# On the Heat Conduction in Natural Porous and Anisotropic Materials

I. Malujda, A. Marlewski

*Abstract*— **On the base of the data gathered in experiments made on the intentionally designed stand there is proposed the arctan-approximation to the dependence of the thermal conductivity in all anatomic directions of the wood, which are longitudinal, radial and tangential. We conclude that this approximation works very well in higher values of the temperature. This range, up to 200°C, has to be taken into account when there are undertaken the optimization problems in the thermal forming of superficial layers of materials which are porous and anisotropic (the wood and sawdust are good examples of such materials).**

*Index Terms*— **anisotropy, Fourier's equation, temperature, conductivity.**

## I. INTRODUCTION

The purpose of compression and thermal plasticization is to obtain desired physical properties of products, especially in the superficial layer (crust) and to convert certain biomasses into an environment-friendly and renewable source of energy. These are primarily porous materials having complex thermo-mechanical properties, such as wood - representing materials with anisotropic properties and sawdust - representing particulate materials. The effect of heat is of primary importance in thermal plasticization and geometrical shaping processes. Therefore, it is necessary to determine the distribution of temperature, especially in the crust during compression and thermal plasticization [6].

The purpose of modeling the processes of compression and thermal plasticization of loose materials, as well as porous and anisotropic materials, is to determine the critical stress condition which initiates plastic flow. This value is critical for the effectiveness of thermal plasticization process and depends on the thermo-mechanical parameters of the material and the key parameters of the process itself [6]. As it has been proven by research, the strength of materials such as wood and sawdust evidently decreases with the increase of temperature, influenced also by the moisture content of the material. Therefore, temperature distribution, especially in the thin superficial layer, is one of the main characteristics taken into account in formulating constitutive equations, especially regarding thermal conductivity and plasticity.

I. Malujda is with the Chair of Basics in Machine Design, Poznań University of Technology, Poland, e-mail: ireneusz.malujda@put.poznan.pl
A. Marlewski is with the Institute of Mathematics, Poznań University of Technology, Poland, e-mail: adam.marlewski@put.poznan.pl

For determining the distribution of temperature in a layer of thermally plasticized material it is indispensable to determine the heat conductivity coefficient $\lambda$.

## II. CHARACTERISTICS OF THE PROCESSES UNDER DISCUSSION

Designing of machines for densification and plasticization of structural and waste materials plays a major role in development of new processing techniques. These are utilized in the production of certain kinds of biomasses as an environment friendly and renewable source of energy. Moreover, specific physical properties may be obtained in the superficial layer of material, an example of which could be improving the quality by refinement of wood surface by hot rolling. For modeling of materials with porous and anisotropic characteristics (Fig.1) and compression and plasticization of loose materials (Fig. 2) the primary parameter is the critical strain at which plastic flow commences.

Refining of wood shall be understood as smoothing its surface and compacting its internal structure [7]. As a result of high temperature influencing the external layer of wood undergoes a hydrolysis process which starts the plasticization process. Under simultaneously applied pressure the plasticized material fills up wood pores, so it contributes to densification of wood structure and smoothness of its external surface. Moreover, a simultaneous application of pressure and temperature, results in a uniformity of wood structure in the layer close to the surface and its consolidation occurring mainly in direction parallel to fibers. The resulting increase in wood strength makes the rolled wood is a better material in some technical applications.

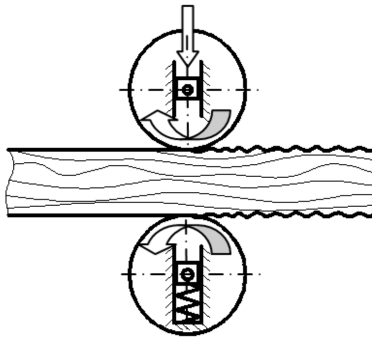

Fig. 1. Rolling process of a veneered furniture element.

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

The smoothness of a layer of wood which has been treated in such a manner, allows for a considerable reduction of sanding and its higher density makes it possible to reduce varnish consumption.

