Modelling Out-Of-Sequence Measurements Using a Grey Relational Anaysis and Copulas Hybrid

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Abstract—One primary concern of researchers, particularly those dealing with multisensory target tracking and filtering, is effectively dealing with out-of-sequence measurements (OOSM). Recently, the use of Kalman filters has proven to be of great practical value in solving a variety of OOSM problems including multi-target tracking prediction. In this paper we argue that delayed and existing measurements are typically correlated and could be described by a joint distribution. We further arugue that the uncertainty of the measurements could be modelled using grey relational analysis (GRA). Thus, the proposed approach deals with the uncertainty of information and combines it with copula in order to model OOSM. Benchmarking results on simulated datasets show the use of GRA coupled with copulas as more robust to handling OOSM as compared to existing methods.

Keywords - out-of-sequence measurements

I. INTRODUCTION

Within the general framework of multi-sensor applications, a difficult step in target tracking and filtering is the handling of out-of-sequence measurements (OOSM). Most of the work on tracking and filtering has been based on the assumption that measurements are immediately available to an agent. However, it is not difficult to conceive situations in which measurements are subject to non-negligible delays such that the lag between measurement and receipt is of sufficient magnitude to have an impact on estimation or prediction. These measurements can be classified as either constant delays or random delays with the resulting occurrence of the latter having the potential to cause OOSM. Say, for example you have a multi target tracking system with two sensors but with different pre-processing times and a fusion centre where the system state is updated by the newest measurement (as depicted in Figure 1). As shown in Figure 1, sensor 2 overtakes the measurement from sensor 1. Thus, at several times, the fusion processor has received measurements from sensor 2 before a measurement from sensor 1 (belonging to an earlier time), arrives.

Handling OOSM represents a challenge for engineers or researchers using multi-sensor data. One easy and normal solution is to simply ignore and discard the OOSM in the tracking process. The appropriateness of this approach has a natural limitation in that information is lost due to the discarded OOSM. To avoid such a drawback, other approaches that include data re-processing or roll back and data buffering have been developed for dealing with OOSM. In the rollback approach, sensor reports are stored in memory and the OOSM is used to re-order the sensor measurements in a track hypothesis. The data buffering approach holds the incoming measurements in a buffer with the size buffer greater than the maximum expected delay of arriving measurements. Both approaches require significant memory and storage measurements.

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Also, since the tracker processing always lags behind the current time, both approaches poses potential problems for real-time target applications.



Figure 1. The out-of-sequence (OOSM) problem

For time delays, one common approach for dealing with the OOSM problem is related to solving a partial differential equation and boundary condition equations which do not have an explicit solution in general [1;2;8;9;19;20]. For random delays, the problem has been investigated via a standard Kalman filtering and by augmenting the system accordingly [3;5]. Matveev and Savkin [13] consider an iterative form of state augmentation for random delays with a random lag. Mallick et al. [12] address the OOSM problem by recalculating the filter through the delayed period. In the same context, Larsen et al. [9] propose a measurement extrapolation approximation using past and present estimates of the Kalman filter (KF) and calculating an optimal gain for this extrapolated measurement (ME-KF). Thomopoulos and Zhang [16] examine the case of random delay under the name of fixed sampling and random delay filter (FSRD-KF) that is shown to be equivalent to constraining the lag to a value of 1. Later, Larsen et al. [9] suggest using delayed measurements to calculate a correction term and adding this to the filter estimate. Zhang et al. [20] proposed algorithms that try to minimise the information storage in an OOSM situation (MS-KF) Challa et al. [4] formulated the OOSM problem in a Bayesian framework (BF-KF). Twala [18] relates OOSM to the incomplete (missing) data problem and uses multiple imputation and later copulas [17] to handle OOSM.

Although the vast majority of the above methods understand the solution, most of them fail to recognise the theoretical basis of the conditional distribution between the delayed measurements and the measurements that are already available (which are sometimes referred to as history). The major contributions and uniqueness of the work presented in this paper are as follows:

- We show the robustness of one of the classical five techniques for handling OOSM in terms of predictive accuracy for multi-target tracking applications;
- We then propose how OOSM can be modelled using grey relational analysis coupled with copulas and further show how it leads to a significant improvement in performance for multi-target tracking (for both single delay).

