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Study of Exponential Thermal Effect on Vibration of Non-Homogeneous Orthotropic Rectangular Plate Having Bi-Directional Linear Variation in Thickness

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ABSTRACT: A mathematical model is constructed with an aim to assist the design engineers for the making of various structure used in the satellite and aeronautical engineering. In this paper effect of exponential variation in temperature is premeditated on an orthotropic rectangular plate whose thickness varies linearly in both directions. Rayleigh Ritz approach is applied for the solution of the problem. Fundamental frequencies and deflection functions are calculated for first mode of vibration of a clamped plate with diverse values of temperature gradient, taper constants and non-homogeneity constants.

KEY WORDS: **Exponential Thermal effect, orthotropic rectangular plate, Rayleigh Ritz method, Vibration, variation in thickness Non- Homogeneity**

1. INTRODUCTION

Vibration effects have always been a principle concern of engineers. In the epoch of science and technologies it is desired to design large machines with smooth operation and unwanted vibrations. Sometimes unwanted vibration causes fatigues.

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Unwanted vibration can damage electronic components of aerospace system, damage buildings by earthquake, bring tsunami, and contribute to toppling of tall smokestacks, collapse of a suspension bridge in a windstorm. There are a multitude of applications where vibration effect is required e. g. in string and percussion instruments, in the design of loudspeakers, space shuttles, satellites where discrepancies in the temperature also affects the vibration effect. Controlled vibration effects are also required in health industry, paper industry, design of structures, building construction, reducing soil adhesion and many more areas engross vibration upshot.

Hence vibrations totally affect our day-to-day life. Thus for design engineers and scientist, it has always been a necessity to optimize or to control the effect of unwanted vibrations as much as possible. Present work is a full-fleshed endeavor to assist the design officers, industry people to come up to the situation.

An initial eloquent essence about the subject of plate vibration is done by Leissa [1] and Leissa [2]. Pradeep and Ganesan [4] worked for thermal buckling and vibration behavior of multi-layer rectangular viscoelastic sandwich plate. The temperature-dependent characteristics of complex shear modulus of viscoelastic core were accounted. An all side clamped (C–C–C–C) plate under thermal loads was analyzed for thermal buckling, frequency and damping behavior. An efficient controller for vibration reduction in a small square plate clamped on all edges was developed by Shimon and Hurmuzlu [3]. Tomar and Gupta [5] studied the effect of exponential temperature variation on frequencies of an

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orthotropic rectangular plate having bi-directional variation in thickness. Effect of thermal gradient on vibration of non-homogeneous orthotropic rectangular plate having bidirectional parabolically varying thickness was estimated by Gupta and Johri [6]. A study regarding the static deflections and natural frequencies of isotropic, orthotropic/laminated composite plates using a Levy-type solution was done by Chen [9]. Mindlin plate theory is applied in conjunction with the state-space concept to find such solutions. Sakata [10] worked on the study of clamped orthotropic rectangular plates with variation in thickness. Singh and Saxena [8] studied the transverse vibration of a rectangular plate with thickness varying in both the directions. Gupta, Johri and Vats [7] studied thermal gradient effect on vibration of non-homogeneous orthotropic rectangular plate having bi-direction linearly variation in thickness.

2. METHODOLOGY

Let the plate is subjected to a steady one dimensional exponential temperature distribution along x- axis, then

$$T = T_0\left[1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right] \tag{1}$$

where T is the temperature excess above the reference temperature at a distance x/a and T_0 is the temperature excess above the reference temperature at the end of the plate i.e. at x=a.

Expressions for Moduli of elasticity as a function of temperature are described as [5],

$$\left.\begin{aligned} E_x(T) &= E_1\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \\ E_y(T) &= E_2\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \\ G_{xy}(T) &= G_0\left[1-\alpha\left(1-\left(\frac{e}{e-1}-\frac{e^{x/a}}{e-1}\right)\right)\right] \end{aligned}\right\} \tag{2}$$

where α is thermal gradient parameter. E_x and E_y are Young's moduli in x- and y- directions respectively and G_{xy} is shear modulus, γ is Slope of variation of moduli with temperature and $\alpha = \gamma T_0\ (0 \le \alpha < 1)$, a thermal gradient parameter.

