Series Active Power Filter Using Generalised Instantaneous Power Theory

M.A. Mulla, Member, IEEE, R. Chudamani, A. Chowdhury, Member, IEEE

Abstract - In the area of active power filtering, with an objective to reduce inverter capacity, the Series Hybrid Active Power Filter (SHAPF) has been taken into account increasingly. Existing method used for controlling SHAPF is either based on detecting source current harmonic or load voltage harmonics. Generalised Instantaneous Power Theory (GIPT) gives simple and direct method of defining power quantities under sinusoidal and non sinusoidal situations. The definition of GIPT is used to decompose voltage vector into different components, which represents different parts of the power quantity. The separated components of voltage vector are used to derive reference for SHAPF. This paper presents simulation study of SHAPF used for harmonic elimination, where the method used for calculating reference is based on GIPT. The mathematical formulation of proposed control scheme with its applications to SHAPF is presented. The validity of the proposed control scheme is verified by the simulation study.

Index Terms— Active filters, instantaneous power, geometric algebra, nonsinusoidal waveforms, power multivector.

I. INTRODUCTION

Load compensation in power engineering is the procedure used to obtain the supply currents that are sinusoidal and balanced. Active power line conditioners (APLCs) make it possible to obtain power-electronic solutions to power quality (PQ) problems. In particular, balanced or unbalanced load compensation in non-sinusoidal supply situations is possible [1].

Active Power Filter (APF) has become the main research direction of load compensation as its filtering characteristic is not affected by system parameters. Various APF configurations and control strategies have been researched during the last decades. So far, a large number of shunt APF have been installed, but there are still some shortcomings like large capacity, high initial investment, harmonic circulation while working with the shunt Passive Power Filter (PPF) together, improving filtering characteristic when the PPF makes the system resonance etc. In order to reduce inverter capacity, the hybrid APF is becoming very popular in recent development. A variety of configurations and control strategies are proposed to reduce inverter capacity [2-4].

To obtain efficient SHAPF performance, it is important to choose proper reference generation algorithm and an appropriate current or voltage control strategy. The publication of the instantaneous reactive power theory caused a great impact in reference generation. Many approaches have been published since then [5–16]. But all of these definitions are computational intensive and do not provide simple expression of instantaneous power quantity. In year 2004, X. Dai [17-20] introduced generalised instantaneous power theory which gives a direct and simple expression for instantaneous power quantities.

This paper presents a reduced rating SHAPF with a novel control algorithm for generating reference voltage using GIPT. It is proposed to decompose multiphase voltage vector into quantities that represents different components of power. Normally the instantaneous power has average component and oscillating component. Using vector algebra it is possible to obtain the voltage vectors corresponding to these average powers. The voltage vector corresponding to oscillating active and reactive power is used to calculate reference voltage for removing harmonics.

This paper is organized as follows. First, a generalized definition of instantaneous active, reactive and apparent power quantity is presented. Then, the proposed decomposition of voltage vector which represents different components of power quantities is defined. Further the use of this decomposition for calculating the reference for SHAPF is discussed for harmonic elimination. Finally, the application of proposed control scheme to SHAPF is presented.

II. DEFINITION OF GENERALIZED INSTANTANEOUS POWER THEORY

This section represents the formulation of basic terms in Generalised Instantaneous Power Theory (GIPT) [17-20]. For a three-phase four-wire system, the instantaneous quantities of load voltage and currents are expressed as

$$v = [v_a, v_b, v_c]^T$$ and $$i = [i_a, i_b, i_c]^T$$.

The instantaneous power multi-vector which is defined as the geometric product of voltage and current vectors can be expressed as

$$s(t) = v(t) \cdot i(t) = v(t) \bullet i(t) + v(t) \times i(t)$$

$$\tilde{s}(t) = p(t) + \tilde{q}(t)$$ .... (1)

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ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)
Load instantaneous apparent power \(s'\) is defined as
\[
\|v\||i||\tilde{i}\|, \quad \|v\| = \sqrt{v_a^2 + v_b^2 + v_c^2} \quad \text{and} \quad \|i\| = \sqrt{i_a^2 + i_b^2 + i_c^2}.
\]

Load instantaneous active power \(p'\) is defined as the inner product of voltage and current vectors.
\[
p(t) = v(t) \cdot i(t) = v_a i_a + v_b i_b + v_c i_c \quad \ldots \quad (2)
\]

