Computational Modelling on the Contact Interface with Boundary-layer Approach

L. Hou, *Member, IAENG*, D.Z. Lin, B.Wang, H.L. Li, L. Qiu

***Abstract*—The boundary layer approach in the contact interface is an important development for the study of the visco-elastic materials in various contact conditions. The information of boundary condition data-base reflect complex properties of visco-elastic material with various impact conditions. In this paper, the application of computer simulation and related numerical result on the contact boundary-layer has been discussed. Numerical results and the asymptotic estimates were calculated. It is presented by the software which are consistent with the boundary-layer analysis in our previous papers. Therefore the contact interface has been analyzed with the asymptotic method in comparison with the stochastic computing.**

Keywords- crash; non-Newtonian; FEA simulations；

Contact Boundary-layer;

I. Introduction

Scientists and engineers have been studying in depth how to create a link between the eco-friendly material test and the celebrated mathematical models with the visco-elastic plastic theory[1] by useful mathematical models and virtual tests(CAE,FEA).

Several articles[2 ， 3, 4] are useful sources of information which are standard texts giving mathematical and engineering perspectives upon the subject. Furthermore, [5] gave specific computational method, in which the rate control of honeycomb strength is based on the non-recoverable crush densification.

In our previous work[6,8,13], the development of recoverable controlled fluid-structure interaction soft solid concept（P2）from 2 dimension to 3 dimension in positive definite FEA schemes was introduced. In our framework, a simple numerical shift can introduce the contribution of the microscopic shell element model towards the currently validated macro-scope non-Newtonian models.

To account for the behavior of the growth rate of the delta-shape functions in the impact interface, when the flow R becomes large, we carry out an asymptotic study which enables us to estimate in the large flow limit analytically.

The associated boundary and jump conditions at the origin are estimated according to conservation law and the numeric computing.

The values of contact interface dynamics at both side of the origin are required to obtain the asymptotic estimate of the eigen-solution of the coupled equations across the contact interface.

II. Computational Model

The coupled PDE equations of the standard visco-elastic equation are the best estimates for the history stress over-shoot. Cauchy conservation equation may be used to calculate the large elastic/plastic deformation resulting from stretching stress (shear thinning). It can be used to describe the velocity and the stress distribution in the auto-crash impact problem [12].

By use of standard mean variables in material sciences, we analyze the special feature of the non-Newtonian P-T/T equation, for the resistance to the extensional and simple shear: For the Cauchy conservation equation, we use the positive definite semi-discrete form of the Euler-Galerkin method, the discrete component form are as follows:

$$\frac{\partial u_x}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) - \left(u_x^n \frac{\partial u_x^n}{\partial x} + u_y^n \frac{\partial u_x^n}{\partial y}\right), \quad (1)$$

$$\frac{\partial u_y}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) - \left(u_x^n \frac{\partial u_y^n}{\partial x} + u_y^n \frac{\partial u_y^n}{\partial y}\right), \quad (2)$$

For contact thin-layer near boundary an anisotropic visco-elastic P-T/T equation is studied to analyze (an exponential impact term has been added to the UCM equation) the following semi-discrete equations, the Galerkin-Runge-Kutta (2nd order or higher) scheme:

$$\{\dot{\tau}_{xx}^{n+1}\} = \frac{1}{2}\left(\{Fp_1\}^n + \{Fp_1\}_{pre}^{n+1}\right) \quad (3)$$

$$\{\dot{\tau}_{yy}^{n+1}\} = \frac{1}{2}\left(\{Fp_2\}^n + \{Fp_2\}_{pre}^{n+1}\right) \quad (4)$$

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$$\left\{\dot{\tau}_{xy}^{n+1}\right\} = \frac{1}{2}\left(\{Fp_3\}^n + \{Fp_3\}_{pre}^{n+1}\right) \tag{5}$$

where *Fp1, Fp2, Fp3* are defined in [8].

The FEA calculation of the moving Maxwell type equation is at least 2nd-order of convergence by use of the Adini-type elements.

On the other hand, the large e/p deformation resulting from stress rate τ known as shear thinning was calculated. That is the Cauchy conservation equation subject to the P-T/T stress effects,

$$\rho\dot{\underline{u}} = \left[\nabla\cdot\underline{\underline{\tau}} - \rho\underline{u}\cdot\nabla\underline{u}\right] \;\; in\; \Omega - \Gamma_0 \tag{6}$$

including the velocity field in the region. The complex initial boundary conditions of stress are decided by static tests. The boundary condition is defined on it which is the contacted surface. We treat the part of the elements (usually singular) between the contacting and non-contacting surfaces as the free surface or flow element from the over-stretched elongation.