The governing differential equation of transverse motion of an orthotropic rectangular plate of variable thickness in Cartesian coordinates is

$$\begin{aligned} &D_x W_{,xxxx} + D_y W_{,yyyy} + 2HW_{,xxyy} + 2H_{,x}W_{,xyy} \\ &+2H_{,y}W_{,xxy} + 2D_{x,x}W_{,xxx} + 2D_{y,y}W_{,yyy} \\ &+D_{x,xx}W_{,xx} + D_{y,yy}W_{,yy} + D_{1,xx}W_{,yy} \\ &+D_{1,yy}W_{,xx} + 4D_{xy,xy}W_{,xy} + \rho h W_{,tt} = 0 \end{aligned} \tag{3}$$

A comma followed by a suffix denotes partial differential with respect to that variable.

Deflection function for free transverse vibrations of the plate can be written as, in the form of Levy type solution,

$$W(x,y,t) = W(x,y)e^{ipt} \tag{4}$$

$$W(x,y) = \left\{\begin{aligned} &\left[\left(\frac{x}{a}\right)^2\left(\frac{y}{b}\right)^2\left(1-\frac{x}{a}\right)^2\left(1-\frac{y}{b}\right)^2 \times\right. \\ &\left.\left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]\right] \end{aligned}\right\} \tag{5}$$

where A_1 and A_2 are constants to be evaluated. Plate is assumed to have linear variation in thickness

$$h = h_0\left(1+\beta_1\frac{x}{a}\right)\left(1+\beta_2\frac{y}{b}\right) \tag{6}$$

where $h = (h_0)_{\substack{x=0\\y=0}}$ and β_1 & β_2 are two taper constants

Density of the plate is assumed to be varying exponentially

$$\rho = \rho_0 e^{\alpha_1 x/a} \tag{7}$$

where $\rho_0 = \rho|_{x=0}$ and α_1 is non- homogeneity constant.

For plate executing transverse vibration of mode shape W(x, y), the Strain and Kinetic energies are, respectively

$$V = \frac{1}{2}\int_0^a\int_0^b \left[\begin{aligned} &D_x\left(W_{,xx}\right)^2 + D_y\left(W_{,yy}\right)^2 \\ &+2D_1W_{,xx}W_{,yy} + 4D_{xy}\left(W_{,xy}\right)^2 \end{aligned}\right] dydx \tag{8}$$

$$T_1 = \frac{1}{2}p^2\int_0^a\int_0^b \rho h W^2 dydx \tag{9}$$

To validate Rayleigh Ritz technique, the maximum strain energy and maximum kinetic energy must be equal i.e. following equation must be satisfied,

$$\delta\left(V - T_1\right) = 0 \tag{10}$$

For clamped plate, the boundary conditions are,

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$W = W,_x = 0$ at x=0, a and

$W = W,_y = 0$ at y=0, b

Using eqn (1), (5),(6),&(7) in eqn (10), after calculating V and T_1 from eqn (8) &(9),one has,

$$\delta\left(V_1 - \lambda^2 T_2\right) = 0 \quad (11)$$

$$\text{where } \lambda^2 = \frac{12a^4\rho_0 p^2(1-\nu_x\nu_y)}{E_1 h_0^2} \quad (12)$$

is a frequency parameter and

Equation (12) contains two unknowns A_1 & A_2 evaluated in the following manner,

$$\frac{\partial}{\partial A_s}\left(V_1 - \lambda^2 T_2\right) = 0 \quad \text{where } s = 1, 2 \quad (13)$$

On simplifying equation (13), one gets,

$$c_{s1}A_1 + c_{s2}A_2 = 0 \quad (14)$$

where c_{s1} & c_{s2} involves the parametric constants and the frequency parameter.

