Comparative Approaches of Calculation of the Back Water Curves in a Trapezoidal Channel with Weak Slope

Fourar Ali, Chiremsel Rachid, Abdessemed Fouzi

***Abstract*—The theoretical development having led to the differential equation governing the back water curves is based on the equation of Bernoulli applied between two cross-sections of the stream discharge running out in a channel of known geometrical profile. The gradient of the pressure loss is evaluated, according to the authors, by calling upon the relations of the normal or uniform flow.**
The most preferred relations are the equations of the type Chézy and Manning-Strickler but seldom that of Darcy-Weisbach. In this last relation, the coefficient of friction depends at the same time on the relative roughness of the walls of the channel considered and the Reynolds number characterizing the flow.
The calculation of the gradually varied flow rests on the knowledge of the nature of the geometrical involved slope. For this reason the classification of the back water curves is a cardinal importance and must be controlled before even approaching the calculation itself depths of the gradually varied flow. The classification of the back water curves consists, in the second time, to locate the depth of the flow compared to the critical level depths and uniform if those had suddenly occurred for the same flow forwarding by the same channel considered. The depth of the flow varies according to conditions' hydraulic and geometrical channel, it can be increased by the lover towards the downstream or on the other hand decreased. The method of classification of the back water curves is common to all the theoretical approaches intended for the calculation of the varied flow.

***Index Terms*—gradually varied flow, back water curves, differential equation, equation of Bernoulli, method of the variations of depth, method of the sections, method of Pavlovski.**

I. INTRODUCTION

The calculation of the back water curves for the flows gradually varied on free face in the open channels makes it possible to deduce the depth at any point from the channel. It consists in following the evolution of the watermark over the entire length of the channel considered. A good inspection of the watermark is regarded as an essential condition in the choice for example of the site of projection of a structure, to envisage corrections of the rivers in order to avoid a possible overflow in the event of rising. Indeed this hydraulic behavior deserves to give him a great interest.

Several methods are proposed for calculation of the movements. These last have the same aim; however their principle of calculation is different.
The principal objective of this study is to propose a comparative approach of calculation between three methods (method of the variations of depth, method of the sections and method of Pavlovski) and to draw the interesting conclusions as for the reliability and to the flexibility of the methods used.
In this study roughness in length of the channel is considered constant, the channel is trapezoidal, prismatic and with weak slope and the curve back water calculated is of type M. [1]

II. FLOW GRADUALLY VARIES

It is a permanent flow for which the flow remains constant in time. On the other hand the changes of cross-section of stream discharge, generally caused by breaks of slopes, return the flow varies.
Moreover if the curve of the watermark is neglected the flow is gradually varies.
The flow gradually varies can extend on a very long distance that's to say that the depth, Dh(x) ≈Dh, as well as the other parameters change only very slowly from one section to another (movement of raising or lowering).

III. DIFFERENTIAL EQUATION OF THE MOVEMENT GRADUALLY VARIES

According to Bernoulli the variation of specific energy is the difference between the slope of the channel and the slope of the line of energy.

$$\frac{dE}{dX} = (I - J) \quad (1)$$

And knowing that specific energy E is related to h and that h is related to X, we can write:

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$$\frac{dE}{dX} = \frac{\delta E}{\delta h}\frac{dh}{dX} = \frac{\delta}{\delta h}\left(h + \frac{Q^2}{2gA^2}\right)\frac{dh}{dX}$$

(2)

$$\frac{dE}{dX} = \left(1 - \frac{Q^2}{gA^3}\frac{\delta A}{\delta h}\right)\frac{dh}{dX}$$

(3)

By equalizing the expressions, (2) and (3) we obtain:

$$\frac{dE}{dX} = I - J = \left(1 - \frac{Q^2 B}{gA^3}\right)\frac{dh}{dX} \tag{4}$$

And finally:

$$\frac{dh}{dX} = \frac{I - J}{1 - \frac{Q^2 B}{gA^3}} \tag{5}$$

In this expression, I and Q are constant and A, B and J are related to h. we consider that the pressure loss has the same value as in uniform mode for the same depth of water and the same flow, therefore according to the formula of Chézy we write:

$$J = \frac{Q^2}{C^2 R_h A^2}$$

The equation (5) is thus the differential equation dx = *f* (h) dh of the watermark moving gradually varies for a flow in permanent mode with appreciably parallel nets moving in block in uniform channel of weak slope.
The general pace of the watermark is a function of the relative values of I and I_c of the channel for the flow considered.

