Proceedings of the World Congress on Engineering 2012 Vol II
WCE 2012, July 4 - 6, 2012, London, U.K.

A Simple Approach for Large Size Digital Off-Axis Hologram Reconstruction

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Abstract— **A new method of digital off-axis hologram reconstruction based on the Fresnel transform is proposed. A combination of composite filtering, Abbe's limitation, and digital lens formulae has been used with an appropriate handling of Fresnel impulse response propagator. A clear image of microscopic object is efficiently reconstructed from hologram by using a plane reference wave with involvement of electric field vector as well as by applying bi-cubic interpolation in the final reconstruction step. In particular, the proposed method automatically suppresses the zero order term and virtual image. Unlike other similar approaches, the image can be reconstructed with large size accordingly. The proposed method facilitate the transverse high resolution of microscopic image with better applicability than others approaches. Moreover, the advantages of the method are its simplicity and convenience in data processing.**

***Index Terms*— Digital Holography, Fresnel Transform, Microscopy, Lens Formula.**

I. INTRODUCTION

THE idea of recording and reconstructing holograms digitally was proposed in early studies by Goodman [1].

Digital holography is a novel imaging technique, which uses a charge-coupled device (CCD) camera for hologram recording and a numerical method for hologram reconstruction. With advances in computer performance and electronic image acquisition devices, digital holography has become more attractive for many applications. However, off-axis holographic configuration suffers from zero order and twin images. Essentially, due to symmetry motives resulting in blurring images when the reconstruction of a hologram object is viewed. In this regard, we analyze previous approaches [2] to [8], for numerical reconstruction of digital off-axis hologram with respect to the above mentioned technique. It has been observed that the spatial resolution of Digital Holographic (DH) is limited by the digital recording device, while the FOV of DH system is restricted by the limited resolution of CCD. Henceforth, the factors contributing to FOV have been investigated.

Manuscript received February 28, 2012; revised April 10, 2012.

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) (NRF-2011-0026245).

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In this paper, we propose an algorithm: the efficient Fresnel impulse response propagation for digital off-axis hologram (FIRP-DOAH) reconstruction method. It reconstructs the hologram image at larger distance and improves the FOV, which has been intrinsically lost by the application of the FIRP [4] and [5], [6]. It not only improves the reconstructed hologram image (RHI) size but also gives better resolution and higher accuracy at a larger distance. Furthermore, due to its simple experimental setup, this method has better resistance to conservational disturbances than other methods. The paper is organized as follows. Section II describes the theoretical expression of an efficient FIRP-DOAH reconstruction method in the cognitive of complementary conditions. Section III evaluates the quality assessment of RHI. Results of Algorithm employed to enlarge the FOV at large distance are discussed in Section IV. Finally, section V concludes the paper.

II. THEORETICAL EXPRESSION

Among the different techniques of digital recording of microscopic holograms; the most common Mach-Zehnder sketch is shown in Fig.1. It is based on the same foundations as optical holography [1].

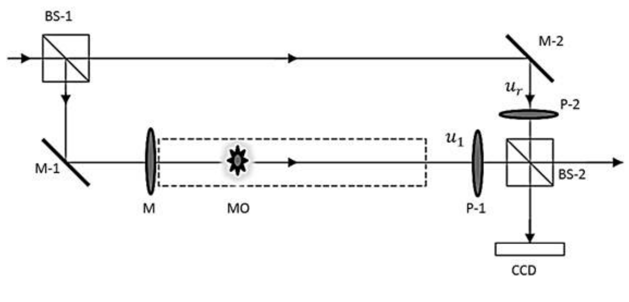

Fig.1 Setup used in the validation of proposed method

Fig. 1 shows the experimental setup used in digital thin lens formulae in validation of proposed method. A collimated and expanded plane wave is divided into two beams by the beam splitter BS-1. One of the beams serves as reference (u_r), and the other is object wave (u_1). Where, A microscopic (MO) collects the object wave (u_1) transmitted by the specimen with the help of beam splitters BS-2, whereas, magnified image of the specimen by using mirror (M) at a distance z behind the CCD. Whereas, two identical polarizer P-1 and P-2 are placed into the path of reference beam and object wave: along with two (beam, wave) turning mirror M-1 and M-2 respectively. As explained in [1], an object wave (u_1) emerging directly from the magnified image of the specimen and not from the specimen itself. In order to improve the sampling capacity of the CCD, a lens can be optionally introduced through a plane reference wave. At the exit of the interferometer, the

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

interference between the object wave and the reference wave creates the hologram intensity. Moreover, in order to achieve high-resolution imaging, the resolution of CCD imaging should be no lower than that of MO imaging. This is appropriately by choosing the recording parameters

Through the adjustment of the following complementary parameters in the proposed method in the first stages, it is possible to enlarge the size of the reconstructed images, at a larger distance.