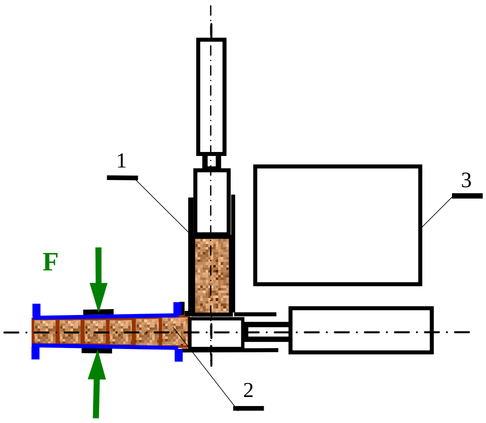


Fig. 2. Kinematics diagram of the sawdust compressing process, where :1 – initial densification chamber 2 – forming sleeve, 3 – hydraulic power unit.

While briquetting of the sawdust and wood wastes without binding agents, in order to achieve a sufficiently durable briquette consolidation, it is necessary to ensure not only adequate reduction of initial volume of compacted material but also its plasticization. Moreover, when the sawdust is pushed through the briquetting chamber (Fig.2) and further on trough the forming sleeve, in a layer of the briquette close to its surface, due to friction, the temperature rises up to about 130 $^0$C. At such temperature the material (just like in case of hot rolling) undergoes the plasticization, thus creating a very thin layer on briquette surface, which becomes, after cooling down, a solid and smooth reinforcing structure. These conditions require appropriate pressure and temperature in the forming sleeve.

Theoretical determination of the thermal conductivity coefficient for these materials is rather difficult and hence the efforts to develop appropriate methods and devices for its empirical determination. The experience from various tests of the above-mentioned materials was used to develop such measuring assembly, designated for testing of wood, particulate, fibrous and other biomass materials ( Fig. 3.).

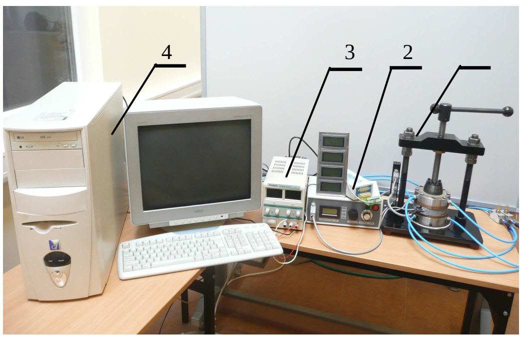


Fig. 3. The station, installed in Laboratory for Material Tests in Poznań University of Technology, to the investigation of the thermal conductivity coefficient; 1- measuring assembly, 2 – power supply unit, 3– signal amplifier, 4 – recording computer with metering card

## III. FLOW OF HEAT DURING COMPRESSION PROCESSES

As it has been proven experimentally the strength of materials such as wood decreases significantly with the increase of temperature [6]. Therefore, the determination of temperature distribution in a layer of material during thermal plasticization is one of the key points in the development of the mathematical model which describes of plasticization processes. The transfer of heat by conduction in unsteady conditions when the layer of material closely abuts on the hot pressing surface is closest to the actual transfer of heat [7].

In thermal plasticization process of the analysed materials, such as wood, the heat is transferred via a unsteady conduction, which is described by the second Fourier's law:

$$\frac{\partial T}{\partial t} = a\nabla^2 T = a\frac{\partial^2 T}{\partial x^2} \qquad (1)$$

where: $a = \dfrac{\lambda}{c_p \rho}$ – thermal diffusivity, $T$ – temperature, $t$ time, ρ – density of material, $c_p$ – specific heat capacity, λ – thermal conductivity coefficient.

The solution of this equation, with appropriately defined initial and threshold conditions, will allow the determination of the temperature as a function of time at any point of the analyzed layer of material. The purpose of invoking the above equation (1) at this point is to draw attention to the significance of the diffusivity coefficient which is critical to the accuracy of the result.

