

Semi-infinite Model of Emissions Limits of Firms in the Areas with Mixture Landscapes

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Abstract—The model and algorithm uses a semi-infinite optimization (SIP) to find the optimal reduction vector of contamination in areas with mixture landscapes. Using the optimization results, contamination maps are generated with MATLAB 7. These maps can serve as a tool for periodically monitoring the territory described in the information model included in the database.

Index Terms—air pollution problem, nonlinear programming, semi-infinite programming.

I. INTRODUCTION

Despite of the tendency to decrease the impact of production on the environment there are many places in which the pollution concentration exceeds critical limits. Especially this occurs in areas in which natural and industrial zones not only are nearby, but also coexist. It is very important to simulate standards compliance of air pollution not only in new areas but also in urbanized areas.

Many models in different regions require the control of limitations to the state of system at each point of geometric region. If the number of functional constraints is infinite and number of variables is finite, semi-infinite programming problems (SIP) appear. In this work semi-infinite programming model was built and an optimization algorithm of the contamination area was proposed. Our model contains the vector of pollution composition, as well as landscape maps, initial pollution and limit contamination norms for a control area.

MATLAB 7 was used to elaborate contamination maps with the results obtained from the optimization process.

II. PROBLEM DEFINITION

In [1] the SIP formulation of the air pollution problem the contamination functions spreads to the entire territory and authors do not take into account the placement of specific pollution sources. Our optimization model included locations of pollution sources, as well as the landscape map of the territory, variable power of pollution sources, emissions from multicomponent

composition, the presence of emissions from different sets of components, the difference in standards of pollution, maps of initial contamination area, the presence of industrial zones in the immediate around of sources, the possibility of different objective functions.

Let $H(j, t)$ be the intensity of release the contamination by j source in the t point of area for reporting period; n the number of sources ($j = 1 \dots n$), located on area d . The emissions composition is like multivariable set of K elements. Then with the symmetrical spread $h(r, k)$ is the value of point pollution at the distance r from k source. $R(k)$ is limit of propagation of k source pollution from $h(r, k) = 0$. About the nature of the torch of the spread of pollution into [2] exponential functions were mentioned, parabolic were applied, and in the simplest version linear functions were used. In practice the linear case is very abstract, for SIP problems it is better to have convex functions. In the model transition to different function of the calculation of curvature with the description of the torch of the spread of pollution is possible.

Semi-infinite model is formulated as a classical optimization problem with constraints. The work of stochastic algorithm consists in the sequential searches of local maximums of unsatisfied constraints. This allows the original semi-infinite problem to replace by a sequence of finite problems. The optimization goal is reduced to finding such a vector \bar{x} , which satisfies all the constraints $g()$, and simultaneously minimized the objective function:

$$\min_{x \in X} f(x), \quad \forall s \in S_d, \quad x \in [0, 1]$$

$$g(1) = \sum_{j=1}^n q(1, j)(1 - x(j))h(t, s, 1, j) + \lambda_0(s, 1, v) - (s, 1, w) \leq 0$$

$$g(2) = \sum_{j=1}^n q(2, j)(1 - x(j))h(t, s, 2, j) + \lambda_0(s, 2, v) - (s, 2, w) \leq 0 \cdot$$

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$$g(k) = \sum_{j=1}^n q(k, j)(1 - x(j))h(t, s, k, j) + \lambda_0(s, k, v) - (s, k, w) \leq 0$$

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here s are coordinates of points from area S_d of infinite points set, $x(j)$ are pollution reduction factors (our decision variables), where $(0 \leq x(j) \leq 1)$, $q(k, j)$ are codes which indicates if source j has k - component of pollution, $h(t, s, k, j)$ is the intensity of disposal k component at point s by source j .

$\lambda_0(s, k, v)$ are levels of the initial pollution of territory at point s by k component taking into account the type of the zone v , $\phi(s, k, w)$ is the permitted norm for k pollution component at point s taking into account the type of the zone w .

