Enhanced Multi-Channel Active Fuzzy Neural Network Noise Control in an Enclosure

Navid Azadi, Abdolreza Ohadi

Abstract—The performance of conventional linear algorithms in Active Noise Control (ANC) applications deteriorates facing nonlinearities in the system, which is mainly due to loudspeakers. On the other hand, fuzzy logic and neural networks are good candidates to overcome this drawback. In this paper, the acoustic attenuation of noise in a rectangular enclosure with a flexible panel and 5 rigid walls is presented using a multi-channel enhanced fuzzy neural network (EFNN) error back propagation algorithm while taking into account the secondary path effect in derivation of equations. The phrase enhanced represents the secondary path effect in derivation of EFNN controller. The primary path in the system comprises a nonlinear model of loudspeaker, parameters of which vary with the input current. Next, the performance of FXLMS and enhanced fuzzy neural network algorithms are compared and it is observed that fuzzy neural network controller exhibits better results in the presence of loudspeakers with nonlinear behavior.

Index Terms—Active Noise Control, Fuzzy Neural Networks, Nonlinear Loudspeaker, FXLMS, 3D Enclosure, Flexible Panel.

I. INTRODUCTION

IN integration with Passive Noise Control techniques, an Active Noise Control system can introduce a more efficient noise cancellation in systems. These systems are widely incorporated in air conditioning systems as well as vehicles and public transportation systems. Active Noise Control systems are based on the principle of superposition of waves, in which the noise sound wave is canceled with anti-noise sound wave as illustrated in Fig. 1. Anti-noise wave should specifically have the same amplitude as noise, but should be in 180 of phase shift. This task is done using an adaptively tuned algorithm in the secondary path of the ANC system.

In most of applications of ANC systems, a linear algorithm is used to generate the anti-noise signal. A vivid example is the FXLMS algorithm, in which an FIR filter is employed in such a way for the error signal to be minimized. On the other hand, in specific ANC systems primary path exhibits nonlinear behavior, the output signal of which a linear algorithm is not designed to cancel. In this paper, an enhanced neuro-fuzzy ANC system is implemented in a 3D enclosure with a flexible panel to minimize the noise introduced in the system by an internal nonlinear noise source, namely, a nonlinear loudspeaker.

Kim and Brenna in [1], [2], [3] discussed the acoustic-structure couple in a rectangular enclosure with a simply supported flexible panel in 1999. They also studied the propagation of a plane wave into the enclosure through the flexible panel. Later in 2003, Jorge introduced an analytical model for the vibration of rectangular clamped plate using virtual work principle and verified his results with finite element method [4]. In addition, in 2007, Wu et al. presented an exact solution for free vibration analysis of rectangular plate using Bessel functions. The formulations introduced here in this article are derived using these two references [5].

Regarding the modeling of nonlinear loudspeaker, Ravaud [6] presented a time-varying nonlinear model of an electrodynamic loudspeaker. In this article, two coupled differential equations, one for displacement of diaphragm and the other for magnetic field in the loudspeaker, are introduced and based on these two equations, a nonlinear model is presented. There is lots of information available in the field of linear Active Noise Control in text books. LMS algorithm and its derivatives like FXLMS and NLMS are widely discussed in [7]. On the other hand, nonlinear ANC is a rather new field of study. In 2000, Carmona et al. [8] implemented an active noise control in a duct using pole placement and sensitivity function reshaping in robust control. Recently, Neural Networks and Fuzzy control are also implicated in ANC application. In 2002, active noise hybrid feedforward/feedback control using neural network compensation was performed, in which error gradient descent is employed to update neural network parameters [9]. Later in 2004, a simplified Fuzzy-Neral Network control algorithm was introduced [11] and was improved in 2006 by the same author as Adaptive Recurrent Fuzzy Neural Network for Active Noise Control. Furthermore, this controller was implemented on the acoustic field in a duct in 2008. The effect of secondary path was not studied in none of these algorithms. As an amendment, Rastegar and Ohadi in 2009 examined the implementation of secondary path in derivation of equations [12]. They showed the effect of secondary path by carrying out simulations on the acoustic field in an enclosure with a flexible panel.

In this article the effect of introduction of secondary path transfer function in derivation of a fuzzy-neural network

![Fig. 1. Schematic illustration of the principle of superposition of waves.](image-url)
controller for the attenuation of acoustic noise in a cubic enclosure with 5 rigid walls and a flexible clamped plate is investigated in simulations.