Load instantaneous reactive power \(q(t)\) is defined as the outer product of voltage and current vector \(q(t) = \tilde{v}(t) \otimes \tilde{i}(t)\). The outer product is defined by tensor product of voltage and current
\[
q(t) = \tilde{v}(t) \times \tilde{i}(t) = \begin{bmatrix} q_{ab} & q_{ca} \\ q_{ac} & q_{bc} \end{bmatrix} \quad \ldots \quad (3)
\]

with each components being defined as,
\[
q_{ab} = v_a i_b - v_b i_a, \quad q_{bc} = v_b i_c - v_c i_b, \quad q_{ca} = v_c i_a - v_a i_c.
\]

\(q(t)\) is denoted as instantaneous reactive tensor and its norm is defined as instantaneous reactive power
\[
\|q\| = \sqrt{q_{ab}^2 + q_{bc}^2 + q_{ca}^2} \quad \ldots \quad (4)
\]

### III. Proposed Method of Reference Generation

In the proposed method, the calculation of reference signal for SHAPF is performed in two steps (i) decomposition of voltage vector into different components, which represents different components of the power quantity and (ii) generating reference voltage corresponding to unwanted components of power quantity.

#### A. Decomposition of Voltage Vector

Using (1) representing voltage with two components, one in phase with current and another quadrature to the current, the apparent power \(s(t)\) can be written as
\[
s(t) = [\tilde{v}_p(t) + \tilde{v}_q(t)] \cdot \tilde{i}(t) + [\tilde{v}_p(t) + \tilde{v}_q(t)] \times \tilde{i}(t).
\]

After simplification we obtain
\[
\tilde{s}(t) = \tilde{v}_p(t) \cdot \tilde{i}(t) + \tilde{v}_q(t) \times \tilde{i}(t) = p(t) + q(t) \quad \ldots \quad (5)
\]

Using the first part of (5), the component of load instantaneous voltage vector \(\tilde{v}_p^*\), corresponding to active power can be expressed as
\[
\tilde{v}_p(t) = \tilde{i}^{-1} p(t) = \frac{\tilde{i}}{\|\tilde{i}\|} p(t).
\]

\[
\tilde{v}_p = [v_{pa}, v_{pb}, v_{pc}] = \frac{\tilde{i}}{\|\tilde{i}\|} p(t) = \left[ \begin{bmatrix} i_a, i_b, i_c \end{bmatrix} \right]^T \tilde{i} \quad \ldots \quad (6)
\]

The inverse of vector \(\tilde{i}(t)\) is expressed using definition given in Appendix A.1.

\(\tilde{v}_p^*\) is denoted as instantaneous active voltage tensor and its norm is defined as instantaneous apparent voltage
\[
\|\tilde{v}_p^*\| = \sqrt{v_{pa}^2 + v_{pb}^2 + v_{pc}^2}.
\]

Using the second part of (5), the component of load instantaneous voltage vector \(\tilde{v}_q^*\), that represents reactive power, is expressed as
\[
\tilde{v}_q(t) = \tilde{v}_q^* \times \tilde{i}(t).
\]

Multiplying both sides by current vector \(\tilde{i}(t)\) and using Appendix A.2 for performing cross product on right hand side, following expression is obtained.
\[
\tilde{i}(t) \times \tilde{v}_q(t) = \tilde{i}(t) \times \tilde{v}_q^* \times \tilde{i}(t) \quad \ldots \quad (7a)
\]

\[
\tilde{i}(t) \times \tilde{v}_q(t) = \tilde{i}(t) \cdot \tilde{v}_q^* \times \tilde{i}(t) - (\tilde{v}_q(t) \cdot \tilde{i}(t)) \tilde{i}(t) \quad \ldots \quad (7b)
\]

\[
\tilde{i}(t) \times \tilde{v}_q(t) = \|\tilde{i}\| \tilde{v}_q(t) - 0 \quad \ldots \quad (7c)
\]

\[
\tilde{v}_q(t) = \frac{\tilde{i}(t) \times \tilde{v}_q(t)}{\|\tilde{i}\|^2} \quad \ldots \quad (7d)
\]

Equation (7d) expresses the load instantaneous active voltage vector \(\tilde{v}_q^*\) which represents reactive power quantity. Using the procedure given in Appendix A.3, the cross product is calculated as
\[
\tilde{v}_q = \begin{bmatrix} v_{qa}, v_{qb}, v_{qc} \end{bmatrix} = \frac{\tilde{v}_q^* \tilde{i}(t)}{\|\tilde{i}\|^2}.
\]

\[
\tilde{v}_q(t) = \begin{bmatrix} 0 & -q_{bc} & q_{ca} \\ -q_{ab} & 0 & q_{bc} \\ q_{bc} & -q_{ac} & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad \ldots \quad (8)
\]

