One Dimensional Generalized Magnetothermoelastic Problem : For a Half-Space

B. Das *and A. Lahiri†

Abstract—**The problem of a magnetothermoelastic interaction of homogeneous and isotropic half-space is considered under magnetic and electric field. Laplace transform for time variable is used and the resulting equations are written in the form of Vector-matrix differential equation. To get the solution in transformed domain, we apply the method of eigenvalue approach. The inversion of Laplace transform is carried out numerically by Bellman method. Finally numerical computation have been done for the expressions of displacement, temperature, stresses and induced magnetic field and also several figures are presented for their comparison.**

Keywords: Eigenvalue Approach, Electromagnetism, Generalized Magnetothermoelasticity, Isotropic, Laplace Transform and Vector-matrix Differential Equation.

1 Introduction

The classical uncoupled and coupled theory of thermoelasticity predicts two phenomena not compatible with physical experiments. The governing equations for displacement and temperature fields in the linear dynamical theory of classical thermoelasticity consists of the coupled partial differential equation of motion and Fourier's law of heat conduction equation. Heat conduction equation does not contain any elastic terms contrary to the fact that the elastic changes produced heat effects. The equation for displacement field is governed by a wave type hyperbolic equation and the temperature field is a diffusion type parabolic equation. The classical theory of thermoelasticity predicts a finite speed for predominantly elastic disturbances but an infinite speed for predominantly thermal disturbances that are coupled together.
At present there are two different theories of the generalized thermoelasticity, the first was studied by Lord and Shulman [1] who obtained a wave type heat equation by postulating a new law of heat equation to replace the classical Fourier's law. This new law contains the heat flux vector as well as time derivative. It also contains a new constant that acts as a relaxation time. Since the heat equation of this theory is of the wave type, it automatically ensures finite speed of propagation for heat wave. The second type of generalization to the coupled theory of elasticity was developed by Green and Lindsay [2]. This theory contains two constants that act as relaxation times and modify not only heat conduction equation but also equation of motion. This theory also known as temperature-rate-dependent thermoelasticity(TRDTE).
The studies of the magneto-thermo-elasticity theory originated before 1960's. The problem of interaction between electromagnetic field, temperature, stresses and strains in a thermo-elastic solid is relevant for a number of applications such as geophysics for understanding the effect of the Earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emissions of electromagnetic radiations from nuclear devices, development of a highly sensitive super conducting magnetometer,electrical power engineering, optics etc.
In two dimensional problems due to mathematical difficulties, numerical techniques have been used. The problem for a half-space was considered by Sherief and Helmy [3]using Laplace and Fourier transforms. Othman [4] applied normal mode analysis to the plane wave problem for a medium of infinite conductivity Extension of magnetothermoelasticity problems in generalized theory are found in the works of many researchers out of which Aboel-nour [5] Santwana Mukhopadhyay [6] are worth mentioning. Recently, Othman and Sang [7]investigates the effect of rotation of magnetothermoelastic problems.
In this paper, we consider a one-dimensional problem for a half-space of generalized thermoelastic medium under a magnetic field with constant intensity and electric field. Then the problem has been solved in Laplace transform domain by eigenvalue approach which is proposed by Lahiri [8]. The inversion of Laplace transform is carried out numerically applying Bellman [9] method and computations have been made numerically and presented graphically.

Nomenclature

λ, μ= Lamè constants. $\mathbf{u}$ = Displacement vector. T= Absolute temperature. T_0 = Reference temperature cho-

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sen such that $|\frac{T-T_0}{T_0}| << 1$. ρ = Density of the medium. γ = Material Constant = $(3\lambda + 2\mu)\alpha_T$ α_T = Coefficient of linear thermal expansion. C_E = Specific heat at constant strain. τ_0 = Thermal relaxation time parameter. t = Time variable. x = Distance x from the bounding plane. **h** = Induced magnetic field vector. **E**= Induced electric field vector. **J** = Electric current density. **B** = Magnetic induction vector. **D** = Electric induction vector. ε_0 = Electric permeability. μ_0 = Magnetic permeability. σ_0 = Electric conductivity. k = Coefficient of thermal conductivity. $H(t)$ = Heaviside unit step function.