II. MODELING OF ENCLOSURE WITH FLEXIBLE PANEL

In this section, modeling of acoustic-structure couple in the enclosure is briefly studied. All the variables used in this section are defined in Table I. As shown in Fig. 2 and according to [1], the governing partial differential equation of vibration of a rectangular plate is (1).

$$D_p \nabla^4 w (y_1, y_2, t) + \rho_b h_p \frac{\partial^2 w (y_1, y_2, t)}{\partial t^2} = F_p (y_1, y_2, t). \quad (1)$$

Boundary limits and boundary conditions for a clamped rectangular plate are defined as (2) and (3), respectively.

$$0 \leq y_1 \leq L_2, \quad 0 \leq y_2 \leq L_3$$

$$w = 0, \quad \partial w / \partial n = 0 \quad (3)$$

On the other hand, the acoustic enclosure is modeled via a partial differential equation as described by (4), which is derived using laws of continuity of mass and continuity of momentum.

$$\nabla^2 P (t, x) - \frac{1}{c_0^2} \frac{\partial^2 P (t, x)}{\partial t^2} = -\rho_0 \frac{\partial^2 W (x_2, x_3, t)}{\partial t^2} \delta (x_1 - L_1) - \rho_0 i u_k (t) P_{0ksp} (x_{sp}). \quad (4)$$

Boundary conditions for the enclosure are defined so that there is no gradient of pressure on the walls, mathematical interpretation of this statement is like (5).

$$\nabla P (t, x) \cdot n = 0 \quad \text{on} \quad \Omega_B \quad (5)$$

Manipulating equations (1) and (4), two coupled differential equations describing this coupled vibro-acoustic problem are derived in (6) and (7).

$$\ddot{\bar{q}}_n (t) + 2 \xi_n \omega_n \dot{\bar{q}}_n (t) + \omega_n^2 \bar{q}_n (t) = \frac{c_0 \rho_0}{V_a} \left[ \sum_{m=1}^{N_p} c_{n,m} \ddot{\bar{q}}_m (t) + g_{sp,n} (t) \right] \quad (6)$$

$$\ddot{\bar{\eta}}_m (t) + \omega^2_{p,m} \dot{\bar{\eta}}_m (t) = \frac{1}{m_p} \int \int \phi_m (y_1, y_2) p_{in} (y_1, y_2, t) \, dy_1 \, dy_2. \quad (7)$$

Defining state space variables like

$$\begin{align*}
\{ \bar{x}_m \}^T &= \{ \bar{\eta}_m (t) \}, \quad \{ \bar{y}_m \}^T &= \{ \bar{q}_m (t) \} \quad (8)
\end{align*}$$

On the other hand, the state space model of the whole system is derived as (9).

$$\dot{\mathbf{x}}_{state} = A_{state} \mathbf{x}_{state} + D_{2sp} f_{sp} \quad (9)$$

In which, $f_{sp}$ is the excitation of this state space model and in related to pressure exerted in the enclosure by a loudspeaker. In this equation states are defined in (10).

$$\mathbf{x}_{state} = \left\{ \begin{array}{c} x_1^1 \ y_1^1 \ x_1^2 \ y_1^2 \ 
\vdots \ x_1^n \ y_1^n \\
x_2^1 \ y_2^1 \ x_2^2 \ y_2^2 \ 
\vdots \ x_2^n \ y_2^n \\
\end{array} \right\}^T$$

The model of the enclosure, from pressure in one point to pressure in another, is now at hand. In the next step, using a nonlinear model of loudspeaker as what defined in [6], transfer function between input voltage of a loudspeaker inside the enclosure and the pressure in another point would be at hand. In the next section, a controller is designed to attenuate pressure fluctuations (or noise) induced in the enclosure by this loudspeaker using a linear loudspeaker.

III. STRUCTURE OF ANC SYSTEM

Schematic diagram of an active fuzzy neural network active noise control system is shown in Fig. 3. Error signal is described as (11).

$$e (n) = d (n) - S (z) y^{(5)} (n) \quad (11)$$

where $e (n)$ is error signal, $d (n)$ is desired signal generated by nonlinear primary path, $P (z)$, and $y^{(5)} (n)$ is control signal at step $n$ handed in to $S (z)$, the transfer function of secondary path. The $z$-transform of (11) is presented as (12).

$$E (z) = P (z) X (z) - S (z) Y^{(5)} (z) \quad (12)$$

In practical ANC applications, $S (z)$ is unknown and must be estimated by an additional filter, $\hat{S} (z)$. Therefore, (12) is modified to (13).