The positive definite [7] semi-discretized form of the Runge-Kutta-Galerkin method is the modified step of the coupled Cauchy， P-T/T equations.

Upon this discrete form of the equations, the numerical results were worked out by the NAG toolbox for MATLAB under the environment of the HPC platform.

Reference [8] presents the discrete form of the P-T/T equation, while [7] and [9] focus mainly the Cauchy equation. Both of the articles use the Lagrange interpolating space. Reference [6] discusses the Hermite-Runge-Kutta scheme, which yields 3rd order convergence, one order higher than the Lagrange- Runge-Kutta scheme [10]. However, the necessary price paid is to further study the uncertainty resulting from the slip boundary condition. A well posted approach has been given by the stochastic data-base analysis based on the high performance computing [14].

III. COMPLEX BOUNDARY CONDITION

Varieties of non-Newtonian fluids are particulate suspensions– Newtonian solvents that contain particles of another material. Furthermore, we studied the behavior of micro-capillary balancing that against gravity in multi-phase properties [11]-[12]. There is more reference [13] analyzed the boundary-layer eigen-solutions for multi field coupled equations in the contact interface which gave the theoretical analysis of the simulation.

The 3-dimensional positive definite framework assured the simulation with the pre-processor. The solid like fluid behavior can be studied effectively by the mesh refinement and adapt. The region between the contacted and non-contacting surface has been the well known free surface problem of the highly visco-elastic plasticity in the non-contact body in auto-crash impact. The resolution of this type of rheological problem is much clearly structured and to be solved in the positive scheme, especially, the impact hardening and shear thinning (IHST) in 3-dimension is in a very sound theoretic background now.

In the numeric transient 3-D scheme, a nonlinear Riccati differential system is kept therefore to keep the LBB positive definite condition in time domain [5,11].

The spatial discretization is restricted to the Finite Element Method [2], [3]. With the direct application of the semi-discretized method to equations(1 & 2) leads to the three dimensional system in space. A further application of the Newtonian contribution of time domain, there is a lot of freedom to choose the approximation spaces for finite difference calculation [1,12], i.e. the Lunge-Kutta method.

In our discussion, the space of lagrange finite element analysis is used as an approximation space for the stress in the first loop of the coupling solver. Even with the freedom of choosing an approximation space for the stress field in regard to the L_B_B condition, the approximation spaces should be chosen carefully for which the keeping positivity is guaranteed. Hence we need to choose the approximation space for the stress for which the positively preserving interpolation scheme is feasible. Especially, the space of the piecewise constant polynomials is such a choice. The stable second loop in the fluid-solid coupling process the Hermite finite element analysis is used to account the sliding or derivatives in space with random impact angles.

The FEA simulation has been obtained by one of the authors for the crash safety analysis. The honeycomb blocks were made of cells[5]. We extended the knowledge base into a new space that is the visco-elastic-plastic rate control concept to modify the material cards in the solver.

If we use polymer honeycomb instead of aluminum, we could have even flexible range of strength for the side crash test. The bending flexibility of the honeycomb gives the friction control in the barrier stability. The clad property and glue bonding strength are also of great importance in the HC bending stability. All the properties can be captured and simulated by use of the stochastic boundary-layer analysis [14] in our model when the elements distributions are well understood. The micro-effects of the elements contributed to the general impact condition if the stress and flow of the elements are well controlled. The piercing and angle/shearing are of the mechanics to control the stability.

The slip control stability is of great importance to overcome the deadly normal impact in side-crash problem. The curtain air-bag (fabric application) will be the secondary protection after magnetic sensor trigger, however statistical important in the CAE. This will also reduce the damage from the glass windows. A simple test can be arranged using honeycomb/press-load interaction impact, i.e. to use rigid press-load to act as second defenses after the honeycomb

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crushing (see figs. 1,2), which delays the bottom out densification..

IV. VARIATION IN THE CONTACT INTERFACE

In finite element model, the numerical results of two crash impact model use two calculation schemes: the normal impact with a crash-barrier; and the variation with impact angles. We use the software of LS-DYNA (Primer) as the pre-processor and get the simulated model as following.