Since both the thermal conductivity coefficient and the specific heat capacity change with temperature [4], the relevant empirically established material functions must be included in equations

$$\lambda = f(T), \quad c_p = f(T) \qquad (2)$$

Moreover, the thermal conductivity coefficient depends also on the anatomic directions of wood and its values were obtained during experimental investigations using testing station (Fig.3).

## IV. THE ARCTAN-APPROXIMATION TO THE THERMAL CONDUCTIVITY DEPENDENCE ON THE TEMPERATURE

In numerous papers there is described the dependence of the thermal conductivity λ on the temperature $T$ (and, moreover, there are taken into account other parameters such that the moisture). The most popular models are polynomial (of the 1st and 2nd degree), rational and power ones, here the approximating functions are of the form:

$$\lambda(T) = A + B\cdot T,\ \ \lambda(T) = A + B\cdot T + C\cdot T^2,$$
$$\lambda(T) = A + \frac{B}{T+C},\ \lambda(T) = A\cdot T^B$$

respectively, where the parameters $A, B$ and $C$ are determined in the way the fit to be as good as possible; see. e.g. [2, chapters 3-17], [3], [12], [13], [14], [16]. Naturally, the description at hand


ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

is valid within the considered range of the temperature $T$, usually it varies from 20 to 200°C. There are also considered other ranges. Let us here cite the paper [15] where there is derived the power law concerning the thermal conductivity of oak and maple in the temperature range 0.02–1°K; Authors conclude that there takes place the power law with the reference thermal conductivity and the exponent:

$$A = (9.3 \pm 1.9)\cdot 10^{-3}\ \mathrm{W/(K\cdot m)},\quad w = 2.7 \pm 0.4.$$

Obviously, this model does not remain valid in a higher temperature, e.g. for 30°C it produces the senseless value $9.3\cdot 10^{-3}\cdot 303.15^{2.7} \approx 25000$, while the real thermal conductivity is less than 0.30.

TABLE 1
VALUES OF THE THERMAL CONDUCTIVITY IN THE LONGITUDINAL, RADIAL AND TANGENTIAL DIRECTIONS ($L_J$, $R_J$ AND $T_J$, RESP.) OF BEECH SAMPLES MEASURED ON THE APPARATUS

| $j$ | $T$ [°C] | $l_j$ [W/(m·K)] | $r_j$ [W/(m·K)] | $t_j$ [W/(m·K)] |
|---|---|---|---|---|
| 1 | 40.6 | 0.3878 | 0.1722 | 0.1556 |
| 2 | 61.2 | 0.4361 | 0.1963 | 0.1558 |
| 3 | 80.2 | 0.4479 | 0.2035 | 0.1648 |
| 4 | 100.1 | 0.4635 | 0.2058 | 0.1636 |
| 5 | 119.3 | 0.4638 | 0.2173 | 0.1667 |
| 6 | 140.4 | 0.4577 | 0.2105 | 0.1594 |
| 7 | 161.9 | 0.4618 | 0.2173 | 0.1594 |

In the Table 1 there are listed the values of the thermal conductivity of beech disks (the diameter of 80 mm, the thickness 5 mm) measured in the longitudinal, radial and tangential directions for seven values of the temperature (equal about 40, 60, ..., 160 °C). We examined several candidates, the four forms listed above included, to be good approximations to the distribution of the points $(T_j, y_j)$, where $y$ stands for $l$, $r$ or $t$. We applied the computer algebra system Derive 5.0 from Texas Instruments, Inc., we worked with 10 decimal digits and we found that the best least-squared fit to the set of points $(T_j, l_j)$ is the function

$$l(T) = a\cdot \mathrm{atan}(T - 273.15)^2 + b$$

where $T$ denotes the temperature [°K], $a = 135.10325$, $b = -211.7495284$ (see Fig.4).
This means that the pair $(a, b) = (135.10325, -211.7495284)$ makes the quantity

$$Q = \sum_{j=1}^{n} \{a\cdot \arctan(T_j - 273.15)^2 + b - l_j\}^2$$

assumes its smallest value. Both values $a$ and $b$ are calculated via the least-squared method (a.k.a. the least square method, la méthode des moindres carrés, first described, independently, by C.F.Gauss in A.-M.Legendre in the turn of 18th and 19th centuries), so they are the solution of the system of the equations