### TABLE I

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1, x_2, x_3$</td>
<td>Coordinates of enclosure</td>
</tr>
<tr>
<td>$y_1, y_2$</td>
<td>Coordinates of plate</td>
</tr>
<tr>
<td>$L_1, L_2, L_3$</td>
<td>Dimensions of enclosure</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Speed of sound</td>
</tr>
<tr>
<td>$c_{n,m}$</td>
<td>Coupling coefficient</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Flexural rigidity of plate</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Young’s modulus of elasticity</td>
</tr>
<tr>
<td>$g_{sp,n}$</td>
<td>Modal pressure of loudspeakers on enclosure</td>
</tr>
<tr>
<td>$h_p$</td>
<td>Flexible panel thickness</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Mass of plate</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure inside the enclosure</td>
</tr>
<tr>
<td>$P_{in}$</td>
<td>Dynamic pressure inside the enclosure</td>
</tr>
<tr>
<td>$p_{in}$</td>
<td>Position of loudspeaker</td>
</tr>
<tr>
<td>$q_n (t)$</td>
<td>Modal coordinates of air</td>
</tr>
<tr>
<td>$V_a$</td>
<td>Volume of enclosure</td>
</tr>
<tr>
<td>$w$</td>
<td>Lateral vibration of plate</td>
</tr>
<tr>
<td>$\eta_m (t)$</td>
<td>Modal coordinates of plate</td>
</tr>
<tr>
<td>$\Omega_B$</td>
<td>Boundary of enclosure</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>Natural frequency of air</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Natural frequency of plate</td>
</tr>
<tr>
<td>$\phi_m$</td>
<td>Modal function of vibration of plate</td>
</tr>
<tr>
<td>$p_p$</td>
<td>Dexterity of plate</td>
</tr>
<tr>
<td>$\nu_p$</td>
<td>Poisson’s ratio of plate</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Dexterity of air</td>
</tr>
<tr>
<td>$\zeta_a$</td>
<td>Damping coefficient of air</td>
</tr>
</tbody>
</table>

Fig. 2. Cubic coupled acoustic-structure enclosure with 5 rigid walls and a flexible panel including coordinate systems.
\[ E(z) = P(z)X(z) - \hat{S}(z)Y^{(5)}(z) \]  
(13)

Assuming \( \hat{S}(z) \) to be a vector of length \( l_f \) of weights \( b_i, i = 1, 2, \ldots l_f \), equation (11) can be presented as (14).

\[ e(n) = d(n) - \left[ b_1 y^{(5)}(n) + b_2 y^{(5)}(n-1) + \ldots + b_{l_f} y^{(5)}(n-l_f) \right] \]  
(14)

Hence, the cost function can be introduced in (15).

\[ E(n) = \frac{1}{2} \left[ d(n) - \left[ b_1 y^{(5)}(n) + b_2 y^{(5)}(n-1) + \ldots + b_{l_f} y^{(5)}(n-l_f) \right] \right]^2 \]  
(15)

Existence of \( \hat{S}(z) \) in (15) insinuates that secondary path effect has to be accounted for in derivation of governing equations of controller. Consequently, \( E(n) \) is to be minimized using steepest gradient method in order for \( e(n) \) to approach zero.

IV. INTRODUCTION TO ENHANCED FUZZY NEURAL NETWORK ANC

In this section, an enhanced fuzzy neural network active noise control system is presented in single channel mode using [11], [12] and then it is developed to multi-channel case. It is worth mentioning that the word “Enhanced” implies that in derivation of controller equations, the secondary path effect in behavior of system is taken into consideration.

