Maximum-revenue Tariff with Different Roles in a Price-setting Competition

Fernanda A. Ferreira and Flávio Ferreira *

Abstract— **In this paper, we study an international duopoly market where firms set prices. The model has two stages. In the first stage, the home government chooses an import tariff to maximize the revenue. Then, the firms engage in a price-setting competition. We study three different roles: (i) simultaneous decisions (Bertrand model); (ii) sequential decisions with home firm as the leader; and (iii) sequential decisions with home firm as the follower. We compare the results obtained in the three different ways of moving on the decision make of the firms.**

Keywords: Industrial Organization, Game Theory, Bertrand model, leadership

1 Introduction

In a standard duopoly, firms choose either prices or quantities in a non-cooperative fashion. If the decisions are made simultaneously, these models are called, respectively, Bertrand model and Cournot model (see [1, 4]). Sometimes, one of the firm has the opportunity to make his decision before the other firm. It is well-know that in a quantity-setting competition the first mover (leader) firm has advantage over the second mover (follower), and in a price-setting competition the follower firm has advantage over the leader.

Here, we consider a two-country, two-good model where a domestic and a foreign good are produced by a home and a foreign monopolist, respectively. Since we assume that the two countries are perfectly symmetric, it is sufficient to describe only the domestic economy. The propose of this paper is to study the maximum-revenue tariff under international price-setting competition, with different possible timings of decisions. Tariff revenue may be an important source of government revenue for developing countries that do not have an efficient tax system. So, the government may use the maximum-revenue tariff. Brander and Spencer [2] have shown that a tariff has a profit-shifting effect in addition to its effect on tariff revenue. Larue and Gervais [14] studied the effect of maximum-revenue tariff in a Cournot duopoly. Ferreira and Ferreira [9] analysed an international quantity-setting competition, with demand uncertainty. Ferreira and Ferreira [12] examined the maximum-revenue tariff under international Bertrand competition with differentiated products when rivals' production costs are unknown. Clarke and Collie [3] studied a similar question, when there is no uncertainty on the production costs.

We should mention that issues related to those of this paper have been studied by Dowrick [5], Ferreira [6, 7, 8], Ferreira and Ferreira [10, 11], Ferreira and Pinto [13] and Spulber [15].

2 The model

We consider an economy made up of a monopolistic sector and a competitive numeraire one. The monopolistic sector comprises two firms, firm F_1 and firm F_2. On the consumption side, there is a continuum of consumers of the same type whose utility function is linear and separable in the numeraire good. The representative consumer maximizes $U(q_1, q_2) - p_1 q_1 - p_2 q_2$, where $q_i \geq 0$ is the production level of the good produced by firm F_i and p_i is its price, $i = 1, 2$; The function U is assumed to be quadratic, strictly concave and symmetric in the variables:

$$U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + 2bq_1q_2 + q_2^2\right),$$

where the parameter $b \in (0, 1)$ measures the degree to which goods are substitutes. Then, demand functions are given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2}, \; i \neq j, \; i, j = 1, 2. \qquad (1)$$

The marginal cost of production of both firms is c.

The model consists in the following two-stage game:

- In the first stage, the domestic government chooses the import tariff t per unit of imports from the foreign firm.
- In the second stage, both firms choose the prices.

Firms' profits, π_1 and π_2, are given by

$$\pi_1 = (p_1 - c)q_1 \quad \text{and} \quad \pi_2 = (p_2 - c - t)q_2.$$

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2.1 Simultaneous decision

In this section, we suppose that, in the second stage of the game, both home and foreign firms play a Bertrand-type game, i.e., each firm F_i independent and simultaneously chooses p_i. Let the superscript B denote the equilibrium outcome of the Bertrand-type game.

Theorem 1. In the Bertrand-type game:

(i) The maximum-revenue tariff is given by

$$t^B = \frac{(a-c)(1-b)(2+b)}{2\left(2-b^2\right)}.$$

(ii) The prices at equilibrium are given by

$$p_1^B = \frac{a(1-b)\left(4+b-2b^2\right) - c\left(4-b-b^2\right)}{2(2-b)\left(2-b^2\right)}$$

and

$$p_2^B = \frac{a(1-b)\left(3-b^2\right) - c\left(1+b-b^2\right)}{(2-b)\left(2-b^2\right)}.$$

(iii) The output levels are given by

$$q_1^B = \frac{(a-c)\left(4+b-2b^2\right)}{2(1+b)(2-b)\left(2-b^2\right)}$$

and

$$q_2^B = \frac{a-c}{2(1+b)(2-b)}.$$

(iv) Home firm's profit is given by

$$\pi_1^B = \frac{(a-c)^2(1-b)\left(4+b-2b^2\right)^2}{4(1+b)(2-b)^2\left(2-b^2\right)^2}$$

and foreign firm's profit is given by

$$\pi_2^B = \frac{(a-c)^2(1-b)}{4(1+b)(2-b)^2}.$$

2.2 Sequential decisions: Home firm is the leader

In this section, we suppose that, in the second stage of the game, the home firm is the leader. Home firm F_1 chooses the price p_1, and foreign firm F_2 chooses p_2 after observing the price p_1. Let the superscript L denote the equilibrium outcome of the game where the home firm F_1 is the leader.