A. Fresnel Number Calculation

The initial distance for propagating object and reference wave towards CCD is considered to be the same in FIRP-DOAH technique, and can be determined with a Fresnel number, described as

$$N_F = \frac{L^2}{\lambda z} \tag{1}$$

B. Fresnel-Kirchhoff Approximation

Under the condition of approximation of the Fresnel-Kirchhoff integral [1] the construction of a Fresnel hologram can be attained by letting the object wave to CCD plane with in the limit of Fresnel number by using Eq. (2):

$$u_2(\xi,\eta) = \frac{jA}{\lambda z}\exp\left[-j\frac{\pi}{\lambda z}\left(\xi^2+\eta^2\right)\right]\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_1(x,y)\times$$

$$\exp\left[-j\frac{\pi}{\lambda z}\left(\xi^2+\eta^2\right)\right]\times\exp\left[-j\frac{2\pi}{\lambda z}\left(x\xi+y\eta\right)\right]dxdy \tag{2}$$

where A is an amplitude constant and λ is the wavelength of wave propagation. Whereas, (u_2) is the diffracted object in complex wave. Whereas, human eyes cannot normally observe complex wave, so the interference of object wave with plane reference wave in the terms of light intensity is recorded on CCD plane.

C. Plane Reference Wave

A plane reference wave (u_r) and the propagated object (u_2) is made to interfere at $(\xi\ ,\eta)$ plane. The plane reference wave can be of any geometry as long as the sampling theorem is fulfilled. In our case, the plane reference wave at the CCD plane is given by Eq. (3)

$$u_r = A\exp\left[ik\left(\xi\cos\alpha+\eta\cos\beta\right)\right] \tag{3}$$

where k is the wave-number and A is the amplitude of constant values [4], which gives rise to a zero order in the FIRP-DOAH reconstruction. To minimize the effect of zero order, we use the composite filter stated in Eq. (12) on hologram recorded intensity. Moreover, the plane reference waves are electromagnetic fields of real functions in the spatial domain. Consequently, the Eq. (3) shows that the electric field does not only provide the part of real object hologram in large, but also reduces the computational time. Hence, we use a very small angle in an off-axis scheme less than 0.0000001^o along vertical direction due to key involvement of electric field vector [7]. Plane reference wave travels along the z-axis $\theta(z=0)=0$, such that the sum of angles α and β should be 90° .

By knowing the recording distances of an object and a plane reference wave, the hologram recording is nothing less than the digitized version of the wave fields that impinge on the CCD surface. Hence, reconstruction becomes more accurate for larger distances when the CCD size is larger than the size of the object aperture. Sampling interval (SI) increases in CCD plane and decreases the resolution intensity. In our proposed method, the limitations regarding the increase in CCD size has been adjusted according to Abbe's theory.

D. Digital Thin Lens Formulae

For determination of reconstruction distance in our proposed FIRP-DOAH method, we supposed the CCD plane as a digital thin lens. The choice of (z_1) in practice is dictated by the required focusing of the real image using digital lens formulae in an appropriate way [4].

