The Random Walk Hypothesis in Emerging Stock Market—Evidence from Nonlinear Fourier Unit Root Test

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Abstract—This study adopts a new unit root test which allows for an unknown number of structural breaks with unknown functional forms and nonlinearity in data generating process of stock prices series to test the random walk hypothesis in an emerging stock market (Iran stock market). The results from nonlinear Fourier unit root test implies that in view of taking into account possible unknown structural breaks and nonlinear behavior in the stock prices series, Iran stock prices index follow random walk theory and is efficient in weak form.

Index Terms: Random walk theory, structural breaks, nonlinearity, Fourier function

I. INTRODUCTION

Stock market plays a crucial role in economic development process through mobilizing national saving, financing new investment projects and channeling investments and financial resources into productive enterprises. In this sense, the efficiency of stock market enhances its role in national economy and provides the suitable conditions to people for contributing in productive economic activity. The efficient stock market hypothesis states that stock prices fully and accurately reflect all available information and securities are appropriately priced at equilibrium level and there is no distortion in the pricing of capital and risk. At the same time, since stock prices are forward looking, they remark a unique feature of investors’ expectations and shifts in investors’ views about the future prospects of companies as well as the economy environment, the efficient stock market indicates the national economics prospect (Green et al., 2005).

In an efficient stock market in weak form stock prices follow a random walk process and the future prices change randomly based on realized new information. Accordingly, successive price changes in individual securities are independent and the stock price fluctuations are unpredictable. In recent years, the study of efficiency of emerging stock markets with respects to their potential capabilities in international portfolio diversification has been significantly regarded (e.g. Chaudhuri and Wu, 2003; Narayan and Smyth, 2004; Asiri, 2008; Ozdemir, 2008; Paytakhti Oskooe et al., 2010; Borges, 2011).

A common feature of these empirical studies is that in efficiency testing methodology have not been take into account the possible nonlinearities specifically in conditional mean of stock prices movements (stock returns). However, it is well known that the data generating process of many economic and financial time series is nonlinear (see Granger and Terasvirta, 1993; Franses and Van Dijik, 2000; Tsay, 2010; Brocks, 2008 and Enders, 2010). In order to avoid spurious results and provide valid implications possible nonlinearities in the behavior of financial time series should explicitly be taken to account in empirical studies.

In financial economic literature there are various sources of nonlinear behavior in financial time series. The most frequently cited reasons of nonlinear adjustment can be summarized as follow: structural changes, high amplitude shocks, technological innovation, changes in regulations, the characteristics of market microstructure, the unreliable and unqualified information, the existence of bid-ask spread, the existence of market imperfections (market frictions e.g. transaction cost, tax, government interventions), the informational asymmetries and lack of reliable information, noise traders. In the case of emerging
stock market another factors such as low liquidity, thin trading, price limits and considerable market volatility also may lead to nonlinear behavior in the data generating process of stock price fluctuations (stock returns).

In view of the possible nonlinear process this paper examines the efficiency of an emerging stock market in weak form using a nonlinear unit root test which captures the possible structural breaks and nonlinearity in the underlying data generating process of series under study. The remainder of the paper is organized as follow: the empirical methodology is described in section. Section 3 represents the data used in the study and preliminary analysis of data. The empirical results are introduced in section 4. The paper ends with conclusion in section 5.

II. Empirical Testing Method

The hypothesis of efficient stock market in weak form states that successive stock price changes are independent and independence of price movements. In this sense, in order to examine the efficiency of stock market in weak form we should test the randomness of stock prices in the basis of the random walk theory. The weak form efficiency that stock prices follow a random walk process such as:

\[ P_t = \mu + P_{t-1} + \epsilon_t \tag{1} \]

where \( \mu \) is the expected price change or drift, \( P_t \) is the natural logarithm of stock price index at time \( t \), \( P_{t-1} \) is the natural logarithm of stock price index at time \( t-1 \) and is \( \epsilon_t \) disturbance term. More precisely, market efficiency under the random walk model implies that successive price changes of a stock are independently and identically distributed, so the past movement or trend of a stock price or market cannot be used to predict its future movement.