The context is organized in the following manner: The problem statement is described first. Then, the grey relational analysis process is presented in detail, followed by a brief description of the copulas strategy. Section 4 empirically explores the robustness and accuracy of the proposed approaches against existing methods for dealing with OOSM using simulated data. We close with conclusions and directions for future research.

II. PROBLEM STATEMENT

The presentation herein is based on the Kalman Filter (KF) equations for a discrete linearized time-varying system with state vector x_k , input vector u_k , and output vector y_k . KF is the optimal recursive data processing algorithm for a discrete linear system corrupted with noise in the states and measurements.

A. System

The KF addresses the general problem of trying to estimate the state $x \in \Re^n$ of a discrete-time controlled process that is assumed to evolve over time t_{k-1} to t_k and governed by the linear stochastic difference equation

$$x(k) = F(k, k-1)x(k-1) + v(k, k-1)$$

where x(k) is the state vector at time k, F(k, k - 1) is the state transition matrix to time t_k from t_{k-1} and v(k, k - 1) represents the (cumulative effect of the) process noise for this interval. The order of the arguments in both F and v is according to the convention for the transition matrices. Typically, the process noise has a single argument, but here the two arguments will be needed for clarity. The time τ , at which the OOSM was made is assumed to be such that

$$t_{k-l} < \tau < t_{k-l+1}$$

This will require the evaluation effect of the process noise over an arbitrary non-integer number of sampling intervals. Note that l = 1 corresponds to the case where the lag is a fraction of a sampling interval; for simplicity this is called the "1-step-lag" problem, even though the lag is really a fraction of a time step.

The measurement $z \in \Re^m$ and thus measurement or observational model is

$$z(k) = \boldsymbol{H}(k)\boldsymbol{x}(k) + \boldsymbol{w}(k)$$

where z(k) is the observation vector, w(k) is the observation noise vector and H(k) is the observation matrix. The noise vector v(k, k - 1) and w(k) are assumed to be independent (of each other), white, and with normal probability distributions

$$p(w) \sim N(0, Q)$$
$$p(v) \sim N(0, R)$$

The process noise covariance Q(k) and measurement noise covariance R(k) are mutually uncorrelated and they are given as

$$E[v(k,j)v(k,j)'] = Q(k,j) \quad E[w(k)w(k)'] = R(k)$$

Similarly to x(k) = F(k, k-1)x(k-1) + v(k, k-1), one has

$$x(k) = F(k,\kappa)x(\kappa) + v(k,\kappa)$$

where κ is the discrete time notation for τ . The above can be written backward as

$$x(\kappa) = F(\kappa, k)[x(k) - v(k, \kappa)]$$

where $F(\kappa, k) = F(k, \kappa)^{-1}$ is the backward transition matrix.

B. Fusion of delayed measurements

Denoting a cumulative set of measurements $Z^k \triangleq \{z(i)\}_{i=1}^k$, the OOSM problem [up to time instance $t = t_k$, and excluding a measurement $z(\tau)$ with a time stamp $t_{\tau} < t_k$ reduces to the problem of computing the conditional mean estimate of the target state

$$\hat{x}(k|k) \triangleq E[x(k)|Z^k]$$

and its associated error covariance

$$P(k|k) \triangleq cov[x(k)|Z^k]$$

Under the assumption that the initial state x_0 is Gaussian, the conditional mean estimate $\hat{x}(k|k)$ of the target state, which is optimal in the minimum variance sense, can be computed recursively using the KF. Also, it is assumed that a measurement z is collected and used to update the track at the time interval h. The basic KF algorithm can then be extended to multi sensor systems where the data is assumed to arrive at known times and in correct time sequence.

Suppose that a given measurement corresponding from time τ (denoted with discrete time notation as κ),

$$z(\kappa) \triangleq z(\tau) = H(\kappa)x(\kappa) + w(\kappa)$$

arrives with a certain delay after $\hat{x}(k|k)$ and P(k|k) have been computed.