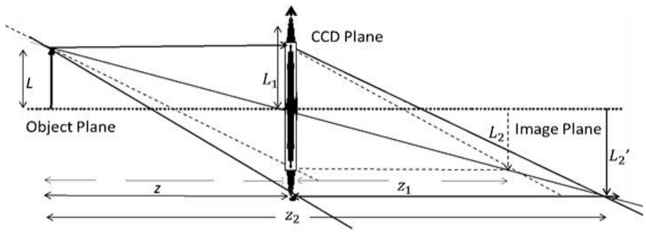

Fig.2. Digital thin lens formulae for the reconstruction distance

Fig. 2 shows the magnification of image plane. This setup is best suited for highly specular micro-size objects [4]. The spatial resolution of CCD plane is expressed with total distance, $z_2=z_1+z$ from object to image plane. The relation of these factors with spatial resolution are investigated and presented in [4] and [6]. Consequently, the spatial resolution in the reconstructed plane can be amended by increasing the CCD aperture size. CCD size is increased relative to object for attaining the desired reconstructed image at larger distance, (z_1) and can be calculated as in Eq. (4). However, this method may lead to under-sampling.

$$z_1 = z\left(\frac{L_1}{L-L_1}\right) \tag{4}$$

E. Abbe's Limitation for Image Formation

Object and plane reference waves approaching at the CCD at a specific angle on the base of optical system converge at points consistent to diffraction peaks [1] and [6]. The optical resolution of reconstructed image is firm by the numerical aperture (N.A) of the object wave confined in the CCD plane. Though, relation $L<L_1<L_2$ corresponding to $z<z_1$ expressed as in Eq. (5) and Eq. (6):

$$Mag = \frac{L_1}{L} = \frac{z_1}{z} = \frac{M_1}{M} \tag{5}$$

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

$$N.A. = \sin\theta \approx \frac{L}{z} \qquad (6)$$

The reconstruction distance (z_1) is required to focus a reconstructed hologram image (RHI).

F. Nyquist Shannon Theorem

Nyquist rate is well known as the lower limit for image sampling, which avoids aliasing. However, to obtain RHI, only a finite number of samples are taken at the Image plane, whereas according to the theorem, Nyquist frequency must be twice the image bandwidth in order to avoid the loss of information [4]. The Nyquist Shannon theorem must be followed for getting an appropriate level of up-sampling. Thus, the composite filtering at CCD plane for enhancing detail and h_s impulse response function for Ideal band-limited interpolation by using Eq. (7) at RHI plane is attained.

$$h_s = sinc\left(\frac{L\xi}{\lambda z}\right) \qquad (7)$$

G. Composite Filtering

In an efficient FIRP-DOAH reconstruction method, the recording device is a CCD camera array. A hologram recorded by a CCD is nothing less than digitized version of the wave fields that impinge on the CCD surface, which is determined by

$$I_{(x.y)} = |u_2|^2 + |u_r|^2 + u_2^* u_r + u_2 u_r^* . \qquad (8)$$

Two of the last terms of the above equation are directly proportional to the intensity distribution $I_{(x,y)}$ from the object wave and reference wave alone. However, only one of these terms, either the real or the virtual image, is generally subject to our interest. In our method, we suppress the virtual and enhance the real term by a high pass filter before the reconstruction process. Its application is helpful to understand what's going on behind or depth in an object details and ultimately reduce the zero order effect at the RHI. In this way eliminating the zero-order diffraction is much more convenient and faster than with other methods. The RHI analysis starts with a calculation of the input and output impedances and the impulse response function. The impedance is one of the main motivations for using the composite filters [6] because of the requirement for better matching of the original object resolution level to the RHI resolution. This is a type of point operation based on a simple approach. Point operations are one to one mapping between object and recorded intensity of hologram where each pixel is independently adjusted and the average value of the surrounding pixel is utilized for anti-aliasing effect. A composite filter being used for suppressing the zero order term and enhancing the information detail of recorded intensity of hologram is given as

$$I_h(x,y) = T\left[I(x,y)\right], \qquad (9)$$

where $I_h(x,y)$ denotes the row and column index of hologram recorded intensity matrix after composite filtering. For a monochrome object, the general formulae of composite filtering can be presented as in Eq. (12):

$$I_h(x,y) = 1 - I(x,y), \qquad (10)$$

$$I_h(x,y) = \begin{pmatrix} 1-I_{x,,x} & 1-I_{x,y} \\ 1-I_{y,x} & 1-I_{y,y} \end{pmatrix}, \qquad (11)$$

$$I_h(x,y) = |u_2|_h^2 + |u_r|_h^2 + \left[u_2^* u_r\right]_h + \left[u_2 u_r^*\right]_h . \qquad (12)$$

After treatment of composite filtering, we get the recorded intensity with improved data rate and proceeded for impinging the same plane reference wave again for reconstruction of hologram image.

