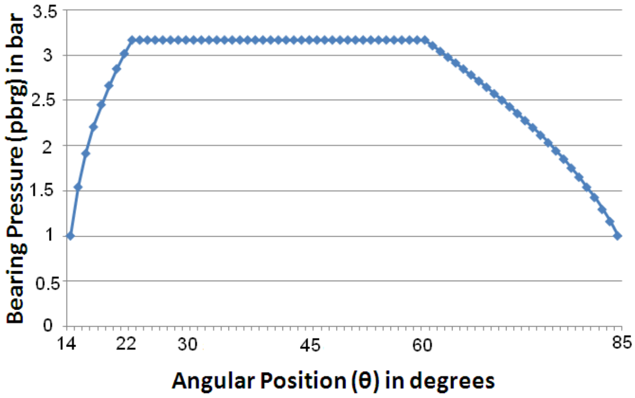

Fig. 4. Pressure profile inside the bearing w.r.t. Angular position.

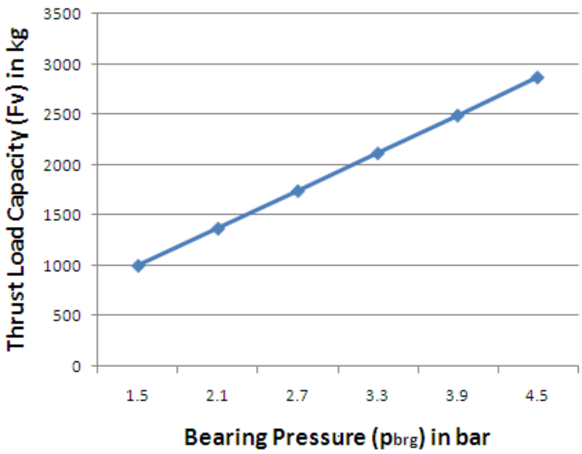

Fig 5. Bearing Pressure Vs Thrust Load Capacity of the spherical bearing.

IX. Design of Bearing Elements

The elements of the hemispherical bearing are stator, rotor, orifices, supporting rings/feed plates, bearing stoppers/linings etc. The bearing parts may be FE modeled and applying the working loads and appropriate boundary conditions, the bearing may be verified for stiffness. The stiffness for the parts are required to be relatively very high compared to that of air film stiffness. Localized deformations which alter the profile of the bearing surface are to be minimized much below the operating film clearances.

X. Crash Proof Design

In the event of pressure drop or failure in air supply, the air film ceases and results in rubbing of metallic stator and rotor due to the external loads. This causes a severe damage to the bearing surface and the metallic debris created due to rubbing may further damage the bearing. The bearing has to be protected against the accidental crash.

Protective Lining Rings made of low friction soft material having least stiction to the bearing rotor and stator materials (Delrin) are used at the outer and inner peripheries. These linings are provided with localized pads protruding in such a way that the bearing clearance at these linings is kept nearly half of the nominal operating clearance given in the bearing. In the event of crash, these linings bear the loads before appreciable damage to the bearing. In addition, the stator bearing surface is coated

ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

with a thin film of Teflon in order to overcome accidental damages.

Further protection is taken at the pneumatic flow circuit by introducing a parallel supply through a high volume accumulator built with pressure relay valve and audio alarm, which will feed the circuit in the event of failure of primary supply. Standard dry nitrogen cylinders with regulated outlet pressure are used for this purpose as a precautionary measure.

XI. PNEUMATIC CIRCUIT

Restricted air flow is provided to orifices through high response flow control devices in order to maintain the pressure ratio. The air supply source need to have filters and a dryer. Filters in various particulate stages are incorporated in the supply line to minimize the entry of contaminants and dust which are likely to damage bearing surface. Dryers avoid condensation due to temperature variations, if any, in the bearing.

XII. FABRICATION

Aerostatic Spherical bearing demands maintaining the geometrical accuracies for principal features and absolute dimensions. Geometric accuracies on spherical profile and concentricity are important from bearing performance point of view. Other geometric relationship when integrated with associated parts of the MPM, influence the accuracy of measurements. The high accuracy on profile and surface finish are obtained within the tolerance limit using high speed diamond turning machines. Fine hole (dia 0.3 to 0.5 mm) drilling for orifices also forms a fabrication limitation which is resolved by laser drilling process or by using ruby orifice inserts press fitted into the bearing stator.

XIII. CONCLUSION

Spherical aerostatic bearing for application in MPM is designed by following the basic steps discussed above. These steps determine the mass flow rate, position of orifices for equal inward and outward mass flow rates, pressure at any position on the spherical bearing surface, the thrust load capacity and stiffness of the bearing. The bearing will be fine tuned for its performance experimentally during its test and evaluation process.

REFERENCES

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[2] Theoretical Analysis of a Hydrostatic Thrust Journal air bearing with Torque Load Capacity by Rizos.I (Ref: ESRO-TN-39-ESTEC)
[3] Hydrostatic & Hybrid Bearing Design by Rowe.W.B

APPENDIX

TABLE I: SYMBOLS, QUANTITIES & UNITS

Symbol	Quantity	Units
C_d	Co-efficient of Discharge	Dimensionless
γ	Ratio of specific heats	Dimensionless
θ_0	Inward fluid exit angle on the sphere	radians
θ_3	Outward fluid exit angle on the sphere	radians
θ_1	Position of first row of orifices	radians
θ_2	Position of second row of orifices	radians
θ	Position at any point on the sphere	radians
R_b	Radius of the stator/bearing	m
R_r	Radius of the rotor	m
h	Film Thickness	m
d	Diameter of the orifice	m
A	Area of the orifices	m^2
Q	Discharge	m^3
T	Temperature at NTP	°K
m_1	Inward mass flow rate	Kg/sec
m_2	Outward mass flow rate	Kg/sec