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A Smooth Transition GARCH Model with Asymmetric Transition Phases

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Abstract—**This paper develops a smooth transition GARCH model with an asymmetric transition function, which allows for an asymmetric response of volatility to the size and sign of shocks, and an asymmetric transition dynamics for positive and negative shocks. We apply our model to the empirical financial data: the NASDAQ index and the individual stock IBM daily returns. The empirical evidence shows that our model outperforms many existing GARCH specifications.**

Keywords: Volatility conditional variance, smooth transition function, GARCH

I. INTRODUCTION

Given its importance in modern pricing and risk management theories, volatility in financial time series is one of the most popular topics in financial econometrics. Over the years, a number of different features of financial volatility have emerged, such as positive dependence in the volatility process, volatility clustering, high persistence, and nonlinearity. The observation of these features has motivated a wide class of volatility specifications, including the autoregressive conditional heteroskedasticity (ARCH by Engle (1982)) and generalized autoregressive conditional heteroskedasticity (GARCH by Bollerslev et al. (1990)) model , which are widely applied in empirical research.

The key criticism of the GARCH specifications comes from the modeling of conditional variance as a function of past squared residuals, which makes the sign of the residuals irrelevant in predicting volatility. The symmetric treatment of positive and negative residuals contradicts the stylized fact, first noted by Black(1976), that stock market returns become more volatile after a negative shock, than they do after a positive shock of the same magnitude. One possible explanation, known as the "leverage effect", is that negative excess returns reduce the equity value, hence raise the leverage ratio, of a given firm, thus raising its riskiness and the future volatility of its assets.

This sign asymmetry effect has motivated a large number of different volatility specifications in the literature. Nelson's (1991) Exponential GARCH (EGARCH) model is one of many specifications, the threshold GARCH (TGARCH) model proposed by Rabemananjara and Zakoian (1993), and the asymmetric power ARCH model developed by Ding et al (1993), to name only a few, all involve asymmetric functions of the residuals. It is well known that the specifications which allow for "leverage effect" dominate the standard GARCH specifications.

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Recently, several authors have introduced smooth transition specifications (Hagerud (1997), Gonzalez-Rivera (1998)), Anderson et al. (1999), and Medeiros and Veiga (2009)), widely used in models of conditional mean, to model the asymmetric response of conditional variance to positive and negative news. The smooth transition model can be thought of as a regime switching model with a continuum of regimes, which overcomes the limitation of linear and binary (Markov switching) switching models. For certain parameter values, the smooth transition model nests with the threshold specifications that only allows finite regimes. The model in some sense generalizes the modeling of asymmetry in variance and empirical evidence in favor of the smooth transition specification are also reported by these authors.

The main purpose of this paper is to propose a new smooth transition GARCH model, which allows for both sign asymmetry and transition asymmetry. The smooth transition specifications in the volatility literature generally assume a transition function that is symmetric around its midpoint, which implies that negative shocks and positive shocks will have the same transition phases. However, the symmetry in the transition phases may be too restricted for practical purposes. Following Nelder (1961) and Sollis et al. (1998), we will make use of the generalized logistic function that allows for both sign asymmetry and transition asymmetry, to model conditional variance.

The remainder of this paper is organized as follows. In section 2, we introduce the asymmetric smooth transition GARCH (ASTGARCH for abbreviation) model. In section 3, we address the statistical properties of the new model. In section 3, we offer an application to NASDAQ index daily returns and IBM daily returns, and in section 4, we conclude the paper and summarize this work.

II. THE MODEL

Let r_t denote the return of a financial asset from time $t-1$ to time t and let Ψ_{t-1} be the investors' information set which contains relevant information at time t. The unexpected shock ε_t is defined as $r_t - E(r_t|\Psi_{t-1})$. The conditional variance of the return, first proposed by Engle(1982), $h_t = Var(r_t|\Psi_{t-1})$, is a measure of volatility.

