Statistical Linear Dynamic Control of Automatic Devices

Jamal Tariq Mian , Akram Muhammad Chaudhry

Abstract—**The product designers of automatic devices always hypothesize their devices to work satisfactorily in real life situations; whereas, in fact their hypotheses are not always true. The devices, developed according to some preset specifications and laboratory conditions, may not work as desired when subjected to nature: an uncertain environment. It is due to the fact that some sorts of variations, random and non-random, are crept into during production and working stages. These needed to be separated, analyzed and filtered by employing appropriate statistical tools (models, techniques or/and procedures) to ensure quality of products at optimum cost.**

For this purpose, a dynamic statistical process control system based on CUSUM control charts of one step ahead forecast errors, generated by a linear dynamic system model is introduced. This control system is expected to enhance quality of automatic devices by minimizing both types of errors in a cost effective manner.

Index Terms—**Statistical process control system, Noise processes, Linear dynamic system models, Recurrence equations, One step ahead forecast errors, CUSUM quality control charts.**

I. Introduction

For statistical process control of automatic devices numerous types of control schemes with and without memory are available, such as, memory less control schemes of Shewhart (1933) and memory (CUSUM) control schemes of Johnson (1961) and Hawkins (1993). Both of these types of schemes have their own merits and demerits in process control such as Shewhart schemes are effective in detecting large process shifts, whereas, CUSUM schemes are capable of detecting small shifts in processes. Further, for identification of small shifts in an effective manner V masks for CUSUM charts may be developed and used. For more discussion, see Bersimis, *et al* (2007), Box-Jenkins (1963), Crosier (1988) and Hawkins (1993).

Keeping in view, merits of both of these schemes, therefore, a hybrid Shewhart-CUSUM scheme is introduced.

Manuscript received October 11, 2010; revised December 15, 2010. submitted to the IMECS 2011 International Multi conference of Engineers and Computer Scientists 2011.

Jamal Tariq Mian is with University of Engineering and Technology, G.T Road, Lahore, Pakistan, Department of Mechatronics and Control Engineering, phone: (+92) 322 435 4810, (+92) 42 9029241; e-mail: jtmian@hotmail.com, jtmian@uet.edu.pk

Akram Muhammad Chaudry is with University of Bahrain, College of Business, P.O.Box #32038, Sakhir, Kingdom of Bahrain, M'East, phone: (+973)17642281, (+973)17438586; e-mail: drakramm@hotmail.com, drakramm@buss.uob.bh

For this purpose, sampled observations on performance of automatic devices are taken during production or/and functioning of automatic devices over some passage of time at certain time intervals and are analyzed using a linear dynamic system models of Harrison-Akram (1983) and Akram (1990, 92, 94). The outcomes of these models are then used in Hybrid scheme for determining control of automatic devices. These aspects and their practical implications are discussed in the following sections.

II. General Linear Dynamic System Model

For analysis and forecasting of time series $\{y_t\}_{t=1,2,\dots,T}$, bearing white noise $\{\delta_t\}_{t=1,2,\dots,T}$, linear growth models of the type:

$$y_t = f\,\theta_t + \delta_t$$
$$\theta_t = G\,\theta_{t-1} + w_t$$

are frequently constructed and applied for analyses and forecasting of discrete time series, where components of this model are defined as follows:

f = (1 x n) vector of some known functions of independent variables or constants.

θ_t = (n x l) vector of unknown stochastic parameters.

G = (n x n) matrix, called, state or transition matrix, of the n - number of nonzero eigenvalues $\{\lambda_i\}_{i=1,\dots,n}$.

δt is an observation noise, assumed to be normally distributed with mean zero and some known constant variance.

w_t= (n x 1) vector of parameter noise, assumed to be normally distributed with mean zero and a constant known variance-covariance matrix $W = \text{diag}\,(W_1,\dots, W_n)$, the components of which are as defined by Harrison-Akram (1983).

Linear Growth Model

A special case of above general linear dynamic system model is a second order (n = 2) model, called a linear growth model. It is the most commonly used member of the family of the linear dynamic system models as in many real life cases it sufficiently represents the underlying processes of many time series in a parsimonious manner. In this paper, therefore this model will be considered for further discussion. This specific model, in a canonical form, is obtained by defining:

$$f = (1 \;\; 0)$$

$\theta_t = (\theta_1 \;\; \theta_2)'$, where the parameter θ_1 is the level of underlying process of time variant data on functioning of automatic devices and θ_2 is the regression parameter.

ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$G = \{g_{ij}\}_{i,j=1,2}$ is a 2x2 transition matrix having non zero eigenvalues $\{\lambda_i\}_{i=1,2}$, such that $g_{11}=\lambda_1$, $g_{12}=1$, $g_{21}=0$, $g_{22}=\lambda_2$.

