Probing Variance Separation on Coordinate Measuring Machines

A. Nafi and J. R. R. Mayer

***Abstract*—This paper presents a method to estimate the non-repeatability of measurement and its sources on a coordinate measuring machine. The approach uses, as input data, the residuals obtained from the separation of machine and probe systematic errors on a CMM using multi-step redundancy probing of the machine's own master ball. The objective is to use data not explained by those error models in order to separate the contributions to the probing results randomness originating from the machine approach direction and from the probe triggering direction. Such information can then be used as input to the estimation of measurement uncertainty.**

***Index Terms*—coordinate measuring machine, touch trigger probe, repeatability, uncertainty, multi-step**

I. INTRODUCTION

SEVERAL sources of error affect the accuracy of CMMs such as errors from the axes of the machine and errors from the probing system [1]. These errors coexist. When measuring an artifact in a small volume to evaluate the probing system, it is usually assumed that the effect of the axes of the machine are not dominant so that the observed errors are essentially caused by the probing system [2]. Probe error can be a dominant error source when using a CMM to inspect small features [3]. It has also been established that the probe errors generally increase with the length of the stylus [4, 5]. Despite these developments there remains an industrial requirement to assess probe performance in a production environment and on the CMM and determine whether the probe or the rest of the machine are causing erroneous readings. A redundancy method was applied to separate the systematic errors of the probing system from those due to the rest of the machine by measuring a test sphere [6]. The non-repeatability present in the data was not treated.

This paper uses data not explained by the previously developed model in order to separate the contributions to the probing results randomness originating from the machine approach direction and from the probe triggering direction. Such results would be useful to assess the uncertainty in CMM measurements using Monte Carlo approaches [7].

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II. REDUNDANCY METHOD

The redundancy method for the separation of machine and probe error uses the machine's own test sphere [6]. Fig. 1 shows a coordinate measuring machine and a close-up on the machine test sphere and the touch trigger probe, stylus and stylus tip. The sphere is used to acquire data about the probing process performance.

a) b)

Fig. 1. Experimental setup for the data acquisition, a) machine LK; b) probe and test sphere.

The target contact points on the sphere are equally spaced on the equator; n points are measured equally spread longitudinally. The sphere is measured for n setups each corresponding to an orientation of the probing system, each setup is defined by rotation angle B around the vertical axis (Renishaw PH10 articulation system) at increments of 360°/n. For every configuration, the probe system remains vertical (A=0°). From one configuration to another, a probe qualification is first performed on the sphere. As a result of the rotation there is a permutation of the probe errors on all the points measured. Fig. 2 illustrates the measurement sequence for the different probe head orientations.

Each measurement is defined by a machine approach direction i and a probe triggering direction j corresponding to set (i, j). A complete measurement collection consists of n^2 possible configurations or sets, i.e. i=1 to n and j=1 to n. The machine approach direction labelled 1 is used n=24 times, each time combined with a different probe triggering direction. Similarly, a particular probe triggering direction is used in turn with each possible machine approach direction.

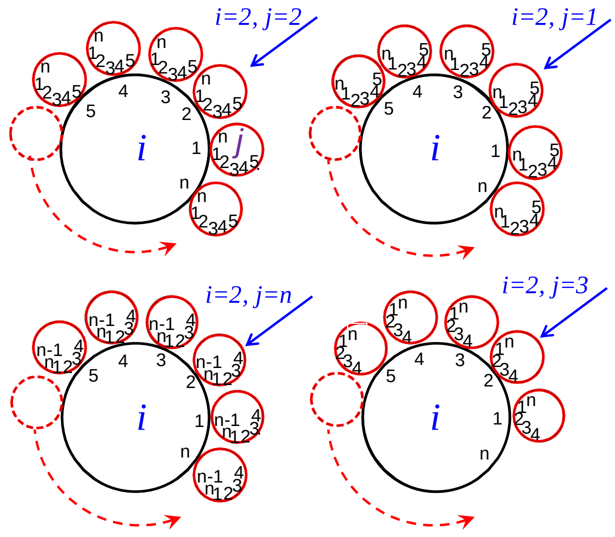


Fig. 2. Illustration of the data acquisition for the multiple redundancy method. Each illustration shows the probing action for a particular orientation of the probe head B angle (and of the probe).