2 Basic Equations and Formulation of the Problem

We now consider a perfectly isotropic,homogeneous and thermoelastic half-space ($x \geq 0$) under a magnetic field with constant intensity H_0 acts tangentially to the bounding plane and an induced electric field **E**. The electric intensity vector is normal to both the magnetic intensity and also displacement vectors. We also assume that both **h** and **E** are small in magnitude in accordance with the assumption of the linear theory of thermoelasticity and the initial magnetic field components are $(0, 0, H_0)$, thus the displacement components have the form

$$u_x = u(x,t), \quad u_y = u_z = 0 \tag{1}$$

The electromagnetic quantities satisfy Maxwell's equations

$$curl\ \mathbf{h} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}\ ;\ curl\ \mathbf{E} = -\mu_0 \frac{\partial \mathbf{h}}{\partial t}\ ;\ \mathbf{D} = \varepsilon_0 \mathbf{E}$$
$$div\ \mathbf{h} = 0\ ;\ div\ \mathbf{E} = 0\ ;\ \mathbf{B} = \mu_0(\mathbf{H}_0 + \mathbf{h}) \tag{2}$$

The above field equation are supplemented by constitutive equation which consists Ohm's law

$$\mathbf{J} = \sigma_0(\mathbf{E} + \mu_0 \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{H}_0) \tag{3}$$

And the equation for the Lorentz force is

$$\mathbf{F} = \mathbf{J} \times \mathbf{B} \tag{4}$$

The constitutive stress components are given by the Hooke-Duhamel-Neumann law[1]

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \gamma(T - T_0)\delta_{ij} \tag{5}$$

where $e = div\mathbf{u}$ is the cubical dilatation, δ_{ij} is Kronecker's delta tensor and e_{ij} is the strain tensor whose components are given by

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{6}$$

Equation of motion are given by

$$\sigma_{ji,j} + F_i = \rho \ddot{u}_i \tag{7}$$

Equation (6) gives the strain components like

$$e_{xx} = \frac{\partial u}{\partial x}\ ,\ e_{yy} = e_{zz} = e_{xz} = e_{yz} = e_{xy} = 0 \tag{8}$$

Equation (5) also gives the stress components

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma(T - T_0)$$
$$\sigma_{yy} = \sigma_{zz} = \lambda \frac{\partial u}{\partial x} - \gamma(T - T_0)$$
$$\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0 \tag{9}$$

After linearization, ohm's law transformed to

$$J = \sigma_0(E + \mu_0 H_0 \frac{\partial u}{\partial t}) \tag{10}$$

Substituting from equation (10) into equation (4),we get

$$F_x = -\mu_0\sigma_0 H_0(E + \mu_0 H_0 \frac{\partial u}{\partial t}), F_y = F_z = 0 \tag{11}$$

With the help of equation (7),(9) and (11),we get the equation of motion as follows-

$$\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x}$$
$$-\mu_0\sigma_0 H_0(E + \mu_0 H_0 \frac{\partial u}{\partial t}) \tag{12}$$

And now the heat conduction equation is

$$k\frac{\partial^2 T}{\partial x^2} = (\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2})(\rho c_E T + \gamma T_0 \frac{\partial u}{\partial x}) \tag{13}$$

From Maxwell's equation and Ohm's law i.e.,equation (2) and (10), we obtain

$$\frac{\partial h}{\partial x} = \varepsilon_0 \frac{\partial E}{\partial t} + \sigma_0(E + \mu_0 H_0 \frac{\partial u}{\partial t}) \tag{14}$$

