Effect of MFD Viscosity and Porosity on Revolving Axi-symmetric Ferrofluid with Rotating Disk

Kushal Sharma, Paras Ram

Abstract - The Effect of magnetic field-dependent viscosity (MFD) along with porosity on the revolving Axi-symmetric steady flow of ferrofluid with rotating disk is studied by solving the boundary layer equations using Neuringer-Rosensweig model. Besides calculating the velocity components and pressure for different values of MFD viscosity parameter k and porosity ε with the variation of Karman's dimensionless parameter α, we have also calculated the displacement thickness of the boundary layer, total volume flowing outward the z-axis and the torque on the disk. The numerical results obtained here for various flow characteristics, are shown graphically.

Index Terms - **Ferromagnetic fluid, porosity, Axi-symmetric, rotating disk, boundary layer, magnetic field.**

I. Introduction

Ferrofluids are stable suspensions of colloidal ferromagnetic particles of the order of 10nm in suitable non-magnetic carrier liquids. These colloidal particles are coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, the investigation on them fascinated the researchers and engineers vigorously since last five decades. One of the many fascinating features of the ferrofluids is the prospect of influencing flow by the magnetic field and vice-versa. Ferrofluid is widely used in sealing of hard disk drives, rotating x-ray tubes under engineering applications. Sealing of the rotating shafts is the most known application of the magnetic fluid. The major applications of ferrofluid in electrical field is that controlling of heat in loudspeakers which makes its life longer and increases the acoustical power without any change in the geometrical shape of the speaker system. Magnetic fluids are used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery as the bio-medical applications.

There are rotationally symmetric flows of the incompressible ferrofluids in the field of fluid mechanics, having all three velocity components; radial, tangential and vertical in space different from zero. In such types of flow, the variables are independent of the angular coordinates and the angular velocity is uniform at large distance from the disk. We consider this type of flow for an incompressible ferrofluid; when the plate is subjected to the magnetic field $(H_r, 0, H_z)$ using, N-R model. This model has been used by Verma and Singh [1] for characterizing magnetic fluid flow behaviour through a porous annulus. Rosensweig [2] has given an authoritative introduction to the research on magnetic liquids in his monograph and studied the effect of magnetization, resulting in interesting information.

A study of flow within the boundary layer and its effect on the general flow around the body, in detail, are given in Schlichting [3]. Karman's [4] rotating disc problem is extended to the case of flow started impulsively from rest and the steady state is solved to a higher degree of accuracy than previously done by a simple analytical method which neglects the resembling difficulties in Cochran's [5] well known solution. The pioneering study of ordinary viscous fluid flow due to the infinite rotating disc was carried by Karman. He introduced the famous transformation which reduces the governing partial differential equations into ordinary differential equations. Cochran obtained asymptotic solutions for the steady hydrodynamic problem formulated by Karman. Benton [6] improved Cochran's solutions, and also, solved the unsteady case. Attia [7] studied the unsteady state in the presence of an applied uniform magnetic field. Attia [8] studied the steady flow of an incompressible viscous fluid above an infinite rotating disk in a porous medium with heat transfer and also discussed the effect of porosity of the medium on the velocity and temperature distribution. Frusteri and Osalusi [9] examined the laminar convective and slip flow of an electrically conducting Newtonian fluid with variable properties over a rotating porous disk.

Using linear instability analysis, Venkatasubramanian and Kaloni [10] discussed the effects of rotation on the onset of convection in a horizontal layer of ferrofluids rotating about its vertical axis, heated from below and in the presence of uniform vertical magnetic field. The effect of an alternating uniform magnetic field on convection in a horizontal layer of a ferrofluid within the framework of a quasi-stationary approach is studied by Belyaev [11]. Ram et al. [12] discussed the various fluid characteristics of ferrofluid flow in the porous medium with rotating disk.

In general, magnetization is a function of magnetic field, temperature and density of the fluid. This leads to convection of ferrofluid in the presence of the magnetic field gradient. Viscosity is also one of the astounding rheological properties of ferrofluid influencing convection flow problems. Detail accounts of magneto viscous effects in ferrofluids have been given in a monograph by Odenbach [13]. Sunil et al. [14] discussed the influence of rotation on

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medium permeability and how MFD viscosity affects the magnetization in ferromagnetic fluid heated from below in the presence of dust particles saturating a porous medium of very low permeability using Darcy model. Nanjundappa et al. [15] studied Benard-Marangoni ferroconvection in a ferrofluid layer in the presence of a uniform vertical magnetic field with magnetic field dependent (MFD) viscosity. Ram et al. [16] solved the non-linear differential equations under Neuringer-Rosensweig model for ferrofluid flow by using power series approximations and discussed the effect of magnetic field-dependent viscosity on the velocity components and pressure profile. Further, the effect of porosity on velocity components and pressure profile in the presence of rotating disk has been studied by Ram et al. [17].