The proposed model took into account several types of areas of infrastructure (landscape zones): 1 - buildings (housing, commercial, cultural), 2 - aquatic areas (marine, rivers, dams), 3 - green areas (forests, parks), 4 - free zones and 5- mountains. In the territory there are some numbers of zones with arbitrary boundaries between them.

In [3] the permissible standard of pollution $\phi()$ was defined as a constant. We proposed to make it $\phi(s, k, w)$ a function that depends on the coordinates, the form of the disposal component and the landscape.

The function $\lambda_0(s, k, v)$ of the initial pollution of territory with the zone vector v is introduced.

The parabolic function of the connection of the intensity of the disposal of the pollution components was selected: $h(r, k, j) = -a(k)r(j)^2 + H(j)$, with the removal $r(j)$ from the arrangement point of j source if $r(j) \leq R(j)$, $a(k)$ are parabolic function coefficient which characterizes the curvature of spread by the source $H(j)$ of component k .

The coefficients of parabola $a(k)$ which impacts $R(j)$, can characterize the specific weight of each component. Then, for example, heavier (dust) components correspond to larger values of $a(k)$, and smaller values of $a(k)$ correspond to light component (gas). The application of a parabolic function of spread subsequently can be refined or substituted, for example, with Gauss function or with data of the direct physical monitoring of sources.

The unequal enumeration (collection) of the components of pollution in different sources was considered, which can be connected with the differences in nature and technology of different productions. These collections are determined by the matrix of the presence code $q()$ (1- if the source has this component for disposal, 0 - if this component is not in the collection).

The costs of cleaning the freed territory are cost coefficients in the objective function. Formally these expenditures are determined proportional to the general pollution reduction according to the components sum for all sources. The volume of parabolic function will be integral estimation for the parabolic function of pollution and the effect of a decrease will be determined by the difference:

$$\Delta V = V_{beg} - V_{end}, \text{ where } V_{beg}, V_{end} \text{ are initial}$$

and final (when $x() > 0$) volumes of parabolic function.

If $V = \pi H^2() / 2 a()$, the objective function is

$$\min \left\{ f(x) = \pi \sum_{j=1}^n \sum_{k=1}^K q(k, j) x(j)(2-x(j))H^2(j) \frac{m(k)}{2a(k)} \right\},$$

where $\pi = 3.14$ and $m(k)$ is the cleaning cost of integral unit (volume) for the component k .

The nature of the objective function is nonlinear then this problem is a nonlinear programming problem.

III. ALGORITHM

We use stochastic outer approximation method [4]. This method replaces the original semi-infinite programming problem to sequence of finite optimization problems ([1] and [3]). Each one of them we solve with finite optimization algorithm. This method uses active and passive searches of active constraints for finite problems.

The diagram of algorithm is given in the Fig. 1. Here $x()$ - is the solution vector, it is the -counter of iterations, re is the counter of the matrix reorganization, sl is the counter of random points, lim is limit base of random points, tz are coordinates of sources zone, eps is tolerance for stop, LMN is maximum local for nonfulfillment, NonLinPr is no linear optimization problem, C is the objective function value and $\Delta C = C(it + 1) - C(it)$.