IV. CURVES BACK WATER STANDARD M

These curves answer the following inequalities: $I<I_c$ and $h_n > h_c$ What corresponds to a river flow.
Three cases can occur:

➢Branch M_1 who corresponds to the following conditions:

$h > h_n > h_c$; $I > J$; $F_r < 1$; $\frac{dh}{dX} > 0$The branch M_1 is a back water curve of raising; it corresponds to a gradually delayed movement.
Upstream this curve tends asymptotically towards the normal depth i.e. can be propagated upstream at an infinitely long distance.
Downstream it tends asymptotically towards the horizontal.

➢Branch M_2 who corresponds to the following conditions:

$h_c < h < h_n$; $I < J$; $F_r < 1$; $\frac{dh}{dX} < 0$

The branch M_2 is a back water curve of lowering which corresponds to a gradually accelerated movement.
Upstream this curve is connected asymptotically to the normal depth for tending towards the downstream while decreasing perpendicularly towards the critical depth.

➢Branch M_3 who corresponds to the following conditions:

$h < h_c < h_n$; $I < J$; $F_r > 1$; $\frac{dh}{dX} > 0$

We can write:

$$\frac{dh}{dX} \approx \frac{J}{\frac{Q^2 B}{gA^3}} = \frac{\frac{Q^2}{C^2 R_h A^2}}{\frac{Q^2 B}{gA^3}} = \frac{gA}{C^2 R_h B} = \frac{gP}{C^2 B} > 0 \tag{6}$$

The branch M_3 is a back water curve of raising which corresponds to a gradually delayed movement, it leads to the projection close the critical depth making it possible to pass from the torrential mode to the river mode.

V. METHODS OF CALCULATION

The calculation and the exact construction of the forms of the free face require the integration of the differential equation:

$$\frac{dh}{dX} = \frac{I - J}{1 - \frac{Q^2 B}{gA^3}}$$

What comes down to saying that the flow, the slope of the bed, its roughness is known. However one does not know the dimension of the free face.
Consequently the variables are the coordinate X and the corresponding depth h. The integration of the differential equation (5) conduit with an indefinite integral. It will thus be necessary to know the characteristics of the flow in a section of reference or control where there is a final relation between the flow and the depth of water.

A .Method of the variations of depth (Δh is fixed)

The method of the variations of depth or method with direct steps is appropriate particularly well for the prismatic channels.
It is used for a light difference in depth. And to reduce the errors, it is advisable to tighten the variation the depths h_2 and h_1
This method consists in seeking the value of the coordinate X_2 for a depth h_2 very near to h_1 using the equation:

$$\Delta X = X_2 - X_1 = \frac{(h_1 + \frac{V_1^2}{2g}) - (h_2 + \frac{V_2^2}{2g})}{I - \frac{J(h_2) + J(h_1)}{2}}$$

The coordinate X_2 is calculated and we will pass then to the following section.

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B. Method of the sections (ΔX is fixed)

The method (implicit) of the sections or with standard step is used for the sections at short distances. It applies to the equation of the not uniform movement in the form:

$$\Delta X = \frac{E_1 - E_2}{I - J_{moy}}$$

It should be announced that calculations with this method are long and complicated.

The method of the sections consists in determining the depth of water h_2 with the coordinate X_2 very near to h_1 with a coordinate X h_1

By choosing a first value h'_2 of section A'_2 we calculate the values I_2, J_1 and A_1 While carrying these values in the equation:

$$h_1 - h_2 = (X_2 - X_1)\left(I - \frac{J(h_2) + J(h_1)}{2}\right) - \frac{Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2}\right)$$

We obtain the value of h_2 which will be probably different from the selected value h'_2.

We start again by successive approximations until the value of h_2 data by this equation is equal to the last selected value h'_2, h''_2…one will pass then to the following section.

C. Method of Pavlovski

Posing:

$$\frac{\alpha Q^2 B}{gA^3} = P_{cin}$$

P_{cin} : is called kinetic parameter.

The equation (5) can be written:

$$\frac{dh}{dX} = \frac{I - J}{1 - P_{cin}} \tag{7}$$

According to the débitance, the equation (7) becomes:

$$\frac{dh}{dX} = \frac{I\left[1 - \left(\frac{K_0}{K}\right)^2\right]}{1 - P_{cin}} \tag{8}$$

The sizes of the second member of the equation (8) present certain functions the depth h. this is why it is rational to write the equation (8) pennies the form:

$$dX = \frac{1 - P_{cin}}{I\left[1 - \left(\frac{K_0}{K}\right)^2\right]} dh \tag{9}$$