W= diag (w_1, w_2) where for a smoothing coefficient $0 < \beta < \min(\lambda_i^2)_{i=1,2}$ the w_1 and w_2 are defined as follows:

$$w_1 = V(1-\beta)(\lambda_1+\lambda_2)(\lambda_1\lambda_2-\beta)/\lambda_2\beta$$
$$w_2 = V(1-\beta)(\lambda_1\lambda_2-\beta)(\lambda_1-\lambda_2\beta)(\lambda_2^2-\beta)/\lambda_2\beta^2$$

The parameters θ_1 and θ_2 of this model are optimally estimated as follows:

For data at time t, $D_t = (y_t, D_{t-1})$, assuming that the prior $(\theta_{t-1} \mid D_{t-1}) \sim N[m_{t-1}; C_{t-1}]$ has the posterior $(\theta_t \mid D_t) \sim N[m_t; C_t]$ an optimum estimate m_t of θ_t is determined by setting f, G, W and priors m_0 and C_0 as explained by Harrison-Akram (1983) and Akram (1992) through the following recursive equations:

$$R_t = GC_{t-1}G' + W$$

$$A_t = R_t f\,[V + f R_t f']^{-1}$$

$$C_t = [I - A_t f]R_t$$

$$m_t = G m_{t-1} + A_t[y_t - f G m_{t-1}]$$

$e_t = y_t - \hat{y}_t = y_t - f^* G m_{t-1}$ are one step ahead forecasts errors.

where at time t, R is a system matrix, I is an identity matrix, A is an updating or gain vector, W is a variance-covariance matrix of parameter noise as defined earlier. The dimensions of all these components are assumed to be compatible with each other. For more discussion see Harrison-Akram (1983).

The estimates m_t of θ_t are then used to generate optimum one step ahead forecasts and fairly accurate medium to long term forecasts on the behavior of automatic devices. These forecasts in turn anticipate automatic devices to respond to real life variations.

Comments

i) In practice more specific version having the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = 1$ is commonly used for a linearly growing phenomena. However, for the exponential growth $\lambda_1 = 1$ and $\lambda_2 < 1$ are considered. Exact value of λ_2 that depends upon the nature of phenomena being studied and their appropriate representation by the exponential functions, such as Logistic and Gompertz functions may be estimated by exponential growth estimation procedure of Akram (1992).

ii) The above model is written in a canonical form. If desired, it may be transformed in to a diagonal form by using inverse transformation of Akram (1988).

iii) The variance V is assumed to be known. If unknown then it may be estimated on line using on-line variance learning procedure of Harrison-Akram (1983).

III. Statistical Process Control Schemes

For statistical process control of automatic devices one step ahead forecast residuals or errors e_t are first standardized as:

$$Z_{et} = \left(e_t - \bar{e}_t\right) \Big/ S_{et}\text{; where } \bar{e}_t = \sum_{i=1}^{T} e_t \Big/ T \text{ and}$$

$$S_{et} = \sum_{i=1}^{T}\left(e_t - \bar{e}_t\right)^2 \Big/ (T-1)$$

where Z_{et} are normally distributed with mean zero and variance one *i.e.* $Z_{et} \sim N(0,1)$ and then used for Shewhart and CUSUM process control schemes.

A. Shewhart Control Limits

For detection of large shifts in process parameters from their preset specifications, the Shewhart control limits are defined as:

$0 \pm Z_{\alpha/2}$; where $Z_{\alpha/2}$ is a standard normal variate at (1- α)% level of confidence *i.e.* upper control limit UCL is $+ Z_{\alpha/2}$ and lower control limit LCL is $- Z_{\alpha/2}$.

B. CUSUM Control Limits

For detection of small shifts in performance of automatic devices from their preset specifications (means and standard deviations) the following control limits, suggested by Hawkins (1993) are defined as follows:

Control Limits for CUSUM

Writing $CUSUM^+$ as accumulation of deviations relative to upper error specifications or reference points of functioning of automatic devices the upper and the lower control limit are defined as follows.

$$CUSUM^+_t = \mathrm{Max}\{0, V_i - k + CUSUM^+_{t-1}\}$$
$$CUSUM^-_t = \mathrm{Min}\{0, V_i - k + CUSUM^-_{t-1}\}$$

whereas $V_i = \{\sqrt{|S_{et}|} - \xi_1\}/\xi_2$ for some known constants ξ_1 and ξ_2. Furthermore, V_i is $\approx N(0,1)$.

For more discussions see Bersimis, *et al* (2007), Hawkins (1993), Lowry, *et al* (1995) and Woodall, *et al* (2004).

$k = \Delta_\mu / 2n_g$ where Δ_μ is the shift in process mean over n_g number of observations in a group or replications at time t. It is the reference value, *e.g.* for shift Δ_μ of mean process, say $\Delta_\mu = 0.02$ and one observation in each subgroup k=0.01.

For using V- mask, if desired, lead distance d may be computed as: d = h/k; where h is the height or distances from the cumulative statistic to the V mask legs.

IV. Practical Aspects of Linear Growth Model and Control Limits

To see whether automatic devices are functioning in accordance with the preset specifications, data on their functions are recorded over some passage of time. These time variant data are then analyzed using the above linear growth model and its parameters are optimally estimated through the stated recursive equations. The estimates in turn

ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

are used to generate one step ahead forecasts residuals or errors. These residuals, as stated earlier, are standardized before drawing charts and computing the control limits of both of these schemes. For computation of Shewhart control limits a value of $Z_{\alpha/2} \geq 2$ is preferred; whereas for CUSUM charts $Z_{\alpha/2} < 2$ may be considered.