One faces the problem of updating the state estimate and its covariance with the delayed measurements, i.e., to compute:

and

$$\hat{x}(k|\kappa) \triangleq E[x(k)|Z^{\kappa}])$$

$$P(k|\kappa) \triangleq cov[x(k)|Z^{\kappa}])$$

where

$$Z^{\kappa} \triangleq \{Z^k, z(\kappa)\}$$

 $P(k|\kappa) \triangleq$ provides a simple, intuitive interpretation of the weight in the time delayed KF. The weight assigned to a measurement is a function of the degree to which the measurement is correlated with the current state of the system Therefore, the difficulty in implementing the time delayed KF is in calculating $P(k|\kappa)$. Solutions to the delay measurement problem are presented in the next section.

III. GREY RELATIONAL STRATEGY

The grey relational analysis technique, proposed by Deng [5;6;7], is a method that can measure the correlation between series and belongs to the category of the data analytic method or geometric method. The measured series can be either time series or index series etc. Usually, researchers will set the target series based on the objective of the studied problem as the reference series. Hence, the purpose of GRA technique is to measure the relation between the reference series and comparison series. Grey prediction power comes from its ability to predict the future value with only a few data.

The Grey relational classifier (GRC) which is based on the grey model (GM) has three basic operations: accumulated generating operation (AGO), inverse accumulated generating operation (IAGO) and grey modelling. AGO is the most important characteristic for the grey system theory and its purpose is to reduce the randomness of data. The GM (1, 1) is the most commonly used model. The first 1 in GM (1, 1) means that there is only one variable, and the next 1 means that the first order grey differential equation is used to construct the model as described below:

Denote the original data sequence by:

$$u^{0} = [u^{(0)}(1), u^{(0)}(2), u^{(0)}(3), \dots, u^{(0)}(n)]$$

where n is the number of years observed.

Then the GM(1,1) is:

$$u^{0}(k) + aZ^{(1)}(k) = b, \qquad k = 1, 2, 3, ..., n$$

where a is the development parameter and b is the grey input. Now we go through the following procedures:

1. The AGO formation of u^0 is defined as:

$$u^{(1)}(k) = AGO\left(u^{(0)}(k)\right) = \sum_{i=1}^{k} u^{0}(k)$$

where

 $u^{(1)}(1) = u^{(0)}(1)$ and

$$u^{(1)}(k) = \sum_{m=1}^{k} u^{(0)}(m), k = 2, 3, 4, ..., n.$$

2. Find $Z^{(1)}(k)$ where

$$Z^{(1)}(k) = 0.5 \left[u^{(1)}(k) + u^{(1)}(k-1) \right]$$

3. Using Least Squares Method, find matrix B and vector.

$$B = \begin{bmatrix} -z^{(1)}(k) & 1 \\ -z^{(1)}(k) & 1 \\ \cdots \\ \vdots \\ -z^{(1)}(k) & 1 \end{bmatrix}$$

and

$$y_n = \left[u^{(0)}(2), u^{(0)}(3), u^{(0)}(4), \dots, u^{(0)}(n)\right]$$

4. Estimate the parameters *a* and *b* by following a set of equations:

$$[a, b]^T = (B^T B)^{-1} B^T y_n$$

5. Find the response equation:

$$u^{(1)}(k+1) = \left[u^{(0)}(k) - \frac{b}{a}\right]e^{-ak} + \frac{b}{a}$$

6. The predicted value of $\hat{u}^{(0)}(k+1)$ is

$$\hat{u}^{(0)}(k+1) = (1+e^a) \left[u^{(0)}(1) - \frac{b}{a} \right] e^{-ak}$$

On receipt of OSSM, there is always a need to update the parameters of the chain (refer to Section II for details). A naive implementation of an OSSM processing algorithm necessitates re-processing of the data from the time of the OOSM to the last time. This is where GRA is incorporated in our proposed strategy (i.e., an attempt to use GRA to process OOSM efficiently).