$$\frac{\partial Q}{\partial a} = 0,\ \frac{\partial Q}{\partial b} = 0.$$

The quality of the approximation is

$$\frac{1}{n}\sqrt{Q} \approx 0.000869;$$

the 100-times zoomed errors of the approximation at each point $P_j$ are visualized in Fig.4. It verifies that the function at hand approximates the distribution $\{ P_j : j = 1, 2, ..., 6\}$ very well. It also appropriately exhibits the behaviour of the real thermal conductivity for the increasing temperature $T$, namely it approaches to circa 0.4702 W/(m·s).

Obviously, we can use this approximation within some limits. A wood starts burning at the temperature about 250°C, so, clearly, this temperature is the definitely upper limit. Therefore we admit the discussed relation when $T$ does not exceed, let's say, 200°C (there is the maximal temperature of the heater working in the experimental stand).

What concerns the lower limit for the temperature $T$, let's admit it to be 20°C. We state it because we gathered the experimental results in the stand installed in the hall where the temperature was about 20°C. Putting $T = 20$ and 300 in our formula we get 0.132 and 0.288 W/(m·K), both values can be well accepted when one looks into the tables where the thermal conductivities of the wood; for instance [14] gives values 0.11-0.45 W/(m·K) at $T = 20$°C and the moisture 0-15%.

Let's call the proposed approximation as the arctan-approximation. The arctan-approximation is a linear fit, as all four popular models mentioned above. It's easy to accept the arctan-approximation as the natural form of the fit description because it relates very well with the natural behaviour of the change in the thermal conductivity $\lambda$, both theoretical consideration and the experiment show that in higher values of the temperature $T$ the conductivity $\lambda$ changes insignificantly (see Fig.4, where the stabilization takes place at the level about 0.466 W/(m·K)). Obviously, this property takes no place when the polynomial or exponential model is applied. For the rational model with assumed value for $C = 0$ the best least-squared fit is of the form

$$l(T) = 0.494741 - \frac{4.04867}{T}$$

with centigrade temperature $T$ [°C] and it shows that the stabilization takes place at the level 0.494741, so it is about 0.0245/0.4702 = 5% higher than the level produced by the arctan-approximation.

Note that the arctan-approximation is very sensitive to the roundings. For the data here reported it admits to have the coefficients rounded to third decimal place,

$$\lambda(T) = 135.103\cdot \arctan(T - 273.15)^2 - 211.749$$

and it exhibits the eye-inspected differences when the pair $(a, b)$ is rounded to (135.10, –211.75) – the graph drops down about 0.01 for $T = 400$ °K, (135.10, –211.74), (135.10, –211.7), (131, –211) – the graph goes up about 0.005, 0.09 and 0.588 (from 0.461 to 1.049).
Four popular fits are not so catastrophic sensible. For instance, the best least-squared fits are

$$l(T) = 0.02728144338\cdot T^{0.471613212},$$
$$l(T) = 0.0005321294619\cdot T + 0.2464546191,$$
$$l(T) = -8.605342471\cdot 10^{-6}\cdot T^2 + 0.006976148824\cdot T - 0.9460821095,$$

in the classes of power functions, of polynomials of 1st and 2nd degrees, resp. When we take the coefficients with less number of digits we get the formulas

$$l(T) = 0.027\cdot T^{0.47},\ l(T) = 0.00053\cdot T + 0.25,$$
$$l(T) = -8.605\cdot 10^{-6}\cdot T^2 + 0.006976\cdot T - 0.9461$$

and it does not essentially affect the quality of approximations; the same good conditioning occurs for the rational fit,

$$l(T) = 0.495 - 4.05/T,$$

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

and no further rounding can be accepted, the graph of the function $l(T) = 0.5–4/T$ is shifted up,at $T = 20°C$ from 0.2915 to 0.3 W/(m·K).