Let us examine the expression of the kinetic parameter now P_{cin}

$$P_{cin} = \frac{\alpha Q^2 B}{gA^3} = \frac{\alpha K_0^2 IB}{gA^3} = \frac{\alpha K_0^2 IB}{gA^2 R_h P}$$

Let us multiply the numerator and the denominator by C^2 Then:

$$P_{cin} = \frac{\alpha IBC^2 K_0^2}{gPA^2 C^2 R_h} = \frac{\alpha IBC^2}{gP} \frac{K_0^2}{K^2} = j\left(\frac{K_0}{K}\right)^2 \tag{10}$$

$$j = \frac{\alpha IBC^2}{gP} \tag{11}$$

Replacing the expression of the kinetic parameter (eq.10) in the equation (9) we obtain:

$$dX = \frac{\left[1 - j\left(\frac{K_0}{K}\right)^2\right]}{I\left[1 - \left(\frac{K_0}{K}\right)^2\right]} dh = f(h)dh \tag{12}$$

To carry out the integration of the equation (12), the sizes J and K_0 / K must be expressed in the form of the explicit analytical functions of h. But this is not possible that for the beds of simple form.

It thus proves that the function *f* (h) is so complex. It is impossible to solve it analytically.

For the integration of the equation (12) we suppose that the term J varies very little on the section subjected to integration. Pavlovski proposed the following process:

That is to say a certain section of the prismatic bed with (I > 0) within the limits of which we observe a not uniform flow of water.

Let us indicate the depth of the current at the origin of the section by h_1 the depth at the end by h_2 and the length of the section by ΔX.

Index 2 will indicate the following section in the direction of the flow. According to an assumption which one already spoke, we adopt for section ΔX:

$$j = j_{moy} = \frac{j_1 + j_2}{2} = c^{te} \tag{13}$$

Let us introduce designations:

$$\eta = \frac{K}{K_0},\ \eta_1 = \frac{K_1}{K_0},\ \eta_2 = \frac{K_2}{K_0}$$

Let us admit that between the variables h and η there is a relation of the type:

h = χ η where :

$$\chi = \frac{h_2 - h_1}{\eta_2 - \eta_1} \tag{14}$$

Or in the differential form:

$$dh = \chi\, d\eta \tag{15}$$

Rewriting the equation (12) by taking account of introduced designations and the assumptions:

$$dX = \frac{\chi}{I} \frac{1 - j_{moy}\frac{1}{\eta^2}}{1 - \frac{1}{\eta^2}} d\eta \tag{16}$$

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The second multiplier of the second member of the éq.16 can be written:

$$\frac{1 - j_{moy}\frac{1}{\eta^2}}{1-\frac{1}{\eta^2}} = \frac{\eta^2 - j_{moy}}{\eta^2 - 1} = \frac{\eta^2 - 1 + 1 - j_{moy}}{\eta^2 - 1} = 1 - \frac{1 - j_{moy}}{1 - \eta^2}$$

And by substitution in the éq.16 we obtain:

$$dX = \frac{\chi}{I}\left(1 - \frac{1-j_{moy}}{1-\eta^2}\right)d\eta = \frac{\chi}{I}\left[d\eta - (1-j_{moy})\frac{d\eta}{1-\eta^2}\right]$$

Carrying out the integration of this last equality enters the limits of the section:

$$\int_{X_1}^{X_2} dX = \int_{\eta_1}^{\eta_2} \frac{\chi}{I}\left[d\eta - (1-j_{moy})\frac{d\eta}{1-\eta^2}\right]$$

Or:

$$X_2 - X_1 = \frac{\chi}{I}(\eta_2 - \eta_1) - \frac{\chi}{I}(1-j_{moy})\left[\frac{1}{2}\ln\frac{1+\eta_2}{1-\eta_2} - \frac{1}{2}\ln\frac{1+\eta_1}{1-\eta_1}\right] \text{If}$$

η_2 and η_1 are lower than 1 and

$$\Delta X = \frac{\chi}{I}(\eta_2 - \eta_1) - \frac{\chi}{I}(1-j_{moy})\left[\frac{1}{2}\ln\frac{1+\eta_2}{\eta_2 - 1} - \frac{1}{2}\ln\frac{1+\eta_1}{\eta_1 - 1}\right]$$

If η_2 and η_1 are higher than 1.

The expressions above can be transcribed definitively in the following form:

$$X_2 - X_1 = \Delta X = \frac{\chi}{I}\left[\eta_2 - \eta_1 - (1-j_{moy})[\Phi(\eta_2) - \Phi(\eta_1)]\right] \tag{17}$$

Where:

$$\Phi(\eta) = 1{,}15\log\frac{1+\eta}{1-\eta} : \grave{a}\,\eta < 1$$

$$\Phi(\eta) = 1{,}15\log\frac{\eta+1}{\eta-1} : \grave{a}\,\eta > 1$$

The equation (17) makes it possible to calculate the back water curves in the prismatic beds of any form, if I > 0.