And also Maxwell's equation i.e.,equation (2) gives

$$\frac{\partial E}{\partial x} = \mu_0 \frac{\partial h}{\partial t} \tag{15}$$

Introducing the non-dimensional variables such as

$$x^* = c_0\eta x\ ,\ u^* = c_0\eta u\ ,\ t^* = c_0^2\eta t\ ,$$
$$\tau_0^* = c_0^2\eta\tau_0\ ,\ \theta^* = \frac{\gamma(T - T_0)}{\lambda + 2\mu}\ ,\ h^* = \frac{\eta}{\sigma_0\mu_0 H_0}h\ ,$$
$$E^* = \frac{\eta}{\sigma_0\mu_0^2 H_0 c_0}E\ ,\ \sigma_{ij}^* = \frac{\sigma_{ij}}{\lambda + 2\mu} \tag{16}$$

Dropping the asterisks, equations (12)-(15) becomes

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \frac{\partial \theta}{\partial x} - \nu\varepsilon_2\left(\nu E + \frac{\partial u}{\partial t}\right) \tag{17}$$

$$\frac{\partial^2 \theta}{\partial x^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\theta + \varepsilon_1 \frac{\partial u}{\partial x}\right) \tag{18}$$

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$$\frac{\partial h}{\partial x} - V^2 \frac{\partial E}{\partial t} = \nu E + \frac{\partial u}{\partial t} \ ; \ \frac{\partial E}{\partial x} = \frac{\partial h}{\partial t} \tag{19}$$

And the non-dimensional stress-components are

$$\sigma_{xx} = \frac{\partial u}{\partial x} - \theta \tag{20}$$

$$\sigma_{yy} = \sigma_{zz} = (1 - \frac{2}{\beta^2})\frac{\partial u}{\partial x} - \theta \tag{21}$$

where

$$\eta = \frac{\rho c_E}{k} \ , \ c_0^2 = \frac{\lambda + 2\mu}{\rho} \ , \ \nu = \frac{\sigma_0 \mu_0}{\eta} \ , \ \varepsilon_1 = \frac{T_0 \gamma^2}{c_E \rho^2 c_0^2} \ ,$$
$$\varepsilon_2 = \frac{\mu_0 H_0^2}{\lambda + 2\mu} \ , \ V = \frac{c_0}{c} \ , \ c^2 = \frac{1}{\varepsilon_0 \mu_0} \ , \ \beta^2 = \frac{\lambda + 2\mu}{\mu} \tag{22}$$

3 Solution Procedure

Formulation of the Vector-matrix Differential Equation

We now apply the Laplace transform defined by

$$\bar{f}(s) = \int_0^\infty f(t) e^{-st} dt \tag{23}$$

to the equation (17)-(21),we get

$$\frac{d^2 \bar{u}}{dx^2} = \frac{d\bar{\theta}}{dx} + \varepsilon_2 \nu^2 \bar{E} + (s^2 + \varepsilon_2 \nu s)\bar{u} \tag{24}$$

$$\frac{d^2 \bar{\theta}}{dx^2} = (s + \tau_0 s^2)(\bar{\theta} + \varepsilon_1 \frac{d\bar{u}}{dx}) \tag{25}$$

$$\frac{d\bar{h}}{dx} = (\nu + sV^2)\bar{E} + s\bar{u} \ ; \ \frac{d\bar{E}}{dx} = s\bar{h} \tag{26}$$

$$\bar{\sigma}_{xx} = \frac{d\bar{u}}{dx} - \bar{\theta} \ ; \ \bar{\sigma}_{yy} = (1 - \frac{2}{\beta^2})\frac{d\bar{u}}{dx} - \bar{\theta} \tag{27}$$

Eliminating $\bar{E}$, we get the system of equations

$$\frac{d^2 \bar{u}}{dx^2} = \left(s^2 + \frac{\nu \varepsilon_2 s^2 V^2}{\nu + sV^2} \right) \bar{u} + \frac{d\bar{\theta}}{dx} + \frac{\varepsilon_2 \nu^2}{\nu + sV^2} \frac{d\bar{h}}{dx} \tag{28}$$

$$\frac{d^2 \bar{\theta}}{dx^2} = \varepsilon_1 \left(s + \tau_0 s^2 \right) \frac{d\bar{u}}{dx} + \left(s + \tau_0 s^2 \right) \bar{\theta} \tag{29}$$

$$\frac{d^2 \bar{h}}{dx^2} = s\frac{d\bar{u}}{dx} + s\left(\nu + sV^2 \right) \bar{h} \tag{30}$$

