

# Determination of Damages in Beams Using Wavelet Transforms

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**Abstract**—There are several techniques of non-destructive damage detection in structures. However, these techniques are expensive and require an accurate analysis of large extension of the structure. The numerical techniques can help in non-destructive tests of structures, showing the possible location of damage and thus decreasing the area of analysis and becoming the non-destructive tests less expensive. Among the numerical methods most used to detect damage stand out the finite element method and the boundary element method. This paper presents numerical applications of the Wavelet Transform for damage detection in a cantilever beam subjected to static and modal analyses. The modeling of damage is done in finite elements using ANSYS program and the damage simulated deleting some finite elements from the mesh. The static response (in displacement) of the structure with simulated damage is used in the analysis to detect the location of damage using the Wavelet Transform. The results of analyses are presented and discussed in this paper.

**Key words**—beams, damage and wavelet.

## I. INTRODUCTION

In recent years, there is a large interest in the scientific community in the researches associated on damage detection in structures using numerical methods with the goal to help in the non-destructive techniques applied to performance monitoring, pathology and structural health of civil structures [1].

Generally, the numerical methods make the comparison of signatures obtained before and after the onset of damage, and these signatures are defined in terms of displacements, frequencies, mode shapes, stress, among others. It should be important that indication of damage position could be showed without using the comparison of signatures but just based on the response of damaged of the actual structure.

Although the literature on damage detection has been so far dominated by studies based on methods that utilize frequency or stiffness variation information, methods based on wavelet transform, a recently developed mathematical theory rooted in signal analysis [2,3,4], are emerging.

These methods are based on discrete measurements in some points of structure. Such measurements could be subjected to equipment errors or human errors and generally follow a normal distribution.

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## II. WAVELETS THEORY

Considering a signal of interest in the time or space domain and  $\psi(t)$  the values of wavelet function in the time and frequency domains. The wavelets are generated from the mother wavelet  $\psi(t)$  by translation and dilation, as follow below:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (1)$$

Where  $a$  and  $b$  are integer numbers which represents the dilation and translation parameters respectively. The wavelet transform of a signal  $f(t)$  is defined by:

$$C_{a,b} = C_{a,b}(t_0) = \int_{-\infty}^{\infty} f(t) \Psi_{a,b}(t_0) dt \quad (2)$$

The results of this transformation are called wavelet coefficients and show how well the function correlates with the signal. These wavelet coefficients are very sensitive to discontinuities and singularities present in the analyzed signal. Considering this property, it was found that damage due to a sudden loss of stiffness can be detected through mode shapes with wavelet coefficients which achieve large amplitudes like a spike or an impulse in the damage location. This perturbation of wavelet coefficients due to this damage is clearer in the finest scales of the wavelet transform. This procedure is the basis of the wavelet transform damage detection [5].

## III. NUMERICAL EXAMPLES

This section presents the finite element model of cantilever beams cracked used in this research subjected to static and modal analyses and also errors noises in the response signals. The beams were modeled using the element PLANE42 of ANSYS11.0 program.

The element PLANE42 has four nodes and three degrees of freedom by node: translations in  $x, y$  and  $z$  directions. The signals of static response obtained in ANSYS were analysed in MATLAB program to compute the wavelet coefficients.

### A. Static Analysis

The cantilever beam was submitted to a load  $F=500\text{kN}$  in the free end and to a transverse crack  $a' = 0,025\text{m}$  positioned at  $d=0,125\text{m}$  from the clamped end, see Figure 1. The material and geometric properties of the beam analyzed are shown in Table 1.

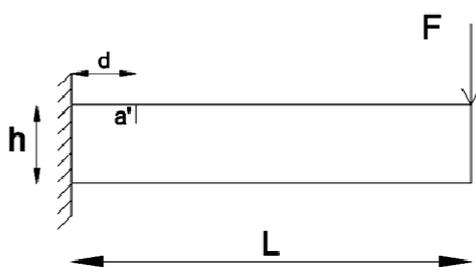


Figure 1: Schematic model of cantilever beam.