VI. SOLUTIONS

b= 10[m] , m =2, n = 0.025[$m^{-1/3}s$], J_f = 0.0025, Q = 30[m^3/s] , h_n= 1.22[m], h_c= 0.911[m], J_{cr} = 0.006929, h_1= 3.2[m].

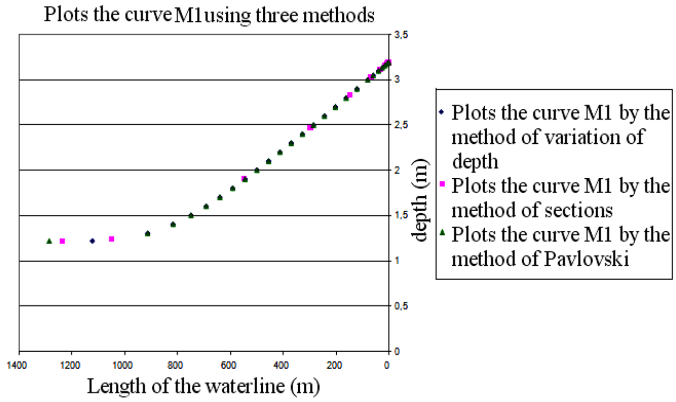

Fig. 1. Plots the curve M1 using three methods(a).

b= 7[m] , m =1.5, n = 0.018[$m^{-1/3}s$], J_f = 0.001, Q = 25[m^3/s] , h_n= 1.459[m], h_c= 1.011[m], J_{cr} = 0.003625, h_1= 2.5[m].

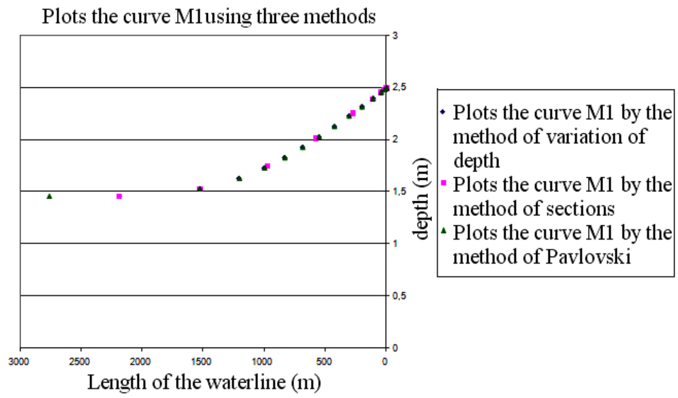

Fig. 2. Plots the curve M1 using three methods(b).

b= 15.2[m] , m =1.5, n = 1.45[$m^{-1/3}s$], J_f = 0.00135, Q = 55[m^3/s] , h_n= 1.687[m], h_c= 1.063[m], J_{cr} = 0.006489, h_1= 1.063[m].

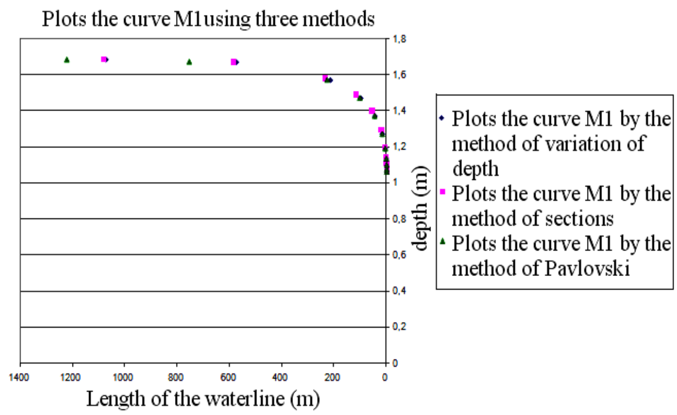

Fig. 3. Plots the curve M1 using three methods(c).

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b= 14[m] , m =1.1, n = 0.021[$m^{-1/3}s$], J_f = 0.00175, Q = 85[m^3/s] , h_n= 1.943[m], h_c= 1.492[m], J_{cr} = 0.004273, h_l= 1.492[m].