Initially i.e., at time t=0, the displacement component,magnetic and electric intensity along with their derivative with respect to t are zero and maintained at the reference temperature T_0, so the following initial conditions hold.

$$u(x,0) = \frac{\partial u(x,0)}{\partial t} = 0 \ ; \quad \frac{\partial T(x,0)}{\partial t} = 0 \ ;$$
$$\frac{\partial h(x,0)}{\partial t} = 0 \ ; \frac{\partial E(x,0)}{\partial t} = 0 \ ; \quad T(x,0) = T_0 \tag{31}$$

As in Lahiri [8], equations (28)-(30) can be written as -

$$\frac{d\underline{V}}{dx} = \underline{A} \ \underline{V} \tag{32}$$

Where

$$\underline{V} = \begin{bmatrix} \bar{u} & \bar{\theta} & \bar{h} & \frac{d\bar{u}}{dx} & \frac{d\bar{\theta}}{dx} & \frac{d\bar{h}}{dx} \end{bmatrix}^T \tag{33}$$

And the matrix $\underline{A}$ is given by

$$\underline{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ C_{41} & 0 & 0 & 0 & 1 & C_{46} \\ 0 & C_{52} & 0 & C_{54} & 0 & 0 \\ 0 & 0 & C_{63} & C_{64} & 0 & 0 \end{bmatrix} \tag{34}$$

$$C_{41} = s^2 + \frac{\nu \varepsilon_2 s^2 V^2}{\nu + sV^2} \quad ; \quad C_{46} = \frac{\varepsilon_2 \nu^2}{\nu + sV^2} \quad ;$$
$$C_{52} = s + \tau_0 s^2 \quad ; \quad C_{54} = \varepsilon_1 (s + \tau_0 s^2) \quad ;$$
$$C_{63} = s(\nu + sV^2) \quad ; \quad C_{64} = s \tag{35}$$

Solution of the Vector-matrix Differential Equation

For the solution of the Vector-matrix differential equation(32), we apply the method of eigenvalue approach as in Lahiri [8]

The characteristic equation corresponding to the matrix $\underline{A}$ is given by

$$\lambda^6 - a\lambda^4 + b\lambda^2 - c = 0 \tag{36}$$

Where

$$a = C_{41} + C_{52} + C_{54} + C_{63} + C_{46}C_{64}$$
$$b = C_{41}C_{52} + C_{41}C_{63} + C_{52}C_{63} + C_{63}C_{54} +$$
$$C_{46}C_{52}C_{64} \ ; \ c = C_{41}C_{52}C_{63} \tag{37}$$

The roots of the characteristic equation (36) are also the eigenvalues of the matrix $\underline{A}$ which are of the form $\lambda = \pm\lambda_i \ ; i = 1(1)3$, where

$$\lambda_1^2 = \frac{2pSinq + a}{3} \ ; \ \lambda_2^2 = -\frac{p(\sqrt{3}Cosq + Sinq) - a}{3}$$
$$\lambda_3^2 = \frac{p(\sqrt{3}Cosq - Sinq) + a}{3} \tag{38}$$

and

$$p = \sqrt{a^2 - 3b}, q = \frac{Sin^{-1} r}{3}, r = -\frac{2a^3 - 9ab + 27c}{2p^3} \tag{39}$$

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The right eigenvectors $\underline{X}$ of the matrix $\underline{A}$ corresponding to the eigenvalue λ are given as-

$$\underline{X} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \tag{40}$$

The components x_i of $\underline{X}(i = 1(1)6)$ can be calculated from the relation

$$\underline{A}\ \underline{X} = \lambda\ \underline{X} \tag{41}$$

Henceforth, we shall denote them as the following:

$$X_i = \begin{cases} [X]_{\lambda=\lambda_i}\ when\ i = (\frac{j+1}{2}) & , j = 1(2)5 \\ [X]_{\lambda=-\lambda_i}\ when\ i = (\frac{j}{2}) & , j = 2(2)6 \end{cases} \tag{42}$$