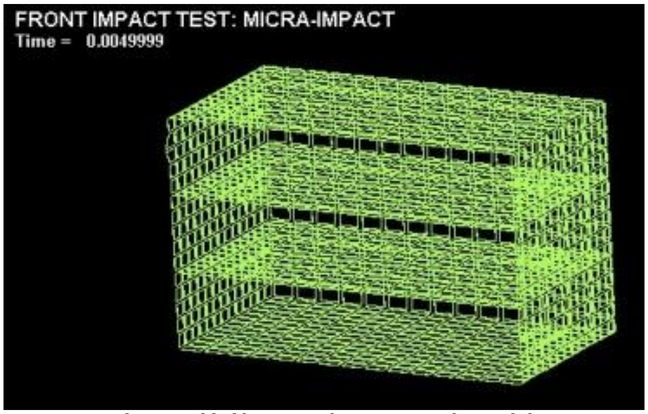

Fig.1.The assembled honeycomb structure in the crash-barrier

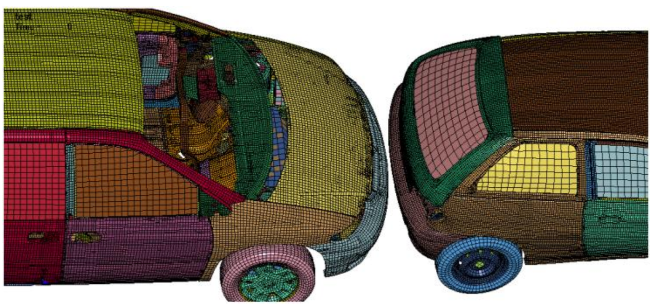

Fig.2. The CAE pre-processing boundary-layer elements of impact model

Fig.3.The CAE simulation of the boundary-layer in the impact model.

The information of crash safety analysis has been compressed into the coupled PDE equations: (1) and (2). Therefore, the analysis of the 3-D FEA simulation is based on equations (1) and (2), with which each node uses Lagrange-Runge-Kutta scheme.

Fig.4. The simulation of the impact with a crash-barrier.

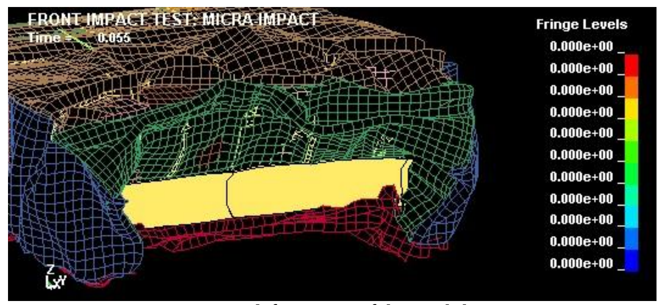

Fig.5. Large deformation of the crash-barrier.

Apart from these results, the information on each element of the FEA were abtained, for example, the effective stress, effective plastic stain, resultant displacement, resultant velocity and so on (see figs.7). The following pictures show the information of four different nodes pitched from the crash-barrier model.

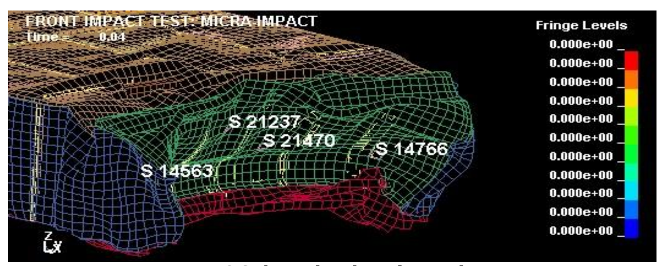

Fig.6.SThe node selected to study

All the results are from the finite element CAE simulation presented in the Figs. 4, 5, 6 and 7 which are validated & comparison with EU experimental regulation (EEVC).