For each measured point, the following equation applies,

$$e_{i,j} = \delta_{m,i} - \delta_{p,j} \quad (1)$$

where i indicates the machine approach direction and j indicates the configuration of the probing system. All data sets can be assembled into a single matrix equation with the following form

$$\boldsymbol{E} = \boldsymbol{P\delta} \quad (2)$$

where $\boldsymbol{E}$ is a column matrix (n^2×1) containing all measured deviations from the perfect circle for all configurations, $\boldsymbol{P}$ is the identification matrix (n^2×2n) (see [6] for more details) and $\boldsymbol{\delta}$ is a column matrix (2n×1) containing the machine and probe error $\delta_{m,i}$ for i=1, n and $\delta_{p,j}$ for j=1, n.

The identification matrix rank is deficient by 1. To solve the system and have the absolute probe errors and the machine errors, one more equation is needed. However, for a probing system what matters is the variation of the probing error, so, the nth machine error is arbitrarily set to zero by removing the last column from the system as well as the last error term in $\boldsymbol{\delta}$. The resulting identification matrix $\boldsymbol{P}$ has full rank with a good condition number of about 10. The system can now be solved for the remaining errors in $\boldsymbol{\delta}$, by pseudo-inversing $\boldsymbol{P}$.

III. Experimental Tests and Results for the Systematic Pre-Travel

Tests are performed on the moving-bridge CMM model LK G90c shown in Fig. 1. It uses a Renishaw TP2 kinematic touch trigger probe mounted on an articulated system model PH10M. The test sphere has a nominal diameter of 20 mm. The measurements are made with an approach speed of 3 mm/sec and an approach distance of 5 mm using a stylus length of 20 mm and n=24. Fig. 3 presents the measured and predicted errors. Predicted errors, for each configuration, are obtained using (2) and the separated machine and probe errors presented in Fig. 4.

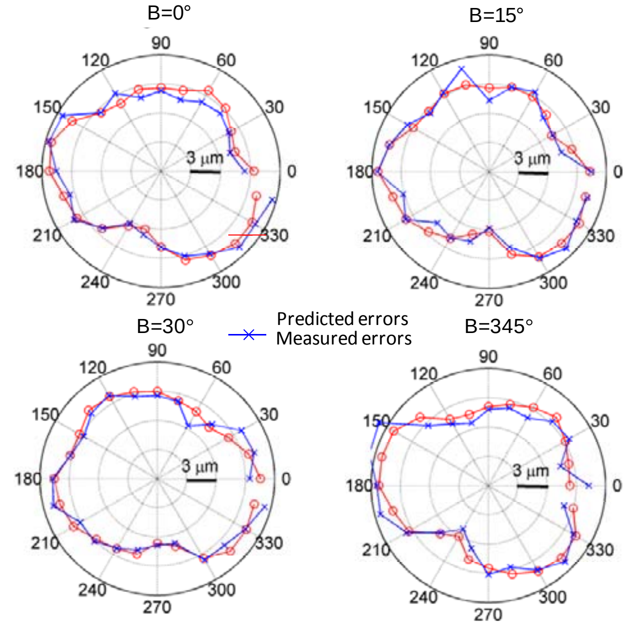
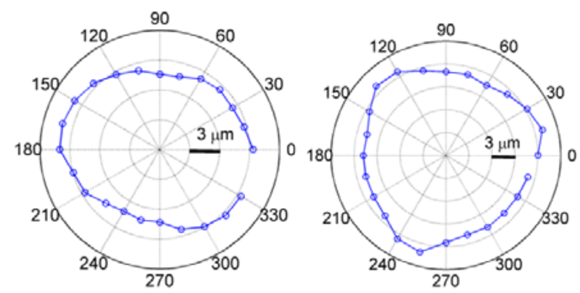


Fig. 3. Measured and predicted errors for some of the experimental setups corresponding to a fixed probe head (B angle) orientation [6].



a) b)

Fig. 4. Separated systematic pre-travel sources for a) Machine and b) Probe [6].

IV. Variance Separation

A method is now proposed to further process the available data in order to distinguish the contributions to the probing results randomness originating from the machine and from the probe. For each configuration, the total randomness effect is here defined as the difference between the actually measured pre-travel and that predicted by the model of the systematic contribution from the machine and probe as described earlier. The separation method can accommodate redundant measurements, i.e. having more measurements then strictly necessary. This occurs naturally when n configurations are used for n probing directions.