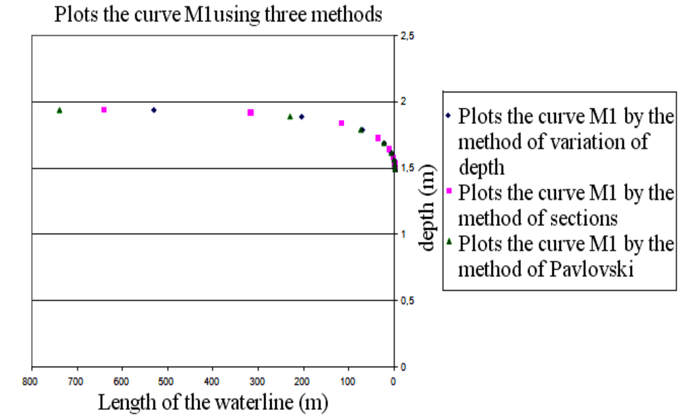

Fig. 4. Plots the curve M1 using three methods(d).

b= 5.5[m] , m =1, n = 0.015[$m^{-1/3}s$], J_f = 0.0015, Q = 12[m^3/s] , h_n= 0.904[m], h_c= 0.749[m], J_{cr} = 0.002832, h_l= 0.22[m].

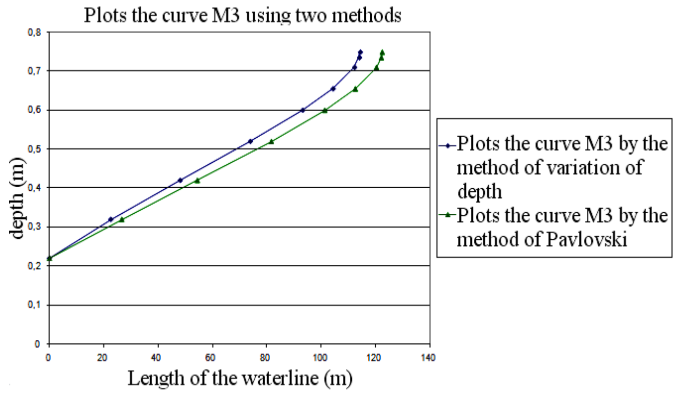

Fig. 5. Plots the curve M3 using two methods(a).

b= 20[m] , m =1.25, n = 0.020[$m^{-1/3}s$], J_f = 0.001, Q = 60[m^3/s] , h_n= 1.464[m], h_c= 0.952[m], J_{cr} = 0.004226, h_l= 0.45[m].

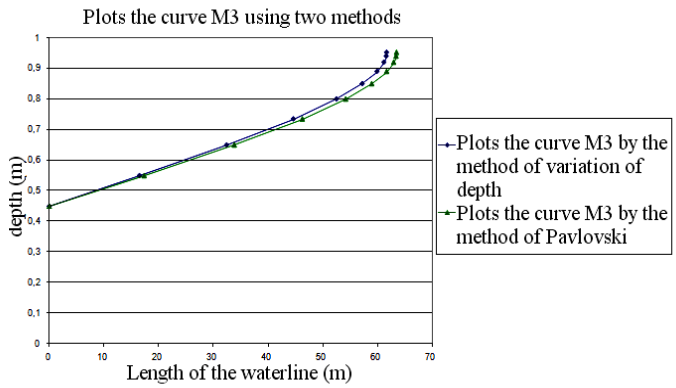

Fig. 6. Plots the curve M3 using two methods(b).

VII. INTERPRETATION

The construction of the back water curves can be carried out by various methods of which those which we used, in fact: method of the sections, method of Pavlovski and method of the variations of depth.
In all the cases this construction is carried out section by section, while passing successively from one section to another upstream according to the current.

The partition of a channel in sections carried out according to the conditions of uniformity of the flow within the limits of each section of which the length can vary few hundreds of meters until tens of kilometers. The most favorable partition is that which is done according to the uniformity of the slope of the watermark.

The calculation and the layout of the free face, taking into account the boundary conditions which we produced, make it possible, a priori, to confirm the assumption according to which all the methods used are asymptotic i.e. they give similar results.

The watermark calculated by the method by direct integration of Pavlovski diverges somewhat from that calculated by the method of the variations of depth.

All the calculated watermarks are more or less similar. Calculations by the method of Pavlovski give a distance appreciably larger, i.e.

The watermark converges more slowly towards the normal depth. The difference of the results between the various branches (M_1, M2 and M_3 is due to the asymptotic nature of the function **$\Phi(\eta)$** used in this method.

VIII. CONCLUSION

The comparison of the results obtained by the three methods (Method of the variations of depth, method of the sections and the method of Pavlovski) makes it possible to reveal the advantages and the disadvantages of each method.

In practice, it is a question of using the method which is appropriate for the problem to be best solved.

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