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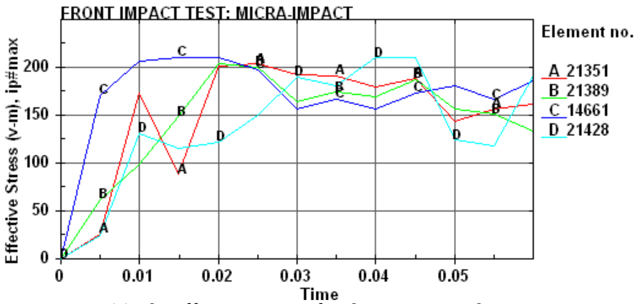
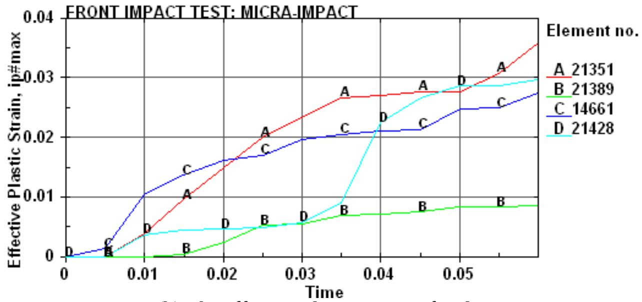
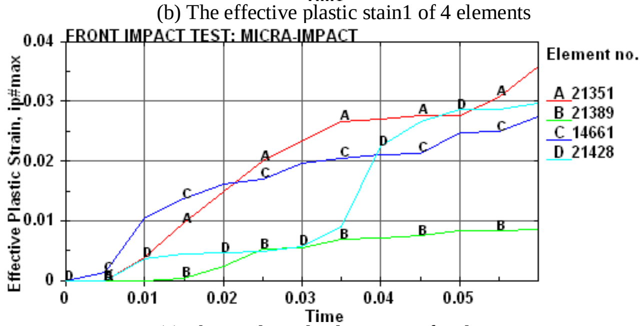


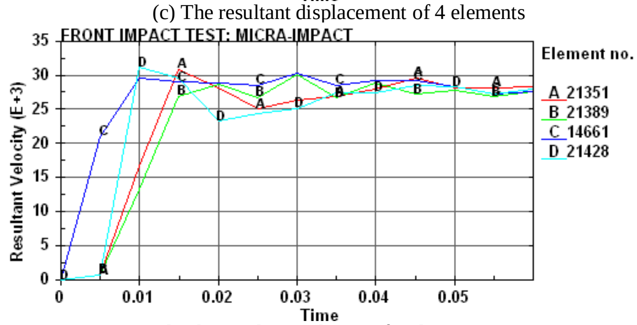
(a) The effective stress of 4 elements on each time step



(b) The effective plastic stain1 of 4 elements



(c) The resultant displacement of 4 elements



(d) The resultant velocity of 4 elements

Fig.7. The information of 4 nodes in the crash simulation.

Apart from solving mathematic model using FEM, to analyze the data from numerical solution, especially the data of boundary layer, is also a better way to further study the non-Newtonian materials in the impact model

V. Asymptotic Analysis supported by the Stochastic Process

The impact angle is defined in this paper, the stochastic analysis concerns mainly about the contact interface in the boundary layer. More than 20,000 data is extracted for the boundary layer analysis in the interface in [8]. The variation of the angles creates the uncertainty in the contact interface.

A histogram graph of the frequency of the impact angle in several time-step is used to study its trend. Fig. 8 shows the frequency function of the impact at the time-steps. Apparently, the impact angle fits the normal distribution [2]. It also shows that the impact angles and the resulting distributions agree with a normal "Fisher's law" supported by the large number theorem[12].

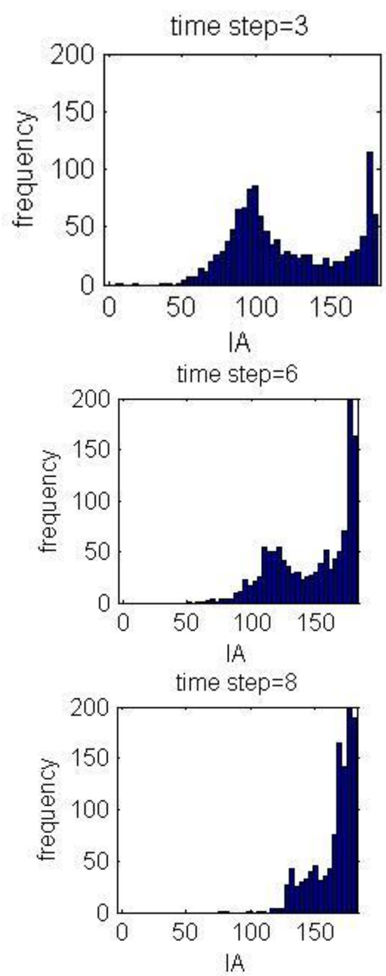


Fig. 8. The histogram graph of impact angle, as time goes by.

Fig. 8 shows the histogram graph of impact angle, as time goes by. A trend towards the angle of 180° (the front impact) is happening more likely. This indicates that a further study and more protection measure needs to be taken.