A different variance value is assigned to each probe triggering direction and also to each machine approach direction. When a measurement is taken both variance sources are combined. The method separates the contributing variances.

A. Machine approach direction

Let's define i the machine approach direction index. The quantity characterising the variability of the measured pre-travel with respect to its mean for a particular machine approach direction is

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$$SSTO_{m_i} = \sum_{k=1}^{n}\left(e_{i,k} - \bar{e}_{m_i}\right)^2 \quad (3)$$

where $e_{i,k}$ is the pre-travel measured for configuration (i, k) corresponding to the probe triggering direction k and the machine approach direction i and $\bar{e}_{m_i}$ is the mean of the measured pre-travel values defined by $\bar{e}_{m_i} = \frac{\sum_{k=1}^{n}(e_{i,k})}{n}$.

The quantity characterising the variability of the model predicted pre-travel with respect to its mean for a particular machine approach direction is defined as

$$SSR_{m_i} = \sum_{k=1}^{n}\left(\hat{e}_{i,k} - \bar{e}_{m_i}\right)^2 \quad (4)$$

where $\hat{e}_{i,k}$ is the systematic pre-travel predicted for configuration (i, k).

The variability for a particular machine approach direction characterising the randomness not associated with the systematic effects predicted by the model is SSE_{m_i}. For each machine approach direction i, the variability decomposition equation is written as:

$$SSE_{m_i} = SSTO_{m_i} - SSR_{m_i}. \quad (5)$$

SSE_{m_i} is composed of two types of variability, $\left(u_{m_i}\right)^2$ due to the machine approach direction i and the sum of variabilities $\left(u_{p_k}\right)^2$ due to the probe and its n triggering directions (k=1,…,n).

As a result, the following variability combinations equation results

$$SSE_{m_i} = \sum_{k=1}^{n}\left(u_{p_k}\right)^2 + n\left(u_{m_i}\right)^2 . \quad (6)$$

Combining (3) to (6) yields:

$$\sum_{k=1}^{n}\left(u_{p_k}\right)^2 + n\left(u_{m_i}\right)^2 = \sum_{k=1}^{n}\left(e_{i,k} - \bar{e}_{m_i}\right)^2 - \sum_{k=1}^{n}\left(\hat{e}_{i,k} - \bar{e}_{m_i}\right)^2. \quad (7)$$

Since the machine is used in i=1, n approach directions, n equations like (7) are generated for a complete test.

B. Probe triggering direction

A similar mathematical development for a particular probe triggering direction j yields the following equation

$$n\left(u_{p_j}\right)^2 + \sum_{k=1}^{n}\left(u_{m_k}\right)^2 = \sum_{k=1}^{n}\left(e_{k,j} - \bar{e}_{p_j}\right)^2 - \sum_{k=1}^{n}\left(\hat{e}_{k,j} - \bar{e}_{p_j}\right)^2 . \quad (8)$$

Since the probe has j=1 to n triggering directions, n equations like (8) are generated for a complete test. Furthermore, the n variances $\left(u_{p_j}\right)^2$ and n variances $\left(u_{m_k}\right)^2$ are unknown. All other values are calculated from the measurements or predicted by the model, itself obtained from the measurements. Furthermore, the n variances $\left(u_{p_j}\right)^2$ and n variances $\left(u_{m_k}\right)^2$ are the same unknowns as for (7).

C. System identification

A complete test will provide a total of n equations like (7), one for each machine approach direction, and n equations like (8), one for each probe triggering direction. Together these 2n equations contain 2n unknowns.