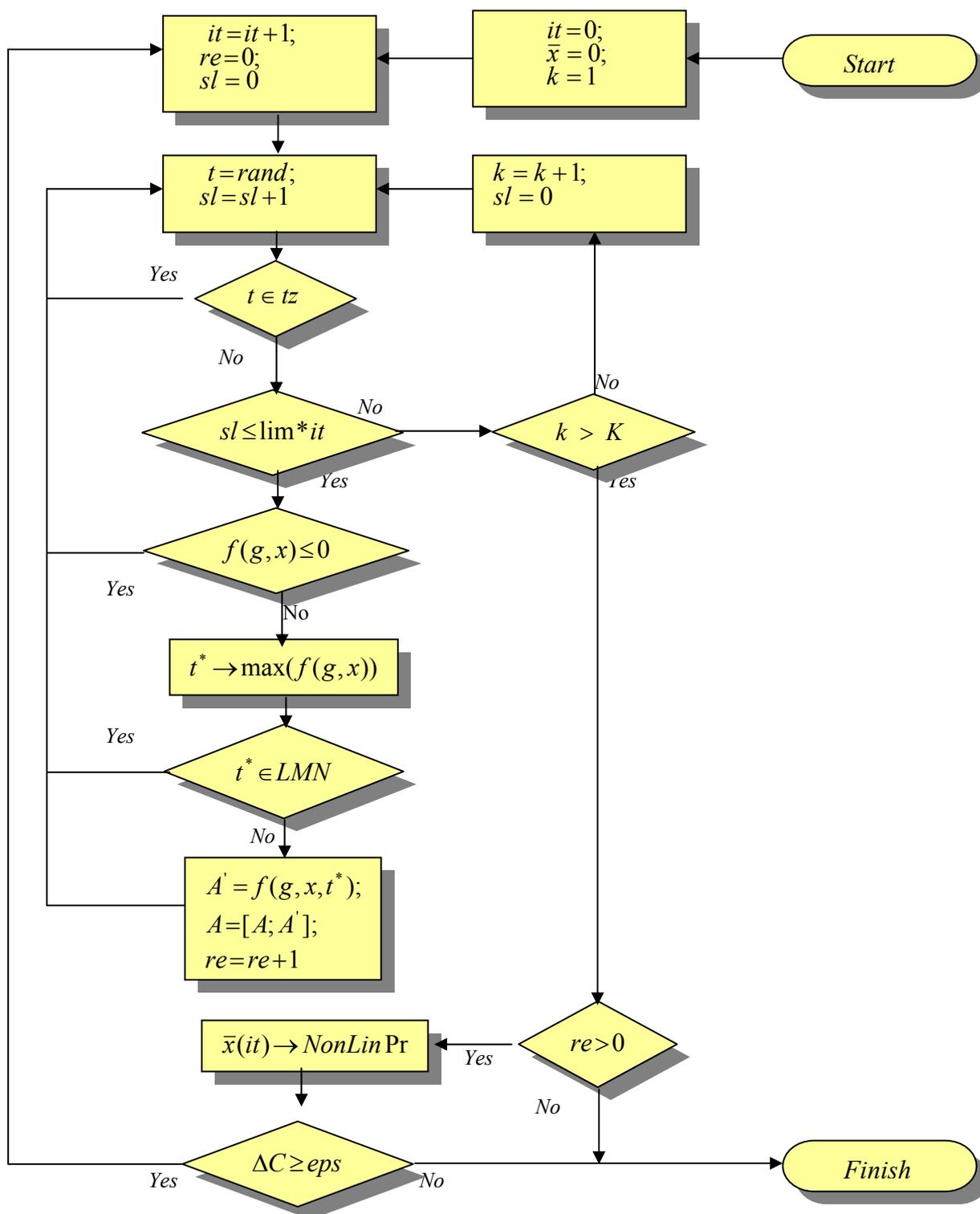


Fig. 1. Algorithm.

IV. NUMERICAL EXPERIMENTS

The initial information simulates a real situation. Number of disposal sources is 15. The power, sources coordinate, the number and the collections of components in the disposal composition are on the Table 1. Vector of the parabola constants of the spread of the pollution

components is (12e-4 to 21e-3). Dimensions of the territory are (600 x 300). Number of zones of the landscape is 5. The matrix of the maximum pollution norms in the zones has the range of 13 to 75. The maps of the initial pollution of area were created with the range of 0 to 79.

TABLE 1

MODEL INFORMATION.