Table 1: Geometric and materials properties of cantilever beam

Properties	Symbol	Value	Unity
Base da viga	B	0,10	m
Beam height	H	0,10	m
Area	S	0,01	m <sup>2</sup>
Moment of inertia	I	8,333x10 <sup>-6</sup>	m <sup>4</sup>
Beam length	L	0,50	m
Modulus of elasticity	E	200,00	GPa
Density	$\rho$	7850,00	Kg/m <sup>3</sup>
Poisson coefficient	$\nu$	0,30	-

The finite element model of cantilever beam was discretized in 4000 elements and 4509 nodes, the crack was simulated deleting some elements from the mesh and the boundary conditions were applied in the all nodes of left end restricting the degrees of freedom x, y and z, see Figure 2.

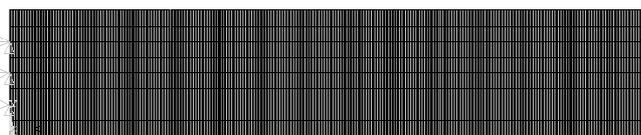


Figure 2: Finite element model of cantilever beam.

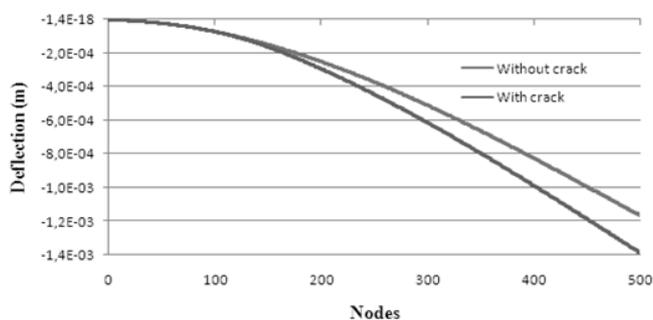


Figure 3: Deflection of cantilever beam.

The wavelet transform was applied in the signal of displacements obtained in the nodes of bottom line of beam using the wavetoolbox of MATLAB to compute the wavelet coefficients for two different mother-wavelet: biorthogonal6.8(bior6.8) and daubechies2(db2). The Figures 4 and 5 shows the results of these wavelet transforms.

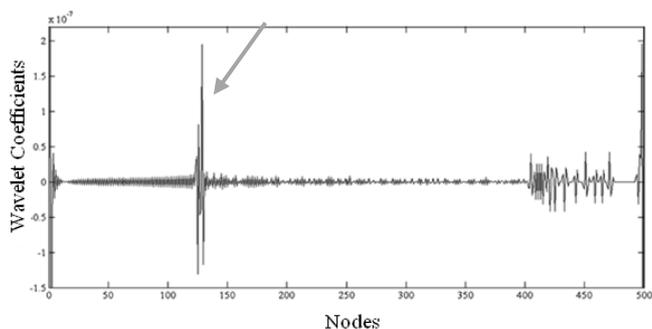


Figure 4: Wavelet coefficients using db2.

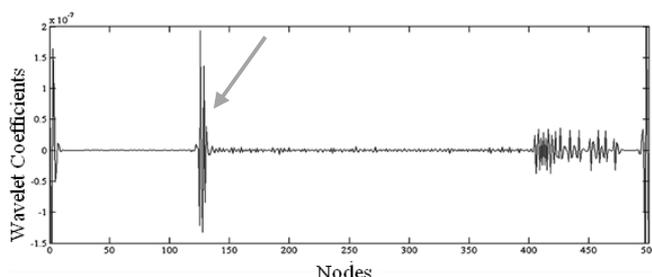


Figure 5: Wavelet coefficients using bior6.8.

The two mother-wavelet were able to detect the exact position of damage (node 125), moreover, the graphics presented little perturbations in the ends due to geometric discontinuities.

