Ship Wake Simulation Based on Particle System in Virtual Test

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Abstract—This paper introduces a simulation method of ship wake based on particle system in virtual test. According to analyze the attributes of the ship wake bubble particle, we simulate ship wake as a collection of primitive and kinetic particles that define its boundaries. The resulting model is able to represent motion, changes of form, and dynamics that are not possible with classical surface-based representations. The geometric parameters of the ship wake simulated in this paper and the experiment data are identical on the whole. The application of this simulation method proves the availability and accuracy of itself, which can also be used for ship wake simulation system in various conditions, and provide realistic debugging environment for the virtual test platforms.

Index Terms—particle system, ship wake, simulation, virtual test

I. INTRODUCTION

In recent years, the virtual test technology gets more and more attentions as an auxiliary testing method. With the integration of virtual test system improving further, the increasing demands put forward, which involve the realism of the virtual test environment, the richness of experiment, the controllability of testing process and the visualization of the test. As a result, ship wake simulation in virtual test greatly affects the research on course design and navigation validation.

Ship wake is the track of ship movement, which is constantly moving and changing shape. At present, many researchers [1] stack some polygons whose parameters are determined by statistics to simulate ship wake. It easily leads to gaps or overlaps between two adjacent polygons as the ship changes course. Xiao Bing *et al.* [2] analyze the ship wake diffusion angle according to the shape characteristic of

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the wake, and then a geometric model is built based on particle

system. However, this method uses the statistics to determine the diffusion angle, thereby it reduces the accuracy of the model.

We establish a geometric model of ship wake according to the particle system theory and the analysis of ship wake bubble dynamics. In addition, we validate the ship wake model of surface ship in different ways of mobile to prove the availabilities and accuracies of the simulation method.

II. PARTICLE SYSTEM

Particle system [3] is considered to be the best method to simulate figures of random objects so far. A mass of moving micro-particles are assembled as the basic element to compose the close particle system. Each particle possesses its own qualities, such as the position, velocity and lifetime. Particle system is a dynamic entity because that the particle's qualities are the functions of time, which are variable with the times. The characteristic of the object which is simulated is defined by qualities of all the particles which constitute the particle system.

Over a period of time, particles are generated into system, move within the system, and die from the system. The resulting model is able to represent motion and dynamics that are not possible with classical surface-based representations. Usually, there are six basic steps as shown in Fig 1 to form a moment appearance [4].



Fig. 1. Basic model of particle system

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III. ANALYZING DYNAMICS OF BUBBLE PARTICLE

A. Particle Initial Velocity and Position

The geometric simulation of ship wake mainly focuses on its edge. Ship wake bubble particles are generated sternwards, and then diffuse vertically to the direction of course. We define diffusion angle is the angle which is between the tangent to point in ship wake edge and the velocity of ship. Diffusion angle close to the stern is about 50° , but the angel decreases to 1° after 20s to 30s [5].

That available in virtual test is called effective ship wake. Actually speaking, the life of effective ship wake is about 2 minutes. To simplify the geometric model of ship wake, only the effective ship wake is simulated. As a result, the lifetime of bubble particles in ship wake is also 2 minutes.

We establish a relative coordinate system as shown in Fig 2 whose origin is in the junction of the ship stern and shipboard. Set the direction of X-axis is east, and the direction of Y-axis is north. Actually, this relative coordinate system is moving along with the ship, but the directions of axes are unchangeable.



Fig. 2. Relative coordinate system

Assume the velocity of ship is $v_s(t)$, particle's diffusion velocity $v_b(t)$ is perpendicular to the direction of $v_s(t)$. The displacement of ship during Δt is $\int_{t_i}^{t_i+\Delta t} v_s dt$, at the same time, the diffusion displacement of the bubble particle is $\int_{t_i}^{t_i+\Delta t} v_b dt$, and the diffusion angle θ is the angle between the tangent to the point that is in the edge of ship wake and the X-axis.

By the trigonometric function, we can get an equation as follows:

$$\tan \theta = \lim_{\Delta t \to 0} \int_{t_0}^{t_0 + \Delta t} v_b dt / (\int_{t_i}^{t_i + \Delta t} v_s dt)$$
(1)

On the time derivative using the L'hospital's rule, the (1) turns to be (2):

$$\tan \theta = v_h / v_s \tag{2}$$

The initial velocity of particle in the system is the sum of average velocity and random velocity, the range of random velocity is $var(v_b)$ which is initial velocity's variance. In (3), rand() is a procedure returning a uniformly distributed random number between -1.0 and +1.0. The actual initial velocity is:

$$v_b(t_0) = v_s(t_0) \cdot \tan \theta + rand() \cdot \operatorname{var}(v_b)$$
(3)

B. Particle Dynamics

To move a particle is a simple matter of adding its velocity vector to its position vector each frame. To add more accuracy, the particle system also uses an acceleration factor to modify the velocity of its particles.