IV. THE COPULAS STRATEGY

A motivation for copulas is that it exists as a multivariate distribution function and allows a consistent and flexible modelling of the dependence structure of dealing with OOSM. It offers a convenient representation of arbitrary joint distribution functions, with the key property being that the specification of the marginal distributions and the dependence structure is separated. This is the most important result in the copula framework and is due to [15]. In recent years, copulas modelling has found many successful applications in actuarial science, survival analysis, hydrology, and with great intensity in finance [14]. The generalized copulas algorithm for handling OOSM is summarised in Figure 2.

- 1. Consider a sequence of measurements up to k instances $X_1, X_2, ..., X_k$ (where k is the delay point) with distribution function $H(x_1, x_2, ..., X_{k-1}) = P(X_1 \le x_1, X_2 \le x_2, ..., X_k \le x_k)$ and univariate marginal distributions $F_1(x_1), F_2(x_2), ..., F_k(x_k)$
- A copulas C represents the joint cumulative distribution function in terms of the margins sucks that H (x₁, x₂,..., X_{k-1})= C(F₁(x₁),..., F_k(x_k)) for all values x₁, x₂,..., x_k(or (X₁, X₂,..., X_k∈ℝ^k).
 - If F_1 , F_2 ,..., F_k are continuous, *C* is unique for every fixed *F* and equals $C(u_1,...,u_k) = F(F_1^{-1}(u_1),..., F_k^{-1}(u_k))$, where F_1^{-1} ,..., F_k^{-1} are the quantiles functions given marginals and are uniform [0, 1] variables;
- 3. Find the conditional distribution $F(x_k|H)$ for delayed measurements conditioned to the history of measurements available as a predictive distribution;
- 4. Predict the delayed measurement from the conditional distribution (copula could be used to find both the joint and conditional distributions even if the joint distribution

Figure 2. The copulas algorithm for dealing with OOSM

A. Experimental set-up

In order to empirically evaluate the performance of the proposed approaches (which we shall now call GREYCOOSM and COOSM) against existing approaches

for dealing with OOSM (FSRD-KF, ME-KF, SARD-KF, MS-KF and BF-KF), experiments are used on simulated datasets in terms of root square mean error (RMSE). GREYCOOSM is the algorithm of GRA coupled with copula while COOSM is the algorithm using copulas on its own. RMSE is a measure of the differences between values predicted by a model (or an estimator) and the values actually observed. The experiment is carried out in order to rank individual OOSM methods and also assess the impact of delayed measurements (at various time and distance intervals) on a single delay against GCOOM and COOSM in terms of position error. Like [4], we assume that the OOSM can only have a maximum of one lag delay and the data delay is uniformly distributed within the whole simulation period with probability $P_{\rm r}$ that the current measurement is delayed.

The CPU times of all algorithm for 1000 Monte Carlo runs are also looked at. Although the measured CPU times represent only imprecise approximations of the computational complexity of the algorithms, they are used in this paper for comparison purposes only.

All statistical tests were conducted using the MINITAB statistical software program. Analyses of variance, using the general linear model (GLM) procedure were used to examine the main effects and their respective interactions. This was done using a three-way repeated measures design (where the effects were tested against its interaction with datasets). The main effects are: OOSM methods; the probability of measurement; and the manoeuvring index.

B. Experimental results

All the main effects were found to be significant at the 5% level of significance (F=18.9, df=5 for methods; F=29.4, df=1 for probability of measurement and F=31.2, df=1 for manoeuvring index; p-value <0.05 for each effect).

As shown in Figure 3a, GREYCOOSM is the best method for handling OOSM (for single delays) with an error rate of 6.9%, closely followed by BF-KF, COOSM, SARD-KF, FSRD-KF, MR-KF with excess error rates of 9.2%, 11.1%, 12.8%, 14.2% and 16.9%, respectively. The worst method is ME-KF, which exhibits an error rate of 20.1%. Tukey's multiple comparison tests further showed significant differences between all the methods at the 5% level of significance.



Figure 3a. Overall means for existing OOSM, COOSM and GREYCOOSM methods (single delay)

For multiple delays, GREYCOOSM (once again) achieves the highest accuracy rates with an error rate of 10.5%, closely followed by COOSM (13.8%), BF-KF 15.2%, FSRD-KF (17.3%), SARD-KF (20.3%) and ME-KF (21.9%). The worst performance is by MR-KF with an error rate of 23.6% (Figure 3b).