Statistically, a process is declared in control if at (1- α)% level of confidence a significant number of residuals or errors are within the control limits of both the schemes and the errors do not reflect non randomness in their distribution. That is, the processes of automatic devices are declared in accordance with the preset specifications. Otherwise the functions of devices are considered not in control.

In case of devices not in control, the distribution of residuals are closely examined for presence of any unwanted noise exhibiting autoregressive structures using ATS of Akram (2001) or AIC of Akaiki (1973). For further discussion see Akram-Irfan (2007).

V. Conclusion

In general, Shewhart and the CUSUM control scheme using one step ahead forecast residuals or errors generated by the linear growth model are expected to perform well for detection of large and small shifts in performance parameters of processes. However, in some cases higher order (n>3) linear dynamic system models with uncorrelated and correlated noise terms may be required to analyze performance data.

For this purpose higher order linear dynamic system models may be constructed and applied using model construction and application procedures of Harrison-Akram (1983) and Akram (1990, 1992 and 1994) and using statistical process control schemes of Harris - Ross (1991). For in depth review of such control schemes, see Bersimis, *et al* (2007).

References

[1] Akaiki H (1973), "Information theory and an extension of the maximum likelihood principle," *Proc. of the 2nd International Symposium on Inference Theory,* Budapest, Hungary, pp. 267-81.
[2] Akram M (2007), "Identification of Optimum Statistical Models for Time Series Analysis and Forecasting using Akaike Information Criterion and Akram Test, Statistic: A Comparative Study," *Proc. of World Congress of Engineers,* London, vol. 2, pp, 956-960.
[3] Akram M (2001), "A Test Statistic for Identification of Noise Processes," *Pak. J. Statistics, 2001,* vol.17(2), pp. 103-115.
[4] Akram M (1994). "Computational Aspects of State Space Models for Time Series Forecasting," *Proceedings of 11th Symposium on Computational Statistics (COMPSTAT-1994),* Vienna, Austria, pp. 116-117.
[5] Akram M (1992). "Construction of State Space Models for Time Series Exhibiting Exponential Growth," *Computational Statistics* vol.1, Physica Verlag, Heidelberg, Germany, pp. 303-308.
[6] Akram M (1990), "State Space Models for Forecasting," *Proc. Third Intl. Conference on Teaching Statistics,* vol.2, ISI Publications, Amsterdam, Netherlands. Pp. 105-6.
[7] Akram M (1988). "Recursive Transformation Matrices for Linear Dynamic System Models," *J. Computational. Stat. & Data Analysis, 6*, pp. 119-127.
[8] Bersimis S-Psaakis S-Panaretos J(2007), "Multivariate Statistical Process Control Charts: An Overview," *Quality and Reliability Engineering International 2007*; pp. 517-543.
[9] Box G.E.P-Jenkins G.M. (1963), "Further contributions to Adaptive Quality Control: Simultaneous Estimation of Dynamics:Non Zero Costs," *ISI Bulletin, 34th Session*, Ottawa, Canada.
[10] Crosier, R.B. (1988), "Multivariate Generalizations of Cumulative Sum Quality Control Schemes," *Technometrics, 30,* pp. 291-303.
[11] Harris T.J– Ross W.H. (1991), "Statistical Process Control Procedures for Correlated Observations," *Canadian J. of Chemical Engineering, 69*, pp. 48-57.
[12] Harrison P J - Akram M (1983), "Generalized Exponentially Weighted Regression and parsimonious Dynamic Linear Modeling." *Time Series Analysis: Theory and Practice 3*, pp. 19-42.
[13] Hawkins, D.M (1993), "Cumulative Sum Control Charting: An Underutilized SPC Tool," *Quality Engineering, 5,* pp. 463-477.
[14] Johnson N.L(1961), "A Simple Theoretical Approach to Cumulative Sum Charts." *JASA, 56,* pp. 835-840.
[15] Keats J.B.-Castillo E.D. Elart V.C.-Saniga E.M. (1997), "Economic Modeling for Statistical Process Control," *J. of Quality Technology, 29,* pp. 144-147.
[16] Lowry C.A-Montgomery D.C (1995), "A Review of Multivariate Control Charts," *IEE Transactions, 27*, pp. 800-810.
[17] Shewhart. W.A (1931). "Economic Control of Quality Manufactured Product," *D. Van Norstrand Co., Inc.,* Princton , N.J, USA.
[18] Wodall W.H -Spitzner D.J–Montgomery D.C-Gupta S (2004), " Using Control Charts to Monitor Process and Product Quality Profiles," J. *Quality Technology; 36*, pp. 309-320.
[19] Vander W.S.A-Tucker W.T-Falten F.W-Doganaksoy (1992), "Algorithmic Statistical Process Control: Concepts and Applications," *Technometrics, 34,* pp. 286-297.

ISBN: 978-988-19251-5-2
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)