In the process of bubble particle diffusion after it is generated sternwards, the particle is mainly forced by the viscous resistance and surface water wind which can be ignored since it is too complicated to determine and not the major force. We mainly consider the viscous resistance of water to simplify the geometric model and improve its applicability.

The viscous resistance F_a suffered by diffusing particle is:

$$F_a = 0.5\pi R^2 \rho_l v^2 C_d \tag{4}$$

Where ρ_l is the density of water, C_d is viscous resistance coefficient which is determined by Reynolds number Re.

$$C_d = 24(1+0.15 \,\mathrm{Re}^{0.687}) \,/\,\mathrm{Re}$$
 (5)

$$\operatorname{Re} = \rho_{i} v R / \eta_{i} \tag{6}$$

Where η_l is the viscosity of water.

We assume that in the process of particle diffusion, it is only affected by the viscous resistance F_a , and F_a is always in the opposite direction to particle velocity. Based on our assumption, we know that particle moving path is a curve under the influence of F_a in the relative coordinate system shown in Fig 2.

First of all, let us analyze the directions of $v_s(t)$, $v_b(t)$, and their relationships are shown in Fig 3.



Fig. 3. Relationships among $v_{e}(t)$, $v_{b}(t)$, and F_{a}

Where $v_b(t)$ is particle diffusion velocity whose direction is vertical to $v_s(t)$, and F_a is in opposite direction to $v_b(t)$ at any time.

If the weight of each bubble particle is m:

$$m = 4/(3\pi R^3 \rho_a) \tag{7}$$

Where ρ_a is the density of air.

According to (4) and (7), we can establish an acceleration equation of bubble particle as follows:

$$a = 3\rho_{\nu}v^{2}C_{d}/(8R\rho_{a})$$
(8)

Herein, *a* is acceleration, direction of *a* is in opposite direction to $v_h(t)$, that is vertical to $v_s(t)$.

We notice that there is a variable R in (8). In practice, the radius of bubble particle is less than 1000 μm , as to large-radius particle, it bursts soon, whereas small-radius one is dissolved in water in short order. As a result, there is a

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critical radius. Bubble particle with the critical radius has the longest life-span and the largest amount. According to theoretical analysis and experimental measurement [6], the critical radius is 44.2 μm . To simplify the algorithm, we focus on the particle with the radius of 44.2 μm to establish its velocity and location equations. If the fidelity of ship wake geometric model is required, we can add some noise in the equations to make the model more credible.

Based on Newton's classical mechanics, the velocity of particle is as follows:

$$v_{i+1} = v_i + a_i \Delta t . (9)$$

v can be broken up into two independent forces in both X-axis and Y-axis. Similarly, *a* can also be broken up in the same way, as shown in Fig 4.



Fig. 4. breaking up acceleration and velocity of particle

Herein, v_s is the velocity of ship; v_b is the velocity of bubble particle; θ is the angle between v_s and X-axis; v_x is a component of v_b in X-axis direction, while v_y is in the Y-axis direction; a_x is a component of acceleration a in the X-axis direction, while a_y is in the Y-axis direction.

We assume that at t_i time, the velocity of bubble particle is $v_i(v_{xi}, v_{yi})$ and the location of particle is $P_i(x_i, y_i)$, while at t_{i+1} time, the velocity is $v_{i+1}(v_{x(i+1)}, v_{y(i+1)})$ and the location is $P_{i+1}(x_{i+1}, y_{i+1})$. We can get the equations as follows:

$$\begin{bmatrix} v_{x(i+1)} \\ v_{y(i+1)} \end{bmatrix} = \begin{bmatrix} v_{xi} \\ v_{yi} \end{bmatrix} + \begin{bmatrix} a_{xi} \\ a_{yi} \end{bmatrix} \cdot \Delta t$$
(10)

$$\begin{bmatrix} x_{i+1} \\ y_{i+1} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} \cdot \Delta t$$
(11)

Where Δt is the duration of each frame.

If we know the ship velocity v_s and its direction θ , it can quickly get the directions of velocity $v_b(t)$ of bubble particle and the viscous resistance F_a that decelerates $v_b(t)$. Based on which, the speed $v_{i+1}(v_{x(i+1)}, v_{y(i+1)})$ and location $P_{i+1}(x_{i+1}, y_{i+1})$ of the next moment are given according to (10) and (11). Finally, we take the particle initial velocity $v_b(t_0)$ shown in (3) into account, the particle acceleration, velocity and location at time *t* are:

$$\begin{bmatrix} a_x(t) \\ a_y(t) \end{bmatrix} = \begin{bmatrix} \frac{3\rho_i v_b^2(t)C_d}{8R\rho_a} \sin(\theta(t_0)) \\ \frac{3\rho_i v_b^2(t)C_d}{8R\rho_a} \cos(\theta(t_0)) \end{bmatrix}$$
(12)

$$\begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix} = \begin{bmatrix} v_b(t_0)\sin(\theta(t_0)) + \int_{t_0}^t a_x(\tau)d\tau \\ v_b(t_0)\cos(\theta(t_0)) + \int_{t_0}^t a_y(\tau)d\tau \end{bmatrix}$$
(13)
$$\begin{bmatrix} x(t) \\ z \end{bmatrix} = \begin{bmatrix} x(t_0) + \int_{t_0}^t v_x(\tau)d\tau \end{bmatrix}$$
(14)