Although the stability of these functions is the advantage of these models, the arctan-approximation fits better to the measured values, namely it better exhibits the values of the thermal conductivity for higher values of the temperature $T$ (and just this behaviour is the point of our interest).

Analogical analysis was made for the radial and tangential directions and, finally, all three arctan-approximations (with 6 decimal digits, after the data listed in Table 1) are

$$l(T) = 135.103 \cdot \arctan(T-273.15)^2 - 211.749$$
$$r(T) = 74.9405 \cdot \arctan(T-273.15)^2 - 117.499,$$
$$t(T) = 53.5509 \cdot \arctan(T-273.15)^2 - 83.9475,$$

where $T$ [°K].The graphs of these dependencies of the thermal conductivity are shown in Fig.5.

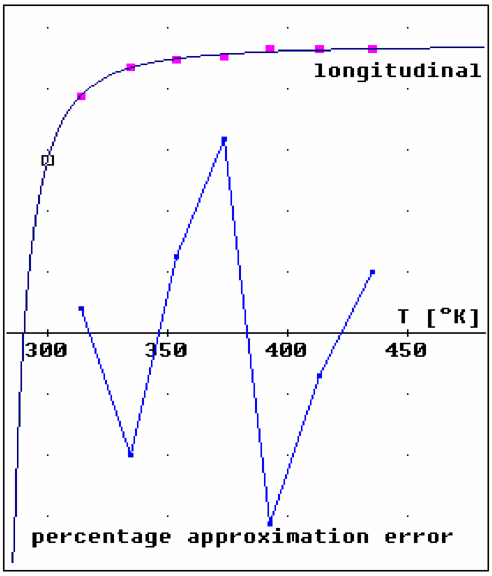


Fig. 4. Seven points $P_j = (T_j, l_j)$, the curve $\lambda = l(T)$, the point (300, 0.283) and the error line magnified 100 times.

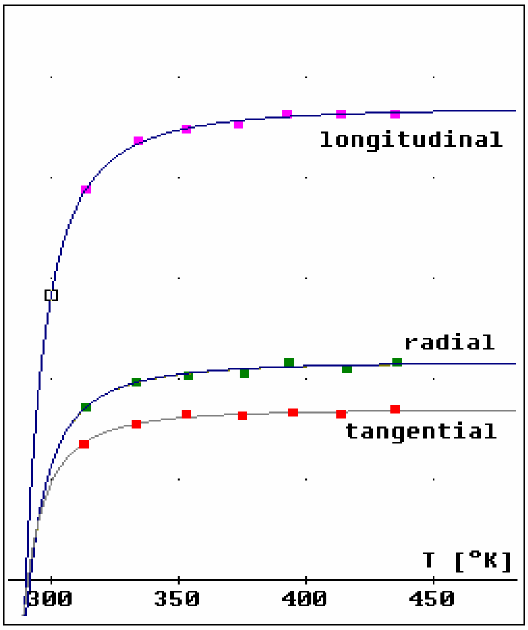


Fig.5. Measured points and the approximating curves for the latitudinal, radial and tangential thermal conductivities

Obviously, the arctan-approximating formulas can be produced in centigraded temperature $T$; then we have

$$l(T) = 133.806 \cdot \arctan T^2 - 209.712,$$
$$r(T) = 57.7231 \cdot \arctan T^2 - 90.7389,$$
$$t(T) = 12.6700 \cdot \arctan T^2 - 19.7389,$$

where $T$ [°C]. The change in coefficients $a$ and $b$ is due to the (double) nonlinearity of the approximating function, but, obviously, the graphs are as that seen in Fig.5 (the only change needed is to scale the horizontal axis in Celsius degrees). After the last three formulas the longitudinal, radial and tangential thermal conductivities stabilize at the level 0.4701, 0.2097 and 0.1631, resp.