H. FIRP-DOAH Reconstruction Model

The reconstructed hologram information of the object is retrieved by impinging the same plane reference wave on the resultant (composite filtering processed) hologram recorded intensity I_h shown in Eq. (12). Henceforward, the plane reference wave from Eq. (3) becomes the reconstruction wave and the reconstructed matrix [2] becomes; $u_3 = u_r I_h$,

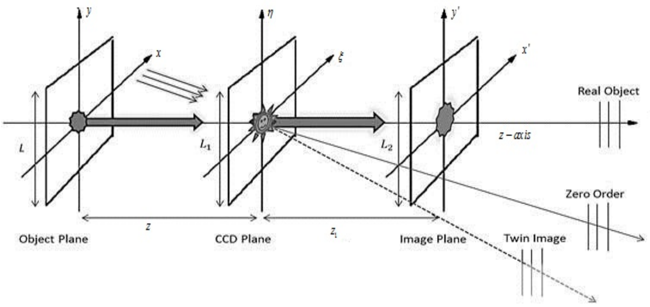

Fig.3. Hologram recording and reconstruction

$$u_3 = u_r |u_2|_h^2 + u_r |u_r|_h^2 + u_r\left[u_2^* u_r\right]_h + u_r\left[u_2 u_r^*\right]_h . \qquad (13)$$

As with classical holography, the reconstructed wave front contains three different terms: a real image, a virtual image and a zero order of diffraction. These terms can be observed separately as a consequence of the FIRP-DOAH geometry. The object embedded in (u_3) matrix given in Eq. (12) is reconstructed by using FIRP-DOAH method at longer distance as shown in Fig. 2 [4].

We noticed three parts in a basic synoptic of the proposed algorithm as given in Fig. 3. The spatial frequencies corresponding to the object image are located in the center of it, and the other two parts are the spatial frequencies corresponding to the zero order. Hence, the real image is shifted away at a larger distance due to the special adjustment of the plane reference wave and the reconstruction distance (z_1) by using a digital lens formula Eq. (4) . The real part of normalized hologram is obtained in the constraint of Shannon's Nyquist theorem discussed in section II. *F.* The spatial resolution of FIRP-DOAH system is limited by pixel averaging: finite CCD plane size, SI and object magnitude.

All these are investigated in different studies and presented in [1], [2], [3] and [5]. The magnified FOV can be obtained by applying a bicubic interpolation technique on RHI is explained in Eq. (20).

I. Holographic Interpolation Technique

Here we use the interpolation technique for fast adjustment of pixel size in the reconstructed image at the rate of the SI of an original object .The SI of an object and a CCD plane are given by Eq. (14) and (15)

$$dx = \frac{L}{M} , \tag{14}$$

$$d\xi = \frac{\lambda z}{M dx} . \tag{15}$$

In the FIRP-DOAH proposed method, the size of CCD is large compare to the object plane. For this reason, the SI is increased at the CCD plane. If the CCD array has different SI in ξ and η directions, or if the extent of the object is different in x and y directions, then there is an optimal manipulation of the pixel array since both the zero orders and twin images are reconstructed simultaneously. As a consequence, we observed the enlargement of object size accordingly.

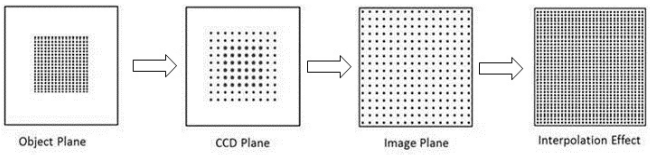

Fig.4.Object expansion in FIRP-DOAH method

If the SI in RHI is expanded into a larger area, as shown above in Fig. 4, the image gets blurred and the perceived sharpness decreases in the image plane because the RHI on the base of Fresnel transform method is always affected by aliasing. Hoever, the aliasing can be reduced by decreasing the SI or by increasing the number of pixels. In this study, we have developed an algorithm that allows us to arbitrarily change in SI of RHI of large size with standard resolution level (resolution level of an object) [9]:

$$M_1 = \frac{\lambda z}{d\xi dx} . \tag{16}$$

It shows by Nyquist criterion, that recording SI of CCD plane ($d\xi$, $d\eta$), needs a padding operation to get the increased number of pixels from $N \times M$ to $N_1 \times M_1$ by employing Eq. (15). Therefore, by decreasing the SI of CCD plane equal to SI of an object plane, the desired resolution of RHI plane can also be achieved. We verify the spatial resolution, by using the Abbe's theorem explained in Eq. (5) and Eq. (6). After applying interpolation technique on RHI, we verified our result for Nyquist theorem stated in Eq. (7). In order to obtain the standard number of pixels in RHI, we calculated by

$$M_2 = M_1 \left(\frac{z_2}{z_1} \right) = \frac{\lambda z}{d\xi\, dx} \left(\frac{z_2}{z_1} \right) , \tag{17}$$

where, M_2 is the number of pixels in RHI plane. From Eq. (17) it is clear that the resolution, and consequently the reconstruction pixel area of RHI, depends on the wavelength, distance, number of the pixels $N \times M$ and $N_1 \times M_1$ of object and CCD array and their physical size. The size of RHI which has a reduced spatial lateral resolution for higher reconstruction distances can be reconstructed seamlessly using an appropriate interpolation technique with standard resolution level according to Abbe's theorem Eq. (5) and Eq. (6):

$$L_2 = M_2 \left(\frac{L}{M} \right) , \tag{18}$$

Thus, an image which is spread over a larger area by using FIRP-DOAH is interpolated and its perceived sharpness arises by filling the empty space around the pixels. This implies that higher number of pixels are required for RHI formation [4] to [7], to get the anticipated *SI (dx',dy')* in the RHI plane *(x',y')*.

$$dx' = \frac{L_2}{M_2} , \tag{19}$$

Hence, images on prime-resolution shown in Eq (19), required the standard mumber of pixels (number of pixels of an object). By using the above analysis, we get the prime resolution of spreading spectrum of Fresnel transform for large area by decreasing SI unto the standard SI of an original object. That is simply achieved by equalizing the Eq. (13) and Eq. (20).

$$dx \approx dx' \tag{20}$$

J. Discussion

It is important that, even if the size of RHI can be improved by zero padding on the digital hologram, the corresponding FOV remains unaffected. In fact, it depends on the real intensity of hologram system by using the generalized plane reference wave Eq. (3) and composite filter Eq. (12) with appropriate reconstruction distance Eq. (4). Even though the padding operation allows only a fictitious enlargement of the aperture of the digital hologram, it is the *sinc* function that coincides with the average speckle size that was calculated in [4] for digital holography with Fresnel reconstruction [1] to [7]. The dc terms do not affect the performance of digital holography since it can be effectively eliminated. Hence, the reason behind the increasing number of pixels is to get large size of hologram. The magnified FOV can be obtained by applying a bicubic interpolation technique on RHI [4] to [7]. The reconstruction process shows that it is possible to control image parameters like focus distance, image size, and image resolution [8]. From Eq. (20), it is clear that the RHI is enlarged or contracted according to the reconstruction distance and that the size of the SI depends on the lateral number of the object standard pixels: $N \times M$.

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

III. IMAGE QUALITY ASSESSMENT

A modified peak signal to noise ratio (PSNR) technique [8] has been used to assess the image quality reconstructed by the proposed FIRP-DOAH method. It is most easily defined through mean-squared error (MSE) shown in Eq. (21):

$$MSE = \frac{1}{MN}\sum_{k=1}^{M}\sum_{l=1}^{N}\left\|u(x,y)-\lambda v(x',y')\right\|^2 , \qquad (21)$$

where λ is introduced for reducing MSE to minimum value, which is defined as

$$\lambda = \frac{\sum_{k=1}^{M}\sum_{l=1}^{N}\left\|u(x,y).v^*(x',y')\right\|}{\sum_{k=1}^{M}\sum_{l=1}^{N}\left|v(x',y')\right|^2} . \qquad (22)$$

where k, l, m and n are integers with ranges to maximum value of M (height of image) and N (width of image). Hence, the performance of PSNR is analyzed by

$$PSNR = 10\times\log_{10}\left(\frac{MAX_I^2}{MSE}\right) . \qquad (23)$$

The bigger the value of PSNR: better the image quality. Moreover, different reconstruction distances have different PSNR values. This result is accordance with subjective assessment on observational plane with respect to large scale of RHI.