In the literature, for ease of exposition, $\varepsilon_t|\Psi_{t-1}$ is generally assumed to follow a normal distribution with mean zero and variance h_t. The distribution, however, can be relaxed to a more general one, such as, the standardized distribution (Bollerslev 1987) or the generalized error distribution (Nelson 1991). We will assume conditional normality of $\varepsilon_t|\Psi_{t-1}$ in this paper. Furthermore, we assume

$$\varepsilon_t = u_t\sqrt{h_t}, \tag{1}$$

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where u_t is a normal $i.i.d$ sequence with zero mean and unit variance. The first volatility model that incorporates the smooth transition specification is by Hagerud (1997) and Gonzalez-Rivera (1998), which is given

$$\begin{aligned} h_t &= w_0 + \sum_{i=1}^{p} \alpha_{0i}\varepsilon_{t-i}^2 \\ &+ \left(\sum_{i=1}^{p} \alpha_{1i}\varepsilon_{t-i}^2\right) F(s_{t-1},\gamma) + \sum_{i=1}^{q} \beta_{0i} h_{t-i}. \quad (2) \end{aligned}$$

The smooth transition model generalizes the modeling in variance with the introduction of a smooth transition specification in the sense that it allows for intermediate transition states. Also, it encompasses a big array of ARCH specifications, such as the DGE model by Ding, Granger, and Engle (1993), the GJR model by Glosten, Jagannathan, and Runkle (1993), and the threshold ARCH model by Rabemananjara and Zakoian (1993).

As addressed by Fonari and Mele(1997), the main restriction of the smooth transition model is that the effects of ε_{t-1} and h_{t-1} on the volatility are additively separable. In other words, the impact of ε_{t-1} on the conditional variance does not depend on past volatility values and is always the same for a given valued of ε_{t-1}. Anderson et al. (1998) introduce an asymmetric nonlinear smooth transition GARCH model that extends the smooth transition GARCH model and allows the nonlinearity in both the GARCH parameters and the ARCH parameters, which is given by

$$\begin{aligned} h_t &= w_0 + \sum_{i=1}^{p} \alpha_{0i}\varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_{0i} h_{t-i} + F(s_{t-1},\gamma) \\ &\times \left[w_1 + \left(\sum_{i=1}^{p} \alpha_{1i}\varepsilon_{t-i}^2\right) + \sum_{i=1}^{q} \beta_{1i} h_{t-i}\right]. \quad (3) \end{aligned}$$

In this paper, we make use of an asymmetric transition function: the generalized logistic function by Nedler (1991) and Sollis et al. (1998) and propose the following new specification.

A smooth transition GARCH model with asymmetric transition phases, is defined by

$$\begin{aligned} h_t &= w_0 + \alpha_0\varepsilon_{t-1}^2 + \beta_0 h_{t-1} \\ &+ F(s_{t-1},\lambda,\gamma)(w_1 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}) \quad (4) \end{aligned}$$

where

$$F(s_{t-1},\lambda,\gamma) = [1 + \exp(\lambda s_{t-1}/\gamma)]^{-\gamma} \quad (5)$$

is the transition function, s_{t-1} is the transition variable and λ is the smooth parameter. Possible transition asymmetry is introduced through the parameter γ, where $\gamma = 1$ implies no asymmetry.

Figure 1 displays the shape of the transition function for several values of γ and λ.

III. Properties of the new model

Positivity of the variance is achieved by imposing restrictions that $w_0 > 0, \alpha_0 > 0, \beta_0 > 0, w_0 + w_1 > 0, \alpha_0 + \alpha_1 > 0$, and $\beta_0 + \beta_1 > 0$.

It is too restrictive for practical purposes to assume that the transition function is symmetric around its mid-point, which would imply that positive and negative shocks will have the same transition phases. Our model departs from most of the existing GARCH specifications by making use of the asymmetric transition function. The intuition behind this assumption resides in two asymmetries found in the volatility literature, i.e, the "leverage effect" and the reversion of asymmetry. With these two features, positive and negative shocks in generally have different impacts on volatility dynamics. Our model can easily capture these two asymmetries. In this paper, we will focus on the asymmetric smooth transition GARCH (1,1) model (abbreviated as ASTGARCH(1,1)). Other variations can also be obtained following the same methodology. For convenience, we denote the parameter vector $\theta \equiv (w_0, \alpha_0, \beta_0, w_1, \alpha_1, \beta_1, \lambda, \gamma,)$.