Figure 3b. Overall means for OOSM, COOSM and GREYCOOSM methods (multiple delays)

We shall now present results for the performances of the six methods for single delay over 1000 runs for one manoeuvring index 0.3.



Figure 4a. RMS performance in the case of highly manoeuvring target with single delay OOSM ($P_r = 0.5$; manoeuvring =0.3)



Figure 4b. RMS performance in the case of highly manoeuvring target with single delay OOSM (P_r = 0.25; manoeuvring = 0.3)

From Figure 4, the following results are observed.

- For manoeuvring target tracking, GREYCOOSM improves estimation accuracy compared to the other methods. This is the case for both probabilities of measurement. However, its performance with BF-KF is comparably when the probability of measurement is 0.25.
- The differences in performance among all the methods are mostly prominent at higher probabilities of measurement. Inconsistent performances are observed for SARD-KF (for $P_r = 0.5$) and FSRD-KF (for $P_r = 0.25$, especially at higher times).
- Increases in probability measurement delay are also associated with increases in performance differences between methods. In fact, the performance of all the

methods degrades with increases in probability of measurement.

From Figure 5, the performances of the six methods for single delay over 1000 runs for a manoeuvring index of 1 are observed.

- For manoeuvring target tracking, GREYCOOSM outperforms all the other methods when the probability of measurement is 0.5. However, its performance with BF-KF is comparably when the probability of measurement is 0.25.
- The differences in performance among all the methods are mostly prominent at higher probabilities of measurement. Poor performances are observed for SARD-KF (for $P_r = 0.5$) and FSRD-KF (for $P_r = 0.25$, especially at higher times).
- Increases in probability measurement delay are also associated with increases in performance differences between methods. In fact, the performance of all the methods degrades with increases in probability of measurement.
- The accuracy of GREYCOOSM is achieved at a higher computational cost in terms of minutes (Table I). Otherwise, the measured CPU times of the algorithms (with the exception of COOSM and BF-KF) are comparable among each other. All the top three methods take about twice (in some situations thrice) to compute compared to the others.



TABLE I COMPUTATIONAL COMPARISON OF GREYCOOSM, COOSM AND EXISTING METHODS (SINGLE DELAY) IN MINUTES

Pr	Methods						
	FSRD- KF	ME- KF	SARD- KF	MR- KF	BF- KF	CO- OSM	GREY- COOSM
0	2.67	2.78	3.71	2.01	5.13	7.00	9.21
0.25	3.15	3.40	4.43	2.57	5.45	6.78	10.01
0.5	3.74	3.97	4.78	2.64	6.10	7.09	10.01

VI. REMARKS AND CONCLUSIONS

Accurate prediction of target tracking given delayed measurements can be very valuable to engineers, especially those dealing with sensoring applications. This is important for minimising cost and improving effectiveness of the software testing process.

The major contribution of the paper has been the application of GRA and copulas to predict multi-target tracking given that some measurements in a multi tracking system application are out of sequence. This was for both single and multiple delays. Simulated datasets were utilised for this task.

Individually, COOSM is the most effective method for handling OOSM (for both single and multiple delays) with BF-KF and COOSM not far behind. Hence, the proposed method can be a valuable choice for multi target tracking applications. The worst performances were observed for SARD-KF.

Our results further show the probability of measurement delays as having an impact on the performance of methods with BF-KF more effective for the smaller probability. Bigger positional error rates were achieved by methods for high probability delays with bigger performance differences among methods. Also, given that the performance of each method varies by probability of measurement delay, it appears that the treatment of delayed measurements not only heavily depends on the probability of measurement delay but on the range of manoeuvring target tracking.

Despite promising preliminary results, the use of GRA coupled with copulas also deserves further investigation on a number of fronts, for example, in terms of the training parameters and the combination rules that can be employed. Also, empirical studies of the application of the copula to real-world dataset should be undertaken to assess its performance across a more general field. Higher central processing unit times were also observed for multiple delays compared to single delays, especially for the proposed strategies. The next step will be to develop an effective strategy that would reduce the CPU for the copula-grey relational-based methods.

We leave the above issues to be investigated in the future.

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