IV. SIMULATION

To prove the veracity of the proposed method, simulations in Matlab were done. The simulated object is a black-and-white film, as shown in Fig. 7 and Fig. 8. The holograms recorded at a distance of 35 cm on a CCD camera with 256 x 256 pixels were illuminated with a coherent light of the 633 nm wavelength [7] to [9]. CCD is assumed to be a thin lens. CCD size is increased relative to object size for attaining the desired RHI size at a larger distance. The side length of square CCD is shown by L_1, whereas the side length of square reconstructed in the hologram image is expressed by L_2. From Eq. (14) to Eq. (20), it has been observed that the SI increases with the reconstruction distance so that the RHI, in terms of resolution, is reduced for a longer distance. For this reason, we would increase the number of pixels to 574×574 in an image by using bicubic interpolation technique to make large size (1.12 cm) hologram. Therefore, well-focused RHI is obtained at larger distance of 91.26 cm. The computer specification which has been used for the experiments are as follows: core i-3 processor with 4 GB RAM.

The technique provides high embedding capacities, allows complete recovery of the original object, and introduces only a small distortion between the object and image bearing the embedded data. In such scenarios, the proposed FIRP-DOAH method has significant advantages over conventional augmenting holographic techniques because it offers finer FOV at a larger distance.

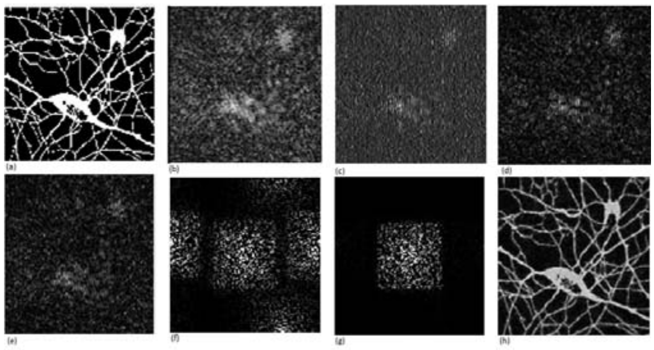

Fig. 11. Proposed simulation stages for test target object 2 (a).Original object (b).Propagated image (c).Hologram intensity (d).Filtering intensity (e).hologram reconstruction (f).FFT of reconstructed hologram (g).FFT of propagated hologram (h).Hologram reconstructed image.

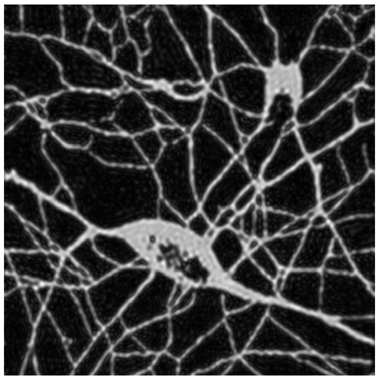

Fig.8. Standard Resolution of an Image

TABLE I
PSNR DATA FOR LARGE SIZE DOAH

Target	Object 1	Object 2	Object 3
PSNR	52.14dB	50.42dB	52.43dB

The results are shown in Fig.7 to Fig. 12 with calculated PSNR value by Eq. (23). Actually, $N\times M$ can be augmented by padding the matrix of the hologram with zeros in both the horizontal and vertical directions such that the net reconstruction image is of large size. The SI of RHI is the same as the SI of the original object. Our future work will be to extend the work of this area by using theory of wavelet. In order to improve the performance of the extracted algorithms, wavelet can be redesigned as a fast simulation algorithm to produce high resolution of large size image and efficient computing in terms of a reduction in processed time.

V. CONCLUSION

A simple method has been developed for enlarging the size of the reconstructed image at a larger distance in digital holography. Our proposed method has the advantage of sweeping out the dc-term from the reconstructed hologram. The results of the simulation are also found to be efficient in terms of computation time achieving half that of previous approaches. The application of this method not only provides double the enlargement of the image size but also produces better resolution and higher accuracy even at a larger distance.

ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

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ISBN: 978-988-19252-1-3
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)