For a non - zero solution, determinant of coefficients of equation (14) must vanish. In this way frequency equation comes out to be,

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = 0 \quad (15)$$

3. RESULT AND DISCUSSION

Frequency equation (15) is quadratic in λ^2 so it will give two roots. But it had already been observed that Rayleigh Ritz method gives best approximation for first mode of vibration as compared to further higher modes of vibrations if two-term deflection function is used. So the frequency parameter and deflection function, both are calculated for the first mode of vibration of the plate. The parameters for orthotropic material have been taken as [7],

$$\frac{E_2}{E_1}=0.32, \nu_x\frac{E_2}{E_1}=0.04, \frac{G_0}{E_1}\left(1-\nu_x\nu_y\right)=0.09$$

Results are plotted in Fig. 1,2,3,4 and 5. Fig 1 depicts the variation of frequency parameter λ with the thermal gradient parameter 'α' for the following two cases:

α_1 =0.0, β_1= 0.0, β_2=0.0

and α_1 =0.0, β_1= 0.2, β_2=0.6.

Fig.2, demonstrates the variation of frequency parameter with non-homogeneity of the plate material. Following two cases are taken into consideration:

α =0.0, β_1= 0.0, β_2=0.0 and α =0.0, β_1= 0.2, β_2=0.6

Fig. 3 and 4 display the variation of taper constant 'β_1' and 'β_2' with frequency parameter 'λ', respectively, for the following cases:

For β_1	**For β_2**
α_1 =0.0, α = 0.0, β_2=0.0	α_1 =0.0, α = 0.0, β_1=0.0
α_1 =0.0, α = 0.0, β_2=0.6	α_1 =0.0, α = 0.0, β_1=0.6
α_1 =0.0, α = 0.4, β_2=0.0	α_1 =0.0, α = 0.4, β_1=0.0
α_1 =0.0, α = 0.4, β_2=0. 6	α_1 =0.0, α = 0.4, β_1=0.6
α_1 =0.8, α = 0.0, β_2=0.0	α_1 =0.8, α = 0.0, β_1=0.0
α_1 =0.8, α = 0.0, β_2=0.6	α_1 =0.8, α = 0.0, β_1=0.6
α_1 =0.8, α = 0.4, β_2=0.0	α_1 =0.8, α = 0.4, β_1=0.0
α_1 =0.8, α = 0.4, β_2=0.6	α_1 =0.8, α = 0.4, β_1=0.6

Fig.5 displays the variation of deflection function W with X for the following cases:

For Y=0.2 and 0.4

α_1 =0.0, α = 0.0, β_1 =0.0, β_2=0.0, a/b=1.5

For Y=0.2 and 0.4

α_1 =0.8, α = 0.4, β_1 =0.2, β_2=0.6, a/b=1.5

Authenticity of result is confirmed by comparing them with those of Tomar and Gupta [5] under following conditions:

α_1 =0.0, α = 0.0, β_1 =0.0, β_2=0.0 and

α_1 =0.0, α = 0.2, β_1 =0.0, β_2=0.0.

4. CONCLUSION

From the above results it is seen that the frequency of vibration reduces on increasing thermal gradient and non-homogeneity, whereas increase in taper constants increases the frequency of vibration. A comparative study was done for the plates in which temperature was varying linearly and exponentially respectively. Thickness of the plate was assumed to be varying linearly in both directions. It was found that plate undergoing linear variation in temperature was more stable as compared to those undergoing exponential variation in temperature, to bear up the thermally induced vibration effects. But in those cases where exponential variation in thickness is a restriction, above said conditions can make them most stable in those situations.

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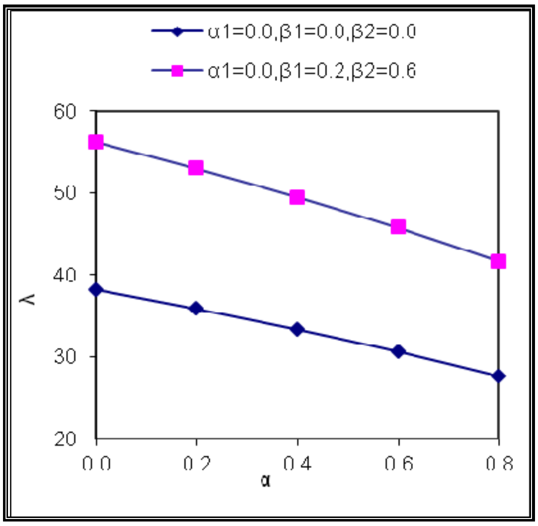

Fig 1: Frequency parameter 'λ' Vs. 'α'