From the econometric point of view, the random walk hypothesis implies that should be difference-stationary (i.e., \( P_t \sim I(1) \)) or has stochastic trend and should be independent and identically distributed (henceforth IID) random variables or a strict white noise. In these conditions, shocks to stock prices will have a permanent effect implying that stock prices will reach a new equilibrium level and future returns cannot be predicted based on historical movements in stock prices.

The well known procedure in econometrics literature to test the random walk hypothesis is the unit root test. This test is designed to investigate whether a series (stock prices) is difference-stationary or trend-stationary as a basic condition for the random walk process. The generally used method to conduct the unit root test is the augmented Dickey–Fuller (ADF) test (Dickey and Fuller 1979). Perron (1989) the structural change in time series can influence the results of tests for unit roots. In this sense, in view of more probability of structural changes occurrence in stock market owing to changes in inter-market or out-market environment the possible structural breaks should be considered in conducting unit root test to examine market efficiency. In empirical studies several methods have been adopted to deal with effects of structural breaks on the unit root test process (Perron, 1989; Zivot and Andrew, 1992; Lumsdaine and Papell, 1997 and Bai and Perron, 2003). The most of these “unit root test in the presence of structural break” assume the number, date or form of the structural breaks. In this condition, the structural breaks occur instantaneously owing to a political or economical event and taking abrupt jumps in the mean or immediate changes in the slope. However, Enders and Lee (2004) state that it may take varying period of time for the economic or financial series to display the effects of realized event. To take into account these feature of structural breaks and control for the effect of unknown forms of nonlinear deterministic terms Enders and Lee (2006) propose a new unit root test that replicates the pattern of structural breaks using a single-frequency Fourier function. In this method, the trigonometric terms is defined to capture unknown nonlinearities in the equilibrium level. More precisely, this test relies on a Fourier approximation for the transition function which captures structural change with one seamless transition regime instead of multiple regime separated by separate structural breaks.
Following Enders and Lee (2006) and (2009); Becker et al. (2006) and Christopoulos and Leo´n-Ledesma (2010) we adopt the Fourier unit root test as follow:

\[ \Delta P_t = \rho P_{t-1} + c_1 + c_2 \sin(2\pi k t / T) + c_3 \cos(2\pi k t / T) + \varepsilon_t \]  

(2)

where \( k \) is the number of frequencies of the Fourier function, \( t \) is a trend term, \( T \) is the sample size and \( \Pi = 3.1416 \).

\[ \{ c_1 + c_2 \sin(2\Pi k t / T) + c_3 \cos(2\Pi k t / T) \} \] is a time-varying intercept and \( \{ c_2 \sin(2\Pi k t / T) + c_3 \cos(2\Pi k t / T) \} \) is the key element. In theory, through selecting an appropriate single frequency \( k \), the corresponding Fourier function \( \{ c_2 \sin(2\Pi k t / T) + c_3 \cos(2\Pi k t / T) \} \) captures structural changes in the sequence \( \{ P_t \} \). Furthermore, since the frequency \( k \) is data-driven and the resulting Fourier function closely replicates the structural change, pre-assuming the number or the date of each structural break is not required. In sum, this unit root test allows for an unknown number of structural breaks with unknown functional forms. In this procedure, the breaks are detected endogenously and specifying a priori the form of the break is not required.

In practice, in order to determine unknown value of \( k \) the equation (2) is estimated for each integer value of \( k \) in the interval 1–5 (1 \( \leq \) \( k \) \( \leq \) 5) and the appropriate value of \( k \) is selected based on the smallest residual sum of squares. After that, a formal test for the presence of unknown breaks in the data generating process of \( P_t \) can then be carried out by testing the null hypothesis \( H_0: c_1 = c_2 = 0 \) against the alternative \( H_1: c_1 = c_2 \neq 0 \) using \( F \) statistic, \( F(k) \) (table 3, Enders and Lee, 2004). If the null hypothesis is rejected the Fourier function captures the nonlinearity. Finally, the test of nonstationarity is carried on by applying OLS to estimate equation (2) and testing the null hypothesis \( H_0: \rho = 0 \). If \( \rho \) is significantly different from zero (using the \( \tau_{DF} \) statistics from table 3, Enders and Lee, 2004), we can reject the null hypothesis of a unit root through taking into account nonlinearity and possible structural breaks and therefore the sequence \( \{ P_t \} \) is stationary. It should be noted that if the errors terms in equation (2) reveal serial correlation, the augmented form of this test is estimated as follow:

\[ \Delta P_t = \rho P_{t-1} + c_1 + c_2 \sin(2\Pi k t / T) + c_3 \cos(2\Pi k t / T) + \sum_{j=1}^{p} \beta_j \Delta P_{t-j} + \varepsilon_t \]  