Parameter s	Pollution sources														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
H	157.2 2	175.0 7	58.12	28.56	154.4 6	190.4 0	149.7 6	123.5 7	140.4 1	165.9 5	98.09	26.62	117.8 9	72.63	81.39
t1	575.0 1	127.9 1	314.6 9	210.0 0	395.0 0	570.1 6	19.00	338.9 2	81.08	80.14	347.0 1	503.9 9	450.9 4	533.2 0	477.7 5
t2	28.87	213.3 4	247.0 0	242.0 0	282.3 7	148.0 0	25.00	276.0 0	240.6 5	107.8 9	241.2 2	262.8 7	190.7 3	211.3 8	134.7 5
1	0	1	0	1	0	1	1	0	1	1	0	1	1	0	1
2	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1
3	0	0	0	1	1	1	1	1	0	1	0	1	0	1	0
4	1	0	0	0	0	1	0	1	1	0	1	1	1	1	1
5	1	1	1	1	0	0	1	0	1	0	1	1	1	1	0
6	1	1	1	0	1	1	0	0	1	1	0	0	1	0	1
7	0	1	0	1	0	0	1	1	0	0	1	0	1	0	1

TABLE 2

INITIAL AND OPTIMUM VECTORS.

Vector \bar{x}	Pollution sources														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Initial	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Optimal	0.1229	0.619	1	1	0.3868	0.3004	0	0	0	1	0.465	0	0	1	1

Initial and optimum vectors \bar{x} are given in Table 2. The model parameters defined the results of optimization, for example, sufficiently rigid parameters required that five firms must leave control area; other five firms must comply to pollution norms; and remaining five must correct their power.

Besides the reduction pollution vector, also the equivalent of the excess summary soiling load of sources was determined (minimum charges according to the cleaning by value 5398408.08). Matrix dimension which ensured the optimal solution are $A[39 \times 15]$. The coordinates of discovered local maxima do not coincide with the coordinates of sources, because the generated multiextremal function of general pollution is totally different.

V. COMPARATIVE FIGURES

Contour figures (Fig. 2) shows the initial fields of the general pollution of territory from the ejections of components 1, 2 (maximum) and 7 (minimum) in a series. The ejections of component 1 practically cover all territory with the high level of the pollution (>356, and locally to 600).

Contour graphs (Fig. 3) shows the fields of the general pollution of territory from the imposition of the ejections of the same components after the optimization by proposed algorithm ($\bar{x} = Opt$).

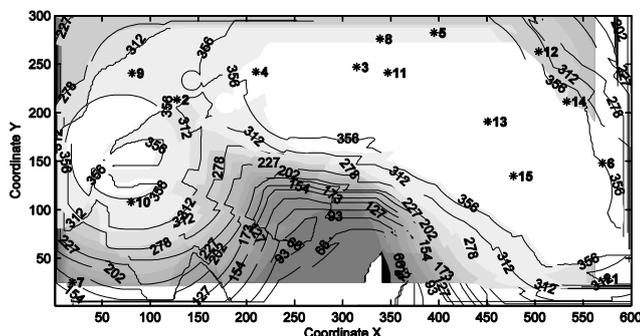


Fig. 2. Initial fields of general pollution.

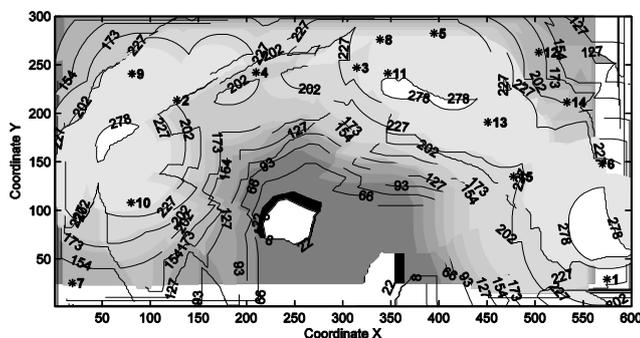


Fig. 3. Pollution fields after optimization.

VI. CONCLUSIONS

1. Semi-infinite programming model for pollution area with mixture landscapes problem is proposed. Optimization algorithm for the optimum search of the

reduction pollution factors is created. It works in the zones of territory with complex landscape.
2. The objective function of problem determines the costs of all pollution sources in control area.
3. Software proposed ensures construction of the pollution levels maps for all territory (or their 3D images). This software can serves as the tool for periodic monitoring of territory according the information base included in model.

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