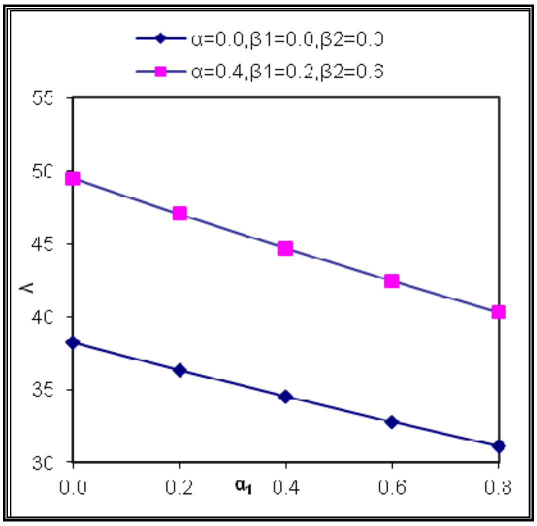

Fig 2: Frequency parameter 'λ' Vs. 'α_1'

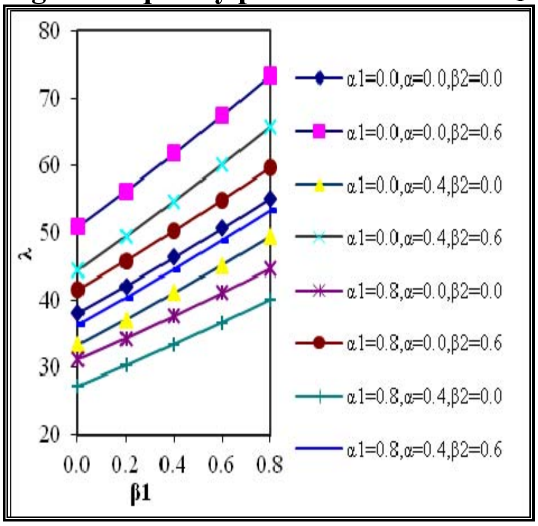

Fig. 3: 'λ' Vs. taper constant ' β_1

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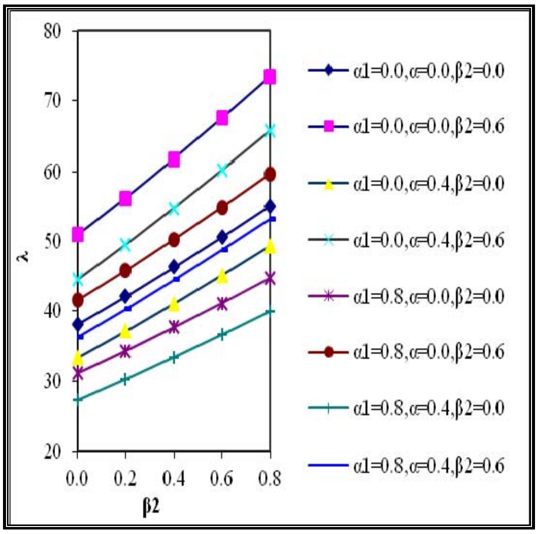

Fig. 4: 'λ' Vs. taper constant ' β_2

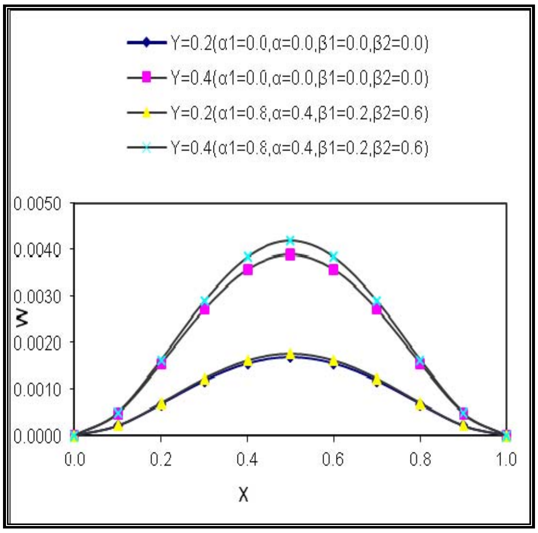

Fig 5: Deflection Vs. X (=x/a)

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