A. Single Channel FNN Structure

The structure of a fuzzy neural network controller is illustrated in Fig. 4. As shown in this figure, this control algorithm comprises 5 layers, namely, input layer, fuzzification layer, rule base layer, inference layer and defuzzification layer. Inference system implemented in this controller is of Mamdani-type. In layer 1, each crisp signal, \( x_i \), is transmitted to the next layer in each node.

\[ y^{(1)}_i = x_i, \quad i = 0, 1, \ldots, m \]  
(16)

Fuzzification is carried out in layer 2 using a Gaussian activation function.

\[ y^{(2)}_{ij} = \exp \left( -\frac{\left( y^{(1)}_j - c_{ij} \right)^2}{\sigma_{ij}} \right), \quad j = 0, 1, \ldots, n \]  
(17)

in which \( c_{ij} \) and \( \sigma_{ij} \) are center and width of this membership function, respectively. Reasoning in fuzzy logic is performed using t-norm or min in layer 3. This is done considering fuzzy logic product introduced in (18).

\[ y^{(3)}_j = C_j = \prod_{i=1}^{m} y^{(2)}_{ij} \]  
(18)

Mamdani inference is what carried out in layers 4 and 5 via a simple center of gravity scheme. Consequently, crisp output signal is generated in layer 5 by (21).

\[ y^{(4)}_1 = \sum_{j=1}^{n} y^{(3)}_j \cdot w_j \]  
(19)

\[ y^{(4)}_2 = \sum_{j=1}^{n} y^{(3)}_j \cdot w_j \]  
(20)

\[ y^{(5)} = \frac{y^{(4)}_i}{\sum_{j=1}^{n} y^{(3)}_j} \]  
(21)

B. Derivation of Updating Rules Considering the Effect of Secondary Path, Single Channel Model

In order to minimize the cost function introduced in (15), parameters like center and width of Gaussian membership function including weights of inference logic, mentioned respectively in (17) and (19), has to be tuned in each time step, \( n \). As a result, using steepest gradient method (22), (23) and (24) are derived.

\[ w_j(n+1) = w_j(n) - \eta_w \frac{\partial E(n)}{\partial w_j}, \]  
(22)

\[ c_{ij}(n+1) = c_{ij}(n) - \eta_c \frac{\partial E(n)}{\partial c_{ij}}, \]  
(23)

\[ \sigma_{ij}(n+1) = \sigma_{ij}(n) - \eta_\sigma \frac{\partial E(n)}{\partial \sigma_{ij}} \]  
(24)

in which \( \eta_c, \eta_\sigma \) and \( \eta_w \) are learning rate coefficients for \( c_{ij}, \sigma_{ij} \) and \( w_j \), respectively. Using chain rule on (15) we can have:

\[ \frac{\partial E(n)}{\partial w_j} = -e(n) \left[ b_1 \frac{\partial y_j^{(5)}(n)}{\partial w_j} + \ldots + b_{l_f} \frac{\partial y_j^{(5)}(n-l_f)}{\partial w_j} \right], \]  
(25)

\[ \frac{\partial E(n)}{\partial c_{ij}} = -e(n) \left[ b_1 \frac{\partial y_j^{(5)}(n)}{\partial c_{ij}} + \ldots + b_{l_f} \frac{\partial y_j^{(5)}(n-l_f)}{\partial c_{ij}} \right], \]  
(26)

\[ \frac{\partial E(n)}{\partial \sigma_{ij}} = -e(n) \left[ b_1 \frac{\partial y_j^{(5)}(n)}{\partial \sigma_{ij}} + \ldots + b_{l_f} \frac{\partial y_j^{(5)}(n-l_f)}{\partial \sigma_{ij}} \right]. \]  
(27)

Again, applying chain rule on (21), unknown parameters in (25), (27) and (26) are derived as followings.

\[ \frac{\partial y_j^{(5)}(n)}{\partial w_j} = \frac{\partial y_j^{(5)}(n)}{\partial y_j^{(4)}} \frac{\partial y_j^{(4)}}{\partial y_j^{(3)}} \frac{\partial y_j^{(3)}}{\partial w_j} = \left( \frac{1}{y_2} \right) \left( y_j^{(3)} \right), \]  
(28)

\[ \frac{\partial y_j^{(5)}(n)}{\partial c_{ij}} = \frac{\partial y_j^{(5)}(n)}{\partial y_j^{(4)}} \frac{\partial y_j^{(4)}}{\partial y_j^{(3)}} \frac{\partial y_j^{(3)}}{\partial c_{ij}} \]  
(29)

\[ \frac{\partial y_j^{(5)}(n)}{\partial \sigma_{ij}} = \frac{\partial y_j^{(5)}(n)}{\partial y_j^{(4)}} \frac{\partial y_j^{(4)}}{\partial y_j^{(3)}} \frac{\partial y_j^{(3)}}{\partial \sigma_{ij}} \]  
(30)
C. Derivation of Updating Rules Considering the Effect of Secondary Path, Multi-Channel Model