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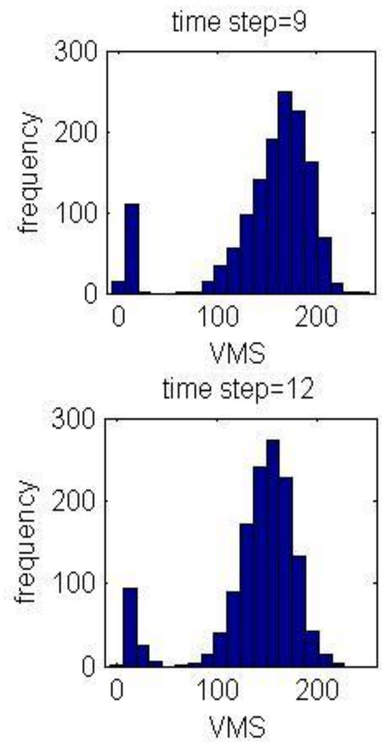

Fig. 9. The delta-shape of the frequency trend of stress (VMS)

The plastic strain (PS) is influenced by multiple impact conditions, such as impact angle, material property, impact proportion, etc. The explicit relation of PS to the influencing factor is useful while studying the properties of non-Newtonian materials. Stochastic analysis is used to find out such relationships. Fig.9 shows the histogram graph of the frequency of strain (VMS) at certain impact angle, which indicates that they fit two-dimensional normal distribution.

In the celebrated results of asymptotic analysis [13] we gave the estimates on the delta functions related to the contact behavior in the interface. To account for the behavior of the growth rate P of the perturbation functions when the impact flow R becomes large, we carry out an asymptotic study which enables us to estimate P in relation with delta-shape distributions from the stochastic process. The viscous G-mode equation takes the form in the large R limit analytically

$$\left(R\tfrac{d}{dk}-P\right)Lh-\tfrac{G}{\left(F'\right)^2}h=0,\quad -\infty<k<0,0<k<\infty \qquad (7)$$

where

$$Lh=\tfrac{d^2h}{dk^2}+R\tfrac{d}{dk}\left(k^2h\right)-\left(k^2P+k^4N\right)h \qquad (8)$$

is the viscous tearing mode operator. The associated boundary and jump conditions at the origin $k=0$ are

$$h\left(\pm\infty\right)=0,h\left(0+\right)=1,h'\left(0\pm\right)=-ie^{\pm i\chi}, \qquad (9)$$

$$h''\left(0+\right)-h''\left(0-\right)=-2\pi i\frac{G}{\left(F'\right)^2}, \qquad (10)$$

together with the non-linear eigen-value relation

$$2\pi P=\frac{h\left(0-\right)}{h'\left(0-\right)}e^{-i\chi}-\frac{h\left(0+\right)}{h'\left(0+\right)}e^{i\chi},\quad -\pi<\chi<\pi. \qquad (11)$$

The values of contact interface dynamics h and h' at both side of the origin are required to obtain the asymptotic estimate of P.

The final form of the asymptotic estimate of the eigen-value for the N-G-mode for large flow is given by

$$P\approx\left(\frac{1}{2\pi\varepsilon}e^{i\chi}\right)^{\frac{1}{2}}+\frac{\varepsilon^{1/3}\Gamma\left(\frac{1}{3}\right)\cos\chi}{2\pi3^{2/3}}+\frac{\varepsilon^{2/3}3^{-1/3}G\Gamma\left(\frac{2}{3}\right)}{2(F')^2}-\frac{\varepsilon^{2/3}N\Gamma\left(\frac{2}{3}\right)}{3^{1/3}}.\quad \varepsilon\to0. \qquad (12)$$

So that we obtained the relationships of eigen-value spectrum P with the parameters of R, G, N and χ. The main parameter χ is the convective coupling angle in the contact interface. The eigen-value spectrum can work out optimization characteristic function-base for the multi-field coupling problem. With the asymptotic forms we may able to analyze the contact interface problems in the mechanical impact model with multi-scale parameters functionality. The asymptotic estimates of the instability growth rate P nonetheless shown the multi field physical coupling effects of perturbation ($\varepsilon=\frac{1}{R}$), contact angle(χ), slip(N), etc with the high sensitivity in the contact interface dynamics.

VI. Conclusions

With the help of the high performance computing platform (ARUP/LS-DYNA v.971 and NAG) in the mathematics department, the auto-crash safety simulations have been carried out for the finite element modeling. After intensive science and technical research since 2000, we are able to engage pre-/post-processing with complex boundary condition analysis for the industrial problems. The mathematical understanding with its engineering application has been achieved by use of the explicit solver which supplies the best stable impact solution for the coupled nonlinear equations.

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