\(\tilde{v}_q^*\) is denoted as instantaneous reactive voltage tensor and its norm is defined as instantaneous reactive voltage
\[
\|\tilde{v}_q^*\| = \sqrt{v_{qa}^2 + v_{qb}^2 + v_{qc}^2}.
\]

It is clear that the quantities defined by \(\tilde{v}_p^*\) and \(\tilde{v}_q^*\) (6) and (8) are representing components of instantaneous voltages that corresponds to active power and reactive power drawn by the load. These components are directly associated with three phase instantaneous voltages and currents. It is also observed that these voltage components are separated without any form of artificial coordinate transformations.

#### B. Reference Generation for SHAPF

In series compensation, the compensator works as a variable voltage source and compensate the unwanted
component of load voltage. Fig. 1 show the SHAPF
equivalent circuit, where load is represented as summation
of different voltages corresponding to power components.
The equivalence circuit of series APLC is represented by
variable voltage source ‘\(v_c\)’ connected in series with the
load.

![Figure 1. Equivalent circuit of the system with SHAPF.](image)

The total instantaneous active power calculated using (2)
is further divided into average active power and oscillating
active power \(p(t) = \bar{p} + \tilde{p}\). The voltages corresponding to
these two components of active powers are expressed as,

\[
\tilde{v}_p = \frac{i}{\sqrt{1}}[\tilde{p} + \tilde{p}] = \tilde{v}_{p_1} + \tilde{v}_{p_2}
\]  

(9)

where,
\(\tilde{v}_p = \) Voltage corresponding to Average Active Power.
\(\tilde{v}_p = \) Voltage corresponding to Oscillating Active Power.

Similarly the total reactive power calculated using (3) is
also divided into average reactive power and oscillating
reactive power \(q(t) = \bar{q} + \tilde{q}\). The voltages

corresponding to these two components of reactive powers
are expressed as,

\[
\tilde{v}_q = \tilde{v}_{q_1} + \tilde{v}_{q_2}
\]  

(10)

where,
\(\tilde{v}_q = \) Voltage corresponding to Average Reactive Power.
\(\tilde{v}_q = \) Voltage corresponding to Oscillating Reactive
Power.

These components of voltage vectors, given by (9) and
(10) are useful for generating reference voltage for SHAPF.
For calculations of reference voltages for harmonic
elimination, the required components of active and reactive
power that need to be compensated are \(p_c(t) = \bar{p}, q_c(t) = \bar{q}\).
The SHAPF reference is calculated as,

\[
\tilde{v}_c = \tilde{v}_{pc} + \tilde{v}_{qc} = \tilde{v}_p + \tilde{v}_q
\]  

(11)

The voltage vector \(\tilde{v}_c(t)\) is a reference for injecting
voltage in series in order to eliminate oscillating active and
reactive component of power from the system.

**IV. SHAPF SIMULATION MODEL**

For evaluating performances of SHAPF, using the
voltage reference calculation with GIPT, simulation study is
performed in PSIM. Fig. 2(a) shows the arrangement of
power circuit configuration which is made up of non-linear
load, Passive Power Filters (PPF) bank, Series Transformer
and Inverter. Typical diode rectifier is modeled as non-
linear load, PPF bank is made up of 5th harmonic, 7th
harmonic, high-pass filter and 100V inverter with 1:1 series
transformer is used for series injection.

![Figure 2. SHAPF (a) power circuit configuration and (b) control circuit.](image)

Fig. 2(b) shows the block diagram of control circuit used
for reference voltage generation. Three phase current and
voltage are sensed and (6), (8) and (11) are evaluated for
calculating reference voltage. Table I shows the system
parameter values with which the simulation study is done in
PSIM software.