(4)

III. Data and preliminary analysis

In order to apply the nonlinear Fourier unit root test for examining the weak form efficiency of an emerging stock market, we use the daily stock price index of Iran stock market (TEPIX). The sample period includes 2632 observations during the period January 2, 1999 to December 30, 2009.

Figure 1 shows the increasing trend of Iran stock market index until December 4, 2004. After that TEPIX has tended to decline and become more volatile. Over the study period, the Iran stock prices index has experienced its highest value in December 4, 2004. Table 1 reports the stochastic properties of the logarithm of Iran stock prices index. The sample mean for stock return series is 7567.161. The high standard deviation (2875.133) in view of the value of mean is an indication of the high volatility in the market prices and the risky nature of the market. Iran stock prices index has a negative skewness, indicating that large negative fluctuations in index are more common than large positive changes in index. Furthermore, the kurtosis or degree of excess, in the Iran stock market index is smaller than the normal value of 3. As a result, the stock prices index series are platykurtic. In line with skewness and kurtosis statistics, the Jarque- Bera statistics reject the null hypothesis that Iran stock prices series is normally distributed.
IV. Empirical results

In this section, we report the results of applying the nonlinear Fourier unit root test to detect stochastic or deterministic trend of Iran stock prices series. In the first place, the equation (2) is estimated with various value of k for interval 1-5 and based on smallest residual sum of squares the number of frequencies of the Fourier function is verified 1(k=1).

In the next step, the null hypothesis of linearity (\( H_0: c_1=c_2=0 \)) is tested. As displayed in table 2, the sample value of the \( F_{\text{statistic}} \) rejects the linearity the 5 percent significance level. After that the augmented form of test (equation (4)) are estimated and appropriate lags based on randomness of error terms are recognized (p=10). The null hypothesis of unit root (\( H_0: \rho=0 \)) is tested using the \( \tau \) statistic for \( \hat{\rho} \) from final equation with 9 lags. Since the yield \( \tau \) statistic (1.62) is smaller than \( \tau_{DF} \) statistic at the 5 percent significance level, the null hypothesis is accepted. This means that Iran stock market index is nonstationary and follows random walk theory.

<table>
<thead>
<tr>
<th>( \hat{\rho} )</th>
<th>( F_{\text{test}} )</th>
<th>K</th>
<th>Lags</th>
<th>( \tau )</th>
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<td>18.02</td>
<td>1</td>
<td>10</td>
<td></td>
<td>1.6289</td>
</tr>
</tbody>
</table>

Note: *denotes significance in 1% level.

Based on table 3, Enders and Lee, (2004) the \( F(k) \) and \( \tau_{DF} \) statistics for \( k=1 \) and \( T=100 \) are 7.137 and -3.816 respectively. These statistics for \( k=1 \) and \( T=500 \) are 6.837 and -3.762 respectively.

V. CONCLUSION

It is well documented that stock prices or stock returns specifically in emerging stock markets behave in nonlinear manner. In this sense, the unknown structural breaks and nonlinearity in data generating process of stock prices should be taken into account in testing the random walk hypothesis in stock markets. This paper adopts a new nonlinear Fourier unit root test to investigate the weak form efficiency of Iran stock market. The empirical results suggest that even in the presence of nonlinearity and unknown structural breaks the Iran stock prices index follows random walk hypothesis and is efficient in weak form. Since the efficiency of stock market is very important in decision making of investor and enhancing the role of stock market in economic development process, the findings of this study have useful implications for individual or institutional domestic and international investors and policymakers.
REFERENCES