Considering the regularity condition at infinity, as in Lahiri [8], the general solution of equation (32) can be written as-

$$\underline{V}(x) = \sum_{i=1}^{3} A_i e^{-\lambda_i x} \tag{43}$$

Then the solutions in Laplace transform domain are as follows-

$$\bar{u} = \sum_{i=1}^{3} a_i(s) e^{-\lambda_i x}\ ;\ \bar{\theta} = \sum_{i=1}^{3} b_i(s) e^{-\lambda_i x}$$
$$\bar{h} = \sum_{i=1}^{3} c_i(s) e^{-\lambda_i x}\ ;\ \bar{E} = \sum_{i=1}^{3} d_i(s) e^{-\lambda_i x} \tag{44}$$

Equation(27) gives the stress components in transformed domain

$$\bar{\sigma}_{xx} = -\sum_{i=1}^{3} [\lambda_i a_i + b_i] e^{-\lambda_i x}$$
$$\bar{\sigma}_{yy} = -\sum_{i=1}^{3} [(1 - \frac{2}{\beta^2})\lambda_i a_i + b_i] e^{-\lambda_i x} \tag{45}$$

From the boundary conditions, we can calculate the arbitrary parameters like a_i's, b_i's, c_i's and d_i's(i= 1(1)3) where

$$a_i = A_i(C_{52} - \lambda_i^2)(C_{63} - \lambda_i^2)\ ;$$
$$b_i = A_i \lambda_i C_{54}(C_{63} - \lambda_i^2) = m_{2i} a_i;$$
$$c_i = A_i \lambda_i C_{64}(C_{52} - \lambda_i^2) = N_{3i} a_i\ ;$$
$$d_i = -\frac{(\lambda_i N_{3i} + s) a_i}{\nu + sV^2}; m_{2i} = \frac{\lambda_i C_{54}}{C_{52} - \lambda_i^2}\ ;$$
$$m_{3i} = \frac{\lambda_i C_{64}}{C_{63} - \lambda_i^2}.(\frac{\delta + V\lambda_i}{\delta}); N_{3i} = \frac{\lambda_i C_{64}}{C_{63} - \lambda_i^2} \tag{46}$$

4 Boundary Conditions

Following two different cases of boundary conditions are studied for calculating the arbitrary parameters. We'll use the boundary conditions in non-dimensional form and the surface of the half-space at x=0.

Case - I
1.Thermal Boundary Condition: The surface of the half-space $x = 0$is subjected to a time dependent thermal shock, i.e,

$$\theta(0,t) = f(t) \tag{47}$$

where $f(t) = \theta_0 H(t)$ and θ_0 is a constant temperature.
2.Mechanical Boundary Condition: The surface x=0 is laid on a rigid foundation, i.e,

$$u(0,t) = 0 \tag{48}$$

3.Electromagnetic Boundary Condition: The electromagnetic quantities h and E must satisfy the following continuity conditions:

$$h = h_0 \quad and \quad E = E_0 \quad at \quad x = 0 \tag{49}$$

where h_0 and E_0 are the magnetic and electric intensities in free space, respectively.
Case - II
1.Thermal Boundary Condition: The surface of the half-space $x = 0$is subjected to a time dependent thermal shock, i.e,

$$\theta(0,t) = f(t) \tag{50}$$

2.Mechanical Boundary Condition: The surface of the half-space x=0 is traction free, i.e,

$$\sigma_{xx}(0,t) = 0 \tag{51}$$

3.Electromagnetic Boundary Condition: The electromagnetic quantities h and E are continuous across the surface of the half-space, i.e,

$$h = h_0 \quad and \quad E = E_0 \quad at \quad x = 0 \tag{52}$$

Using equation (23), we get the transformed boundary condition as follows:-
Case - I
Equation (47) transformed to -

$$\bar{\theta}(0,s) = \bar{f}(s) \tag{53}$$

where $\bar{f}(s) = \frac{\theta_0}{s}$
Equation (48) transformed to -

$$\bar{u}(0,s) = 0 \tag{54}$$

and equation (49) becomes

$$\bar{h} = \bar{h}_0 \quad and \quad \bar{E} = \bar{E}_0 \quad at \quad x = 0 \tag{55}$$