It is straightforward to establish the equivalence of our ASTGARCH(1,1) with several well-known specifications that follow:

- The GARCH(1,1) model if $w_1 = 0$, $\alpha_1 = 0$, and $\beta_1 = 0$.
- The ANTSGARCH(1,1) model if $\gamma = 1$.
- The ST-GARCH(1,1) model if $\alpha_1 = 0$ and $\beta_1 = 0$.
- The threshold GARCH(1,1) model if $\gamma \to \infty$.
- The asymmetric power model of DGE with power equal to 2 if $\gamma \to \infty$.

IV. Application

In this section, we apply the asymmetric smooth transition GARCH model to financial data, as well as the smooth transition GARCH and GARCH(1,1), which are used as benchmarks. The first data set contains daily returns of the valued weighted NASDAQ index from January 2, 1990 to December 31, 2007, consisting of 4540 observations. The second data set is comprised of 4792 daily observations for IBM stock, from January 2, 1990 to December 31, 2008. These data are extracted from the Center for Research on Stock Prices(CRSP) database.

Table 1 Summary Statistics

	Mean	Median	Skewness	Kurtosis
NASDAQ	0.00	0.00	0.26	6.64
IBM	0.00	0.00	0.16	6.38

Table 1 reports the summary statistics for the NASDAQ index daily returns and the IBM daily returns. We find common results that the distribution of the daily returns departs from the Gaussian distribution by their skewness and leptokurtosis, two key stylized facts of stock returns.

Table 2. NASDAQ Index

	ASTGARCH	STGARCH	GARCH
ω_0	0.00	0.00	0.00**
α_0	0.06**	0.05**	0.08**
β_0	0.90**	0.77**	0.92**
ω_1	0.00**		
α_1	0.04**	0.04**	
β_1	0.00		
λ	495.69	133.00**	
γ	0.53**		
Log Likelihood	13785.09	13459.10	13761.42

Notes: ASTGARCH denotes our new model. STGARCH denotes the smooth transition GARCH model. Asterisks indicate parameters are statistically significant at the 5% level under robust standard error.

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Table 2 presents the estimated coefficients and likelihoods of the NASDAQ index for our new model, as well as the smooth transition model and the GARCH(1,1) model. It is apparent that there is a smooth transition between volatility regimes. We also test for the significance of the coefficients λ and γ and find that the null $\lambda = 0$ and $\gamma = 1$ are both rejected at the 5% significance level.

Table 3. IBM daily returns

	ASTGARCH	STGARCH	GARCH
ω_0	0.00^{**}	0.00^{**}	0.00^{**}
α_0	0.05^{**}	0.05^{**}	0.05^{**}
β_0	0.92^{**}	0.77^{**}	0.94^{**}
ω_1	0.00^{**}		
α_1	0.02^{**}	0.08^{**}	
β_1	0.01^{*}		
λ	56.21^{*}	133.00^{**}	
γ	0.09^{**}		
Log Likelihood	12667.60	12397.97	12650.30

Notes: See notes for Table 2.

Table 3 presents the estimation of the conditional variance of IBM stock's daily returns. In this case, the null hypotheses $\lambda = 0$ and $\gamma = 1$ are again rejected at the 5% significance level, which is in support of a smooth transition between volatility regimes.

V. Conclusion

The asymmetric response of volatility to positive and negative shocks, best known as the ”leverage effect” has been well addresses in the financial econometrics literature. A lot of empirical models have been proposed to capture this effect with applications to stock returns, exchange rates, and other financial data. In this paper, we have introduced an asymmetric smooth transition model, which permits both asymmetric responses and asymmetric transition dynamics for the shocks on volatility. This model is a generalization of the asymmetric nonlinear smooth transition model of Anderson, Nam and Vahid(1999) and the smooth transition model of Hagerud (1997) and Gonzalez-Rivera (1998). Under certain conditions, many existing specifications can be nested within our model, such as the threshold model of Zokanian (1993), the widely used asymmetric power model of DGE, and the GJR model. The empirical results also show the advantage of our new model, which is more flexible in capturing dynamic features of stock return volatility.

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