In the present problem, we take cylindrical coordinates (r, θ, z) where z-axis is normal to the plane and this axis is being considered as the axis of rotation. The viscous effects are dominant over a region at a small distance from the disk, if Reynold number is large, which gives rise to a boundary layer over the surface of the disk. The system is subjected to uniform rotation with angular velocity $\vec{\Omega} = (0,0,\Omega)$ about a vertical axis. We have presented the boundary layer equations together with boundary conditions under the influence of porosity. These equations together with the Maxwell equations are solved theoretically as well as numerically. Also, it is found that in the present problem, there is a large variation in the boundary layer thickness as compared to the ordinary viscous flow case. We have also calculated the total volume flowing outwards the axis taken over a cylinder of radius R around the z-axis. The effects of MFD viscosity and porosity parameter in a circular layer of revolving ferrofluid with rotating disk are studied within the framework of the Neuringer-Rosensweig approach, and various types of ferrofluid responses are considered. This problem, to the best of our knowledge, has not been investigated yet.

II. MATHEMATICAL FORMULATION AND SOLUTION

The basic governing equations are:

Equation of continuity $\vec{\nabla}.\vec{V} = 0$ (1)

Momentum equation for an incompressible ferromagnetic fluid with variable MFD viscosity in a frame of reference rotating with angular velocity $\vec{\Omega}$:

$$\frac{\rho}{\varepsilon}\left[\frac{\partial\vec{V}}{\partial t} + \frac{1}{\varepsilon}(\vec{V}.\vec{\nabla})\vec{V}\right] = -\nabla p' + \mu_0(\vec{M}.\vec{\nabla})\vec{H} - \vec{\nabla}\psi + \frac{\mu_f(1+\vec{\delta}.\vec{B})}{\varepsilon}\nabla^2\vec{V} + 2\frac{\rho}{\varepsilon}(\vec{\Omega}\times\vec{V}) + \frac{\rho}{2}\nabla\left|\vec{\Omega}\times\vec{r}\right|^2 \tag{2}$$

In (2), $p' - \frac{\rho}{2}\left|\vec{\Omega}\times\vec{r}\right|^2 = p$ is the reduced pressure, where p' stands for fluid pressure. The effect of rotation includes two terms: (a) Centrifugal force $-\frac{1}{2}grad\left|\vec{\Omega}\times\vec{r}\right|^2$ (b) Coriolis acceleration $2(\vec{\Omega}\times\vec{V})$.

Here, the velocity component w is less as compared to u and v, and also the flow is steady and axi-symmetric i.e. $\frac{\partial}{\partial t}(\;)=0, \frac{\partial}{\partial\theta}(\;)=0$. Cylindrical polar co-ordinates (r, θ, z) are used, with the disk in the plane $(z=0)$, and the fluid occupies the region $(z>0)$. Assuming the similarity solution, with the dependent variable in the form

$$u = r\,\omega\,E(\alpha),\; v = r\,\omega\,F(\alpha), \quad w = \sqrt{\nu\,\omega}\,G(\alpha),\; p = \rho\omega\nu P(\alpha). \tag{3}$$

where $\alpha = z\,(\omega/\nu)^{\frac{1}{2}}$, and boundary layer approximations $-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu_0}{\rho}\left|\vec{M}\right|\frac{\partial}{\partial r}\left|\vec{H}\right| = -r\omega^2$, and very less variation of magnetic field along the z-direction, we find that the Navier-Stokes equations become

$$\varepsilon\frac{\nu_1}{\nu}E'' - GE' - E^2 + F^2 + 2\varepsilon F - \varepsilon^2 = 0 \tag{4}$$