## V. FINAL CONCLUSIONS

The research results presented in this paper are part of a wider testing programme focusing on strength properties of compressed materials and covering also determination of the In the investigation of the dependence of the thermal conductivity $\lambda = \lambda(T)$ of the wood on the temperature $T$, there have to be considered three natural directions of the wood, so there have to be described the longitudinal $l$, the radial $r$ and the tangential $t$ thermal conductivities. In the paper we report the experiment providing the values of these conductivity coefficients for six values of the temperature $T$. For the ovendry disks made of beech these values were gathered on the stand intentionally designed for this purpose and installed in Laboratory of Material Tests in Poznań University of Technology. On the base of this data we propose to describe the discussed dependence $\lambda = \lambda(T)$ via the arctan-approximation. Within the standard range of the temperature, let's say varying from 20 to 200°C, this approximation fits very well to the experimental data and coincides with the stabilization taking places when the temperature is high enough. The description of the discussed relation is essential to a future research concerning the heat transfer (in all three directions) in the plasticization processes of natural materials.

## REFERENCES


[1] Davis D.S., *Thermal conductivity nomographs for wood*, Ind. Eng. Chem., 1943, 35 (6), 673.

[2] *Encyclopedia of wood*, US Department for Agriculture, Skyhorse Publishing Inc., 2007 – 496.Hobler T.: *Ruch ciepła i wymienniki*, WNT, Warszawa, 1968,

[3] Hong-mei Gu, A.Zink-Sharp, *Geometric model for softwood transverse thermal conductivity.*

[4] Hobler, T., *Heat transfer and heat exchangers*, WNT Press, Warszawa 1968.

[5] Hunt J.F., H.Gu, Lebow P.K., Theoretical thermal conductivity equation for uniform density wood cells, USDA Forest Service, Forest Products Laboratory 2006.

[6] Kania S., *Heat flow in wood materials*, W.Pol.Wrocławskiej, Wrocław 1990.

[7] Kubler H., YI – Ren Wang., Barkalow D., *Generation of heat in wood between 80 and –130 $^0$C*, Holzforschung, 39, 1985, s. 85 – 89.

[8] Malujda I., *Simulation of heat conduction in plasticization process of non-classical materials*, Sbornik vedeckych praci Vysoke skoly banske – Technicke university Ostrava, cislo1, 2008, rocnik LI, rada hutnicka, clanek c.1368, ISSN 0474 – 8484.

[9] Malujda I., *Plasticization of a bounded layer of anisotropic and porous materials*, J. of Mechanics of Materials and Structures, PACAM X: Mechanics Pan America 2008, vol. 12, 386-390.

[10] Malujda I., *Plastic yield of plasticization materials under the effect of temperature*. Progress in Industrial Mathematics at ECMI 2008, Copyright Springer-Verlag Berlin Heidelberg 2010, vol. 15, pp. 939 – 944. ISBN 978-3-642-12110-4.

[11] Malujda I., Mielniczuk J., *Field criterion of the soft wood with variable humidity and temperature*, Pan American Congress of Applied Mechanics, XI PACAM' 10, Area - composite materials,

PAC 508, CD Edition, ISBN 978-85-85205-97-3, Sao Carlos SP, Brasil 2010 .

[12] Rice R.W., Shepard R., *The thermal conductivity of plantation grown white pine (Pinus strobus) and red pine (P.resinosa) at two moisture content levels*, Forest Products J. 2004-1, art. no. 9730.

[13] Simpson W., TenWolde A., *Physical properties and moisture relations of wood*, Chapter 3 in *Wood handbook—Wood as an engineering material*, Gen. Tech. Rep. FPL–GTR–113. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory 1999.

[14] Steinhagen H.P., *Thermal conductive properties of wood, green or dry, from –40° to +100° C: a literature review*, Forest Products Laboratory, Forest Service U.S. Dep. of Agriculture, FPL-9.

[15] Wagner Th., Götz S., Eska G., *Thermal conductivity of wood at low temperatures*, Cryogenics Vol.34, Issue 8, 1994, 655-657.

[16] Yang Qing-xian, *Theoretical expressions of thermal conductivity of wood*, Journal of Forestry Research, 12(I): 43-46 (2001).

ISBN: 978-988-18210-6-5
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)