**TABLE I**

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Supply Voltage</td>
<td>440V, 50Hz (line-line)</td>
</tr>
<tr>
<td>2</td>
<td>Source Impedance</td>
<td>(R_s = 0.5 \text{ Ohm}, L_s = 0.1 \text{ mH})</td>
</tr>
<tr>
<td>3</td>
<td>DC Capacitor</td>
<td>1000 \text{ uF}</td>
</tr>
<tr>
<td>4</td>
<td>DC Link Voltage</td>
<td>100V</td>
</tr>
<tr>
<td>5</td>
<td>EMI Filter</td>
<td>(L_f = 1.35 \text{ mH}, C_f = 50 \text{ uF})</td>
</tr>
<tr>
<td>6</td>
<td>Tuned PPF (5th)</td>
<td>(L = 12.32 \text{ mH}, C = 32.88 \text{ uF})</td>
</tr>
<tr>
<td>7</td>
<td>Tuned PPF (7th)</td>
<td>(L = 6.29 \text{ mH}, C = 32.88 \text{ uF})</td>
</tr>
<tr>
<td>8</td>
<td>High Pass PPF</td>
<td>(L = 2.36 \text{ mH}, C = 29.88 \text{ uF}, R = 17.75)</td>
</tr>
<tr>
<td>9</td>
<td>Series Transformer</td>
<td>1:1</td>
</tr>
<tr>
<td>10</td>
<td>Load</td>
<td>Diode Rectifier supplying constant 20A DC current to load</td>
</tr>
<tr>
<td>11</td>
<td>Switching Frequency</td>
<td>20 kHz</td>
</tr>
</tbody>
</table>

**V. RESULTS OF SIMULATION**

The SHAPF model of section IV is tested for
compensating 22 kVA diode rectifier for harmonic
elimination. This load is drawing supply current having
21.83A fundamental, 4.2A 5th harmonic, 2.8A 7th harmonic,
1.67A 11th harmonic, 1.25A 13th harmonic, 0.82A 17th
harmonic, 0.64A 19th harmonic etc. The total harmonic
The performance of the system without any filter, with PPF and with SHAPF are simulated and compared.

Fig. 3 shows the load current and improved supply current waveform after connecting PPF bank. Fig. 4 shows the harmonic spectrum of these current waveforms. Since the PPF bank is made up of 5th harmonic tuned filter, 7th harmonic tuned filter and high pass filter, it is observed that 5th and 7th harmonics are completely removed and other harmonics are attenuated. The THD of supply current which is 29.46% before compensation is reduced to 13.60% after applying PPF and individual harmonic components are also reduced.

Fig. 5 shows the waveforms of load current, improved supply current and injected voltage waveforms when compensation is done with SHAPF. Fig. 6 shows the harmonic spectrum of load current and source current, it is observed that all the harmonics are considerably removed.

As tabulated in Table II, the THD of supply current which is 29.46% before compensation is reduced to 1.58% after applying SHAPF. It is also observed from the value of injected voltage that the required value of compensating voltage is small as compared to the system voltages.

Table II shows the overall performance of the system. The THD of supply current is 29.46% without compensation, which reduces to 13.60% after connecting PPF and which further reduces to 1.58% after connecting SHAPF. The required capacity of series APF in this system is calculated to be 1.75 kVA, while compensating non-linear load of 22 kVA, which is quite small compared to a similar rating shunt APF.

VI. CONCLUSIONS

In this paper, a novel control scheme based on decomposing voltage vector using Generalised Instantaneous Power Theory for SHAPF is presented. Formulation of decomposing different power quantities in terms of voltage components is presented. These components are directly associated with three phase instantaneous voltages and currents and are separated
A.1 Inverse of a vector

Considering current vector $\vec{i}$, its inverse $\vec{i}^{-1}$ into the geometric algebra framework, can be defined as follows:

$$\vec{i}^{-1} = \frac{\vec{i}^+}{\|\vec{i}\|^2} = \frac{\vec{i}^+}{\|\vec{i}\|^2} = \frac{\vec{i}}{\|\vec{i}\|^2}$$

where $\|\vec{i}\|$ is the instantaneous norm of vector $\vec{i}(t)$ and the fact that $\vec{i}^{-1} = \vec{i}$ has been considered [16].

A.2 Cross Product of Three Vectors

In the geometric algebra framework, the cross product of three vectors can be performed using expression,

$$\vec{a} \times \vec{b} \times \vec{c} = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$$

A.3 Conversion of Cross Product into Matrix Multiplication

In the geometric algebra framework, the conversion of this outer product multiplication to matrix multiplication is done using following relations.

$$\vec{a} \times \vec{b} = [\vec{a}]_0 \vec{b} = [\vec{b}]_0 \vec{a}$$

and if

$$\vec{a} = \vec{c} \times \vec{d}$$

$$[\vec{a}]_0 = (\vec{c} \cdot \vec{d}^T)^T - \vec{c} \cdot \vec{d}$$

REFERENCES


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