$$\varepsilon F'' - GF' - 2EF - 2\varepsilon E = 0 \tag{5}$$

$$\varepsilon^2 P' - \varepsilon\frac{\nu_1}{\nu}G'' + GG' = 0 \tag{6}$$

$$G' + 2E = 0 \tag{7}$$

where $\nu_1 = \frac{\mu_f(1+\vec{\delta}.\vec{B})}{\rho}$ = MFD kinematics viscosity and the dash denotes differentiation with respect to α. The approximate initial and boundary conditions for the flow due to rotation of an infinitely long disk $(z=0)$ with constant angular velocity ω are given by

$$\begin{aligned} &at \quad z=0;\; u=0, \quad v=r\omega, \quad w=0. \\ &at \quad z=\infty;\; u=0, \quad v=0. \end{aligned} \tag{8}$$

w does not vanish at $z=\infty$, but tends to a finite negative value and, in view of the similarity assumption (3), these are equivalent to

$$\left.\begin{aligned} &E(0)=0,\; F(0)=1,\; G(0)=0,\; P(0)=P_0 \\ &E(\infty)=F(\infty)=0 \end{aligned}\right\} \tag{9}$$

G must tend to a finite limit, say $-c$ as $\alpha\to\infty$, i.e.

$$G(\infty) = -c,\; (c>0) \tag{10}$$

Following Cochran formal asymptotic expansion (for large α), the solution for equations (4), (6), (7) and (5) are as power series in $\exp\left(-\frac{\nu c}{\nu_1\varepsilon}\alpha\right)$ and in $\exp\left(-\frac{c}{\varepsilon}\alpha\right)$, respectively. i.e.

$$E(\alpha) \approx \sum_{i=1}^{\infty} A_i \exp\left(-\frac{\nu c}{\nu_1\varepsilon}i\alpha\right) \tag{11}$$

$$F(\alpha) \approx \sum_{i=1}^{\infty} B_i \exp\left(-\frac{c}{\varepsilon}i\alpha\right) \tag{12}$$

$$G(\alpha) \approx -c + \sum_{i=1}^{\infty} C_i \exp\left(-\frac{\nu c}{\nu_1 \varepsilon} i\alpha\right) \quad (13)$$

$$(P - P_0)(\alpha) \approx \sum_{i=1}^{\infty} D_i \exp\left(-\frac{\nu c}{\nu_1 \varepsilon} i\alpha\right) \quad (14)$$

Take $k = \frac{\nu_1}{\nu} =$ MFD viscosity parameter and lettng $E'(0) = a$, $F'(0) = b$. Using this supposition and equation (9), we get the additional boundary conditions for the approximate solution for first four coefficients involved in (11) - (14):

$$E''(0) = \frac{1}{k}\frac{(\varepsilon^2 - 2\varepsilon - 1)}{\varepsilon},\ E'''(0) = \frac{-1}{k}\frac{2b(1+\varepsilon)}{\varepsilon};$$

$$F''(0) = 0,\ F''(0) = \frac{2a(1+\varepsilon)}{\varepsilon};$$

$$.G'(0) = 0,\ G''(0) = -2a,\ G'''(0) = \frac{-2}{k}\frac{(\varepsilon^2 - 2\varepsilon - 1)}{\varepsilon};$$

$$P'(0) == -k\frac{2a}{\varepsilon},\ P''(0) = -2\frac{(\varepsilon^2 - 2\varepsilon - 1)}{\varepsilon^2},\ P'''(0) = \frac{4b(1+\varepsilon)}{\varepsilon^2};$$

Using these additional boundary conditions and the values $a = 0.54, b = -0.62$ and $c = 0.886$ from Cochran [5], we calculate the values of the coefficients involve in the system of equations (11) - (14). We draw the graphs of the velocity components and asymptotic pressure with the dimensionless parameter α under the effect of MFD viscosity and porosity. The boundary layer displacement thickness is given by

$$d = \frac{1}{r\omega}\int_0^{\infty} v\,dz = \int_0^{\infty} F(\alpha)\,d\alpha$$

And hence the boundary layer displacement thickness become $d_1 = 0.023399$ and $d_2 = 0.069507$ for $\varepsilon = 0.01$ and $\varepsilon = 0.03$ respectively. Various characteristics of the flow patterns can be computed, in particular the torque on the disk. Assuming for the present a finite radius R, and assuming the similarity solution is accurate over an appreciable part of the disk, we find that the moment is given by

$$M = -2\pi\int_0^R r^2 \mu(\partial v/\partial z)_{z=0} dr = -\pi R^4 (\nu\omega