In multi-channel mode, the structure of ANC system is evolved to what illustrated in Fig. 5. This is evident in Fig. 5 that in multi-channel model there are 4 secondary paths between sensors and actuators; One from error microphone no. 1 to secondary path loudspeaker 1 or \( S_{11}(z) \), second, from error microphone no. 1 to secondary path loudspeaker 2 or \( S_{12}(z) \), Third, from error microphone no. 2 to secondary path loudspeaker 1 or \( S_{21}(z) \) and fourth, from error microphone no. 2 to secondary path loudspeaker 2 or \( S_{22}(z) \).

The respective identified FIR filter of these paths are \( S_{11}(z) \), \( S_{12}(z) \), \( S_{21}(z) \) and \( S_{22}(z) \).

\[
E(z) = P(z)X(z) - Y_1^{(5)}(z)S_{11}(z) - Y_2^{(5)}(z)S_{22}(z), \tag{31}
\]

\[
e_1(n) = d_1(n) - b_1y_1^{(5)}(n) + b_2y_2^{(5)}(n-1) + \ldots + b_{l_{f11}}y_1^{(5)}(n-l_{f11}),
\]

\[
b_1'Y_2^{(5)}(n) + b_2'Y_2^{(5)}(n-1) + \ldots + b_{l_{f12}}Y_2^{(5)}(n-l_{f12}). \tag{32}
\]

And the cost function for the first channel will be derived like (33).

\[
E_1(n) = \frac{1}{2} \left( d(n) - b_1y_1^{(5)}(n) + b_2y_2^{(5)}(n-1) + \ldots + b_{l_{f11}}y_1^{(5)}(n-l_{f11}) \right)^2. \tag{33}
\]

Again, like what was done to derive (25), in multi-channel model using steepest gradient method correction terms of \( w_j \) in channel 1, \( \frac{\partial E_1}{\partial w_{1j}} \) and \( \frac{\partial E_2}{\partial w_{2j}} \), are reproduced like (34) and (35).

\[
\frac{\partial E_1}{\partial w_{1j}} = -e_1(n) \left[ \frac{\partial y_1^{(5)}}{\partial w_{1j}} \right] \text{Filtered by } \hat{S}_{11}. \tag{34}
\]

\[
\frac{\partial E_2}{\partial w_{2j}} = -e_2(n) \left[ \frac{\partial y_2^{(5)}}{\partial w_{2j}} \right] \text{Filtered by } \hat{S}_{21}. \tag{35}
\]

Thus, using chain rule updating formulas take a form like (36) for channel 1.

\[
w_{1j} (t + 1) = w_{1j} (t) - \eta_{w_1} \frac{\partial E_1}{\partial w_{1j}} - \eta_{w_2} \frac{\partial E_2}{\partial w_{2j}}. \tag{36}
\]

In a likewise manner as what was followed for channel 1, updating rule of \( w_j \) for channel 2 is derived like (37).

\[
w_{2j} (t + 1) = w_{2j} (t) - \eta_{w_1} \frac{\partial E_1}{\partial w_{2j}} - \eta_{w_2} \frac{\partial E_2}{\partial w_{2j}}. \tag{37}
\]

Derivation of updating rules for \( c_{ij} \) and \( \sigma_{ij} \) for channel 1 and channel 2 are almost like the process followed in (36) and (37) and are not mentioned here.

![Fig. 5. Structure of multi-channel Fuzzy Neural Network control algorithm.](image-url)
TABLE IV
SOUND ATTENUATION IN 420 Hz SINE INPUT.

<table>
<thead>
<tr>
<th>Input Freq.</th>
<th></th>
<th>Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>420 Hz</td>
<td>Attenuation in channel 1</td>
<td>Attenuation in channel 2</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>27</td>
</tr>
</tbody>
</table>

TABLE V
SOUND ATTENUATION IN 90 Hz SINE INPUT.

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Freq.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90 Hz</td>
<td>7</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>31</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Algorithm</td>
<td>FXLMS</td>
<td>EFNN</td>
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REFERENCES