 $\begin{bmatrix} y(t) \end{bmatrix}^{=} \begin{bmatrix} y(t_0) + \int_{t_0}^{t} v_y(\tau) d\tau \end{bmatrix}$ (14) Where $\theta(t_0)$ is the angle between $v_s(t_0)$ and X-axis at the

initial time t_0 .

IV. USING PARTICLE SYSTEM TO SIMULATE SHIP WAKE

The particle system theory provides a powerful algorithm to simulate irregular and dynamics objects such as ship wake.

In this paper, a geometric model of ship wake is established based on particle system theory, according to which, the attributes of bubble particle including acceleration, velocity and position at any time are determined.

A. Generate New Ship Wake Bubble Particles

Particles are generated into a particle system by means of controlled processes. One process determines the number of particles entering the system during each interval of time, that is, at a given frame. The model designer control the mean number of particles generated at a frame and its variance.

The system particle generates two bubble particles that are respectively in the larboard or the starboard edges in order to simplify calculation since the geometric simulation of ship wake mainly focuses on its edge. If each frame costs 20*ms*, and the lifetime of particle is 120*s*, then there are 12000 particles in particle system and set the variance zero.

B. Initiate Ship Wake Bubble Particle Attributes

Initially, ship wake bubble particles are generated sternwards. We can get the initial particle position by the position of ship. The initial particle velocity is also given which is shown in (3).

C. Update Ship Wake Bubble Particle Attributes

According to the model of viscous resistance and particle system model that we established above-mentioned, ship wake bubble particles move with a path like parabola in the relative coordinate system moving along with the ship, due to the action of viscous resistance and the movement of ship. The relative coordinate system is built in chapter, shown in Fig 2. We update bubble particle attribute according to the motion feature of itself. Proceedings of the World Congress on Engineering 2011 Vol I WCE 2011, July 6 - 8, 2011, London, U.K.

D. Delete Lifetime-over Ship Wake Bubble Particle

Bubble particles can not move unlimitedly, otherwise, when the particles exceed the range of its lifetime, we can indicate that particles may have died, which can be deleted from the particle system. We have assumed that the particle system generate two new bubble particles each frame, at the same time, there are two particles disappear from the system.

E. Outline Shape of the Object Composed by Particles

Once the positions of all particles have been calculated for a frame, the outlining algorithm makes the shape of ship wake [7]. The particle systems are implemented on a line-drawing display. We joint the particles in the larboard or the starboard edges in turn by using linear interpolation method. As a result, the motion of ship wake still looks real and the sequence gives the viewer the impression that something is smooth and moving. This is because the model is dynamic.

V. SIMULATION

To simulate ship wake, we use the particle system theory and bubble particle kinetics equations. Ship wake geometric models of the ship in various ways of moving are established as shown in following text.

We assume that the ship is moving in straight line with invariable speed, the ship wake geometric model as shown in Fig 5(a). And Fig 5(b) shows wake of the ship that changes sailing direction.



The ship wake geometric models shown in Fig 5 are continuous, whose edges are smooth as well. There is no inflexion point or gap in this model. Bubbles particles stop diffusing when the wake duration is about 20s. The widths of far wakes are about three times the width of the ship. Therefore, we can get a conclusion that the geometric

parameters of the ship wake simulated in this paper and the statistical experiment data are identical on the whole. It is obvious that the simulation method can solve the problems effectively. The problems are mainly the distortions of ship wake geometric model, including inflexion point and gap. All of the problems take place by using traditional methods such as putting some polygons with certain characteristics together.

The ship wake model can approximate the geometry of the actual wake. The profile of ship wake homeopathic bend as the ship changing course. In addition, the spatial position of the geometric model changes with the moving ship without gaps and overlaps. All of discussed above prove the availability and accuracy of ship wake's geometric model and the simulation method.

VI. CONCLUSION

We bring forward a simulation method of ship wake based on the particle system theory and the analysis of ship wake bubble dynamics. We establish a ship wake flow geometry model by solving the acceleration, velocity and position of the bubble particle. It improves the accuracy and fidelity of the ship wake model and solves the distortion problem of simulation caused by classical simulation method in virtual test.

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