With the help of this boundary conditions (53) - (55) and equation (44), the constants $a_i's(i = 1,2,3)$ can be determined as -

$$a_1 = \frac{(m_{32} - m_{33})\bar{f}(s)}{\Delta}, a_2 = \frac{(m_{33} - m_{31})\bar{f}(s)}{\Delta}$$
$$a_3 = \frac{(m_{31} - m_{32})\bar{f}(s)}{\Delta} \tag{56}$$

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where $\Delta = (m_{32} - m_{33})m_{21} + (m_{33} - m_{31})m_{22} + (m_{31} - m_{32})m_{23}$

Case - II

Equation (50) transformed to -

$$\bar{\theta}(0, s) = \bar{f}(s) \tag{57}$$

Equation (51) transformed to -

$$\bar{\sigma}_{xx}(0, s) = 0 \tag{58}$$

and equation (52) becomes

$$\bar{h} = \bar{h}_0 \quad and \quad \bar{E} = \bar{E}_0 \quad at \quad x = 0 \tag{59}$$

With the help of this boundary conditions (57) - (59) and equations (44)and(45), the constants $a_i's(i = 1, 2, 3)$ can be determined as -

$$\begin{aligned} a_1 &= \frac{[(\lambda_3 + m_{23})m_{32} - (\lambda_2 + m_{22})m_{33}]\bar{f}(s)}{\Delta_a} \\ a_2 &= \frac{[(\lambda_1 + m_{21})m_{33} - (\lambda_3 + m_{23})m_{31}]\bar{f}(s)}{\Delta_a} \\ a_3 &= \frac{[(\lambda_2 + m_{22})m_{31} - (\lambda_1 + m_{21})m_{32}]\bar{f}(s)}{\Delta_a} \end{aligned} \tag{60}$$

where $\Delta_a = (m_{22}m_{33} - m_{23}m_{32})(\lambda_1 + m_{21}) + (m_{31}m_{23} - m_{21}m_{33})(\lambda_2 + m_{22}) + (m_{21}m_{32} - m_{22}m_{31})(\lambda_3 + m_{23})$

5 Numerical Solution

For this inversion of Laplace transform we follow the method of Bellman, Kalaba and Lockett [9] and choose seven values of the time $t = t_i : i = 1(1)7$ as the time range at which the displacement, temperature, induced magnetic field and stresses are to be determined where t_i are the roots of the Legendre polynomial of degree seven.

For this purpose, copper is taken as the thermoelastic and transversely isotropic material which has the following physical constants in SI units given as Dhaliwal and Singh[10]

$\varepsilon_1 = 0.0168$; $\varepsilon_2 = 0.0008$; $\beta = 2$ $\theta_0 = 293K$

$\nu = 0.008$; $\tau_0 = 0.02$; $V = 1.39 \times 10^{-5}$

6 Concluding Remarks

In order to study the characteristic of displacement, temperature and stresses, we have drawn several graphs for different values of the space variable and at times $t_1 = 0.025775, t_2 = 0.138382, t_3 = 0.352509, t_4 = 0.693147, t_5 = 1.21376, t_6 = 2.04612, t_7 = 3.67119$.