Theorem 2. In the case of sequential decisions, with the home firm as the leader:

(i) The maximum-revenue tariff is given by

$$t^L = \frac{(a-c)(1-b)\left(4+2b-b^2\right)}{2\left(4-3b^2\right)}.$$

(ii) The prices at equilibrium are given by

$$p_1^L = \frac{a(1-b)\left(16+12b-10b^2-7b^3\right) + c\Gamma}{4\left(2-b^2\right)\left(4-3b^2\right)}$$

and

$$p_2^L = \frac{a(1-b)\left(4+2b-b^2\right)\left(12-7b^2\right) + c\Theta}{8\left(2-b^2\right)\left(4-3b^2\right)},$$

where $\Gamma = \left(16+4b-18b^2-3b^3+5b^4\right)$ and $\Theta = \left(16+24b-16b^2-26b^3+3b^4+7b^5\right)$.

(iii) The output levels are given by

$$q_1^L = \frac{(a-c)\left(16+12b-10b^2-7b^3\right)}{8(1+b)\left(4-3b^2\right)}$$

and

$$q_2^L = \frac{(a-c)\left(4+2b-b^2\right)}{8(1+b)\left(2-b^2\right)}.$$

(iv) Home firm's profit is given by

$$\pi_1^L = \frac{(a-c)^2(1-b)\left(16+12b-10b^2-7b^3\right)^2}{32(1+b)\left(2-b^2\right)\left(4-3b^2\right)^2}$$

and foreign firm's profit is given by

$$\pi_2^L = \frac{(a-c)^2(1-b)\left(4+2b-b^2\right)^2}{64(1+b)\left(2-b^2\right)^2}.$$

2.3 Sequential decisions: Home firm is the follower

In this section, we suppose that, in the second stage of the game, the home firm is the follower. Foreign firm F_2 chooses the price p_2, and home firm F_1 chooses p_1 after observing the price p_2. Let the superscript F denote the equilibrium outcome of the game where the home firm F_1 is the follower.

Theorem 3. In the case of sequential decisions, with the home firm as the follower:

(i) The maximum-revenue tariff is given by

$$t^F = \frac{(a-c)(1-b)(2+b)}{2\left(2-b^2\right)}.$$

(ii) The prices at equilibrium are given by

$$p_1^F = \frac{a(1-b)\left(8+6b-b^2\right) + c\left(8+2b-b^2-b^3\right)}{8\left(2-b^2\right)}$$

and

$$p_2^F = \frac{3a(1-b)(2+b) + c\left(2+3b-b^2\right)}{4\left(2-b^2\right)}.$$

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(iii) The output levels are given by

$$q_1^F = \frac{(a-c)\left(8+6b-b^2\right)}{8(1+b)\left(2-b^2\right)}$$

and

$$q_2^F = \frac{(a-c)(2+b)}{8(1+b)}.$$

(iv) Home firm's profit is given by

$$\pi_1^F = \frac{(a-c)^2(1-b)\left(8+6b-b^2\right)^2}{64(1+b)\left(2-b^2\right)^2}$$

and foreign firm's profit is given by

$$\pi_2^F = \frac{(a-c)^2(1-b)(2+b)^2}{32(1+b)\left(2-b^2\right)}.$$

2.4 Comparisons

In this section, we are going to compare the results obtained in each model. First, we observe that, independently of the role, both prices are increasing in the tariff. Next corollary states that the domestic government imposes a higher tariff in the game where the home firm is the leader; and the tariffs are equal in the Bertrand-type game and in the game where the home firm is the follower.

Corollary 1. The tariffs in the different games are related as follows:

$$t^B = t^F < t^L.$$

The total sales in the home market are lower in the game where the home firm is the leader; and they are higher in the Bertrand-type game, as stated in the following corollary.

Corollary 2. The total sales in the home market are related as follows:

$$Q^L < Q^F < Q^B.$$

In the next two corollaries, we compare the profits of the firms obtained in each game. We note that the home firm profits less when it plays a Betrand-type game than in any other game. However, the preference to be leader or follower depends upon the value of the degree b to which goods are substitutes. For low values of the parameter b, the home firm prefers to be follower; and for high values of the parameter b, the home firm prefers to be leader. Furthermore, the foreign firm prefers to be leader, and the worse situation is to play a Betrand-type game.

Corollary 3. Let

$$f(b) = 128 + 32b - 336b^2 - 176b^3 + 172b^4 + 107b^5,$$

and let $b_0 \in (0.65, 0.66)$ such that $f(b_0) = 0$.

(i) If $b \leq b_0$, then home firm's profits are related as follows:

$$\pi_1^B < \pi_1^L \leq \pi_1^F.$$

(ii) If $b > b_0$, then home firm's profits are related as follows:

$$\pi_1^B < \pi_1^F < \pi_1^L.$$

Corollary 4. Foreign firm's profits are related as follows:

$$\pi_2^B < \pi_2^F < \pi_2^L.$$

3 Conclusions

We studied the maximum-revenue tariff under international price-setting competition, with different possible timings of decisions. We showed that the domestic government imposes a higher tariff in the game where the home firm is the leader; and the tariffs are equal in the Bertrand-type game and in the game where the home firm is the follower. Furthermore, we got that a Betrand-type game is the situation where both firms profit less. However, the preference of the home firm to be leader or follower depends upon the value of the degree b to which goods are substitutes. For low values of the parameter b, the home firm prefers to be follower; and for high values of the parameter b, the home firm prefers to be leader. In contrast, the foreign firm always prefers to be leader.

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