These 2n equations can be presented in matrix form as follows:

$$\boldsymbol{A}.\boldsymbol{x} = \boldsymbol{b} \quad (9)$$

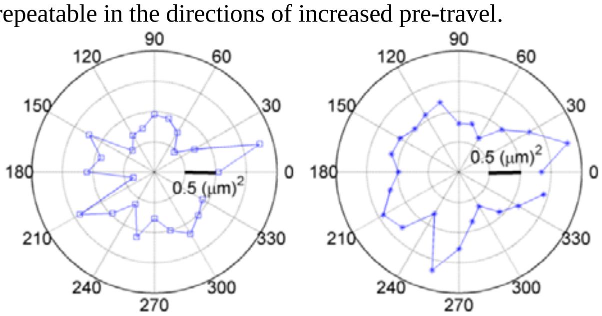
where matrix $\boldsymbol{A}$ is an identification matrix of size 2n x 2n made of zeros, ones and n terms, $\boldsymbol{x}$ is the column matrix of size 2n x 1 containing the unknown variances $\left(u_{m_i}\right)^2$ and $\left(u_{p_j}\right)^2$ and $\boldsymbol{b}$ is a column matrix of size 2n x 1 calculated from the available data.

So, it would appear that a solution can be obtained for the 2n unknowns. However, the rank of $\boldsymbol{A}$ is 2n-1 so that it is said to be deficient by one. As a result, it becomes impossible to obtain absolute values for the unknown variances. However, relative variance values can be estimated. This represents how the variance changes with the probing triggering direction and with the machine approach direction.

V. Experimental results for the variance separation

The experimental data are those presented earlier in the paper. The difference between the model predicted pre-travel and the measured pre-travel are calculated for every n^2 configurations. These values are then introduced into (9).

Fig. 5 shows the variance variation for the 24 probe triggering directions and for the 24 machine approach directions. It can be observed that maximum variations of 1.19 µm^2 and 1.42 µm^2 are obtained for the probe and machine directions respectively. It is noticeable that the shape of the probe variation is faintly trilobed which matches that of the probe systematic pre-travel variation. This indicates that the probe triggering is likely less repeatable in the directions of increased pre-travel.



a) b)

Fig. 5. Variation of variance a) as a function of machine approach direction for the machine and b) as a function of probe triggering direction approach direction for the machine.

VI. Total variability during actual measurements

The previous analysis revealed that only variance variations can be estimated from the test results. However, a method is now proposed to enrich the data and provide at least some knowledge about the "absolute" variance, i.e. the variance itself. An additional test is conducted consisting in repeatedly probing a surface with a specific reference probe triggering direction and machine approach direction, such as configuration (r, q). Fig. 6 shows the setup used. A series of 10 repetitions yielded a variance of 0.8 µm^2.

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Fig. 6. Setup for the single configuration repeatability test.

The combined variance associated with any configuration, ${u_{c_{i,j}}}^2$, can be expressed as follows:

$$ {u_{c_{i,j}}}^2 = {u_{c_0}}^2 + ({u_{p_{j_{\mathrm{v}}}}}^2 - {u_{p_{\mathrm{q_v}}}}^2) + ({u_{m_{i_{\mathrm{v}}}}}^2 - {u_{m_{\mathrm{r_v}}}}^2) \qquad (10) $$

where ${u_{m_{i_{\mathrm{v}}}}}^2$ and ${u_{p_{j_{\mathrm{v}}}}}^2$ are the relative variance for the machine approach direction i and the probe triggering direction j respectively and ${u_{c_0}}^2$ is the combined bias constant.

Substituting the combined variance value obtained experimentally for configuration (r, q) in (10) provides a means to estimate ${u_{c_0}}^2$. Subsequently, absolute variance can be calculated using (10) for any measurement configuration. Such capability is highly desirable in the wider context of uncertainty estimation of actual measurements made on a coordinate measuring machine. Equation (10) allows the generation of an actual measurement variance associated with the probing process. With the increasing interest in the use of Monte Carlo methods for uncertainty estimation in metrology, this result is useful.

VII. CONCLUSION

The method presented in this paper extends the use of the data gathered for the separation of the systematic machine and probe error sources during probing. It processes the residual, i.e. the data not explained by the systematic machine and probe errors, in order to separate the contributions to the probing results randomness originating from the machine approach direction and from the probe triggering direction. The analysis is based on the decomposition of the total variability of the measurement errors relative to the predicted errors. Experimental results are processed to estimate the variance variation for both the machine approach directions and for the probe triggering directions. The result indicates that the probe triggering is less repeatable in the directions of increased pre-travel. A single probing configuration is further repeated to estimate a combined variance bias which can be associated to any probing configuration and provides a means to estimate the measurement variance for any measurement configuration, a result useful for the estimation of measurement uncertainty in coordinate metrology.

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