### B. Modal Analysis

In the modal analysis, the three mode shapes were computed and the first mode shape was used in the application of wavelet transforms, as can be seen in Figures 6,7 and 8.

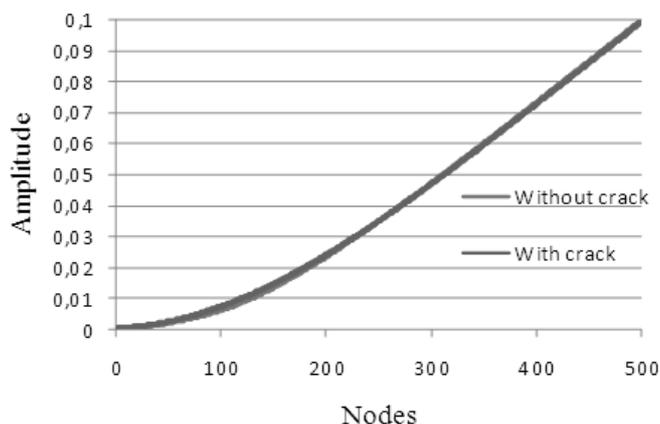


Figure 6: First mode shape of cantilever beam.

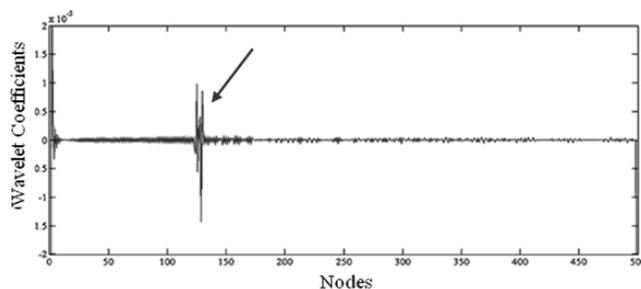


Figure 7: Wavelet coefficients using db2.

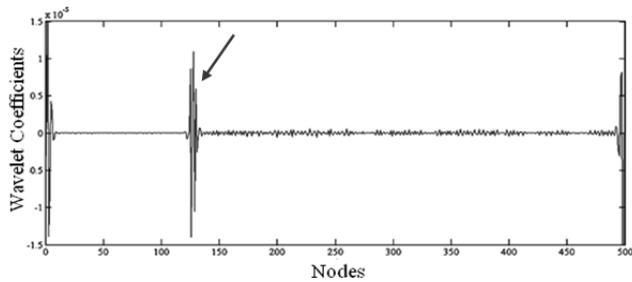


Figure 8: Wavelet coefficients using bior6.8.

The results of the modal analysis show that the application of wavelet transforms in the modal signals, like mode shapes, could be used in the damage detection process, furthermore, the modal signals transformed presented less disturbances than static ones.

#### IV. CONCLUSION

This paper presented a recent methodology applied to the inverse problem of damage detection using static and modal signals.

The wavelet transform applied in the static and modal signals could detect the exact position of damages and also, singularities due to geometric discontinuities.

The main advantage of the the methods based on wavelets, is that it can detect the position of damage just from the response of damaged structure.

#### REFERENCES

- [1] Silva, R. S. Y. C. "*Determinação de patologias estruturais utilizando modelagem numérica e transformadas de wavelet*".MS. Thesis. University of Brasilia. Department of Civil Engineering, Brasilia, DF,2011.
- [2] Daubechies, I., "*Orthonormal bases of compactly supported wavelets*". Comm. Pure and Applied Maths", XLI, 909-996, 1988.
- [3] Daubechies, I., "*The wavelet transform, time-frequency localization and signal analysis*". IEEE Trans. On Information Theory 36,961-1005,1990.
- [4] Mallat, S. A., "*Theory for multiresolution signal decomposition: the wavelet representation*". IEEE Trans. Pattern Anal and Machine Intell. 11, 674-693,1989
- [5] Estrada, E. S., "*Damage detection methods in bridges trough vibration monitoring:evaluation and application*". PhD. Thesis, University of Minho, 298p, 2008