Case - I

1. Fig.(1)-(2) exhibit the variation of displacement and

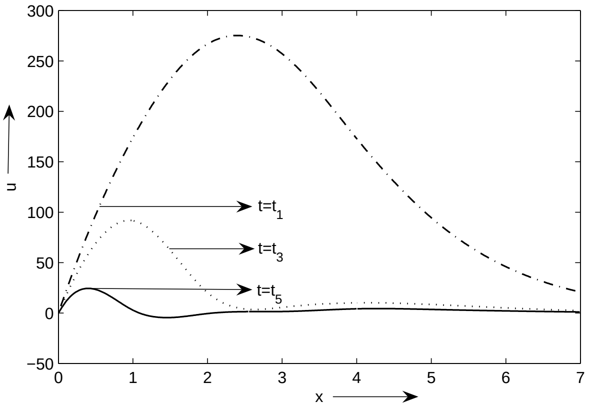

Figure 1: CaseI:Displacement vs.x

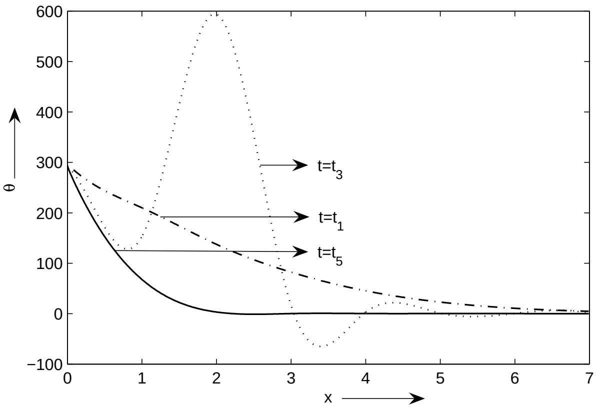

Figure 2: CaseI:Temperature vs. x

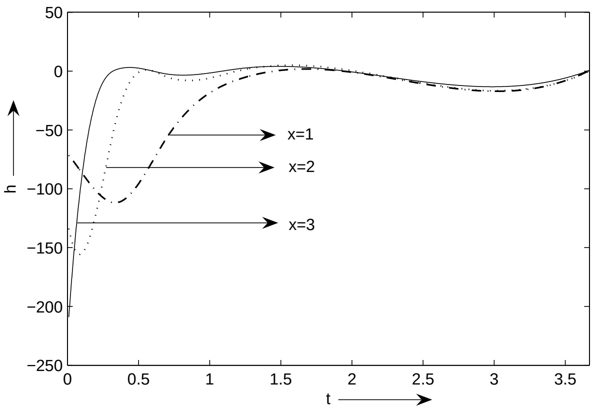

Figure 3: CaseII:Magnetic field vs.time

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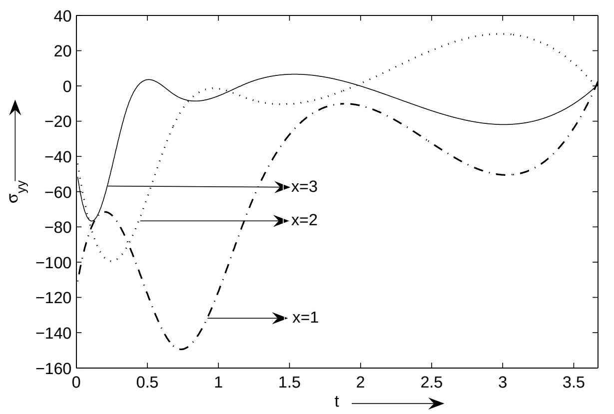

Figure 4: CaseII:Stress(σ_{yy}) vs.time

temperature with space variable x for fixed values of the time variables $t = t_1$, $t = t_3$ and $t = t_5$.

(i) From fig.(1), we notice that the magnitude of displacement u gradually decreases for fixed value of space variable x as time t increases.

(ii) From fig.(2), we notice that the temperature θ is maximum for $t = t_3$ when $x = 2$. For all time $t = t_1$, $t = t_3$ and $t = t_5$ the values of temperature θ gradually decreases as x increases,finally vanish when $x = 6$.

Case - II

2. Fig.(3)-(4) exhibit the variation of magnetic field and stress with time variable t for fixed values of space variable x. We observe the following -

(i) From fig.(3), it is clear that the absolute value of magnetic field gradually decreases with greater wave length as t increases and finally vanishes. For fixed time, the absolute value of magnetic field gradually increases as space variable x changes from $x = 1$, $x = 2$ and $x = 3$.

(ii) From fig.(4),we notice that the absolute value of stress σ_{xx} gradually decreases with greater wave length as t increases. The absolute value of σ_{xx} and σ_{yy} are maximum for $x = 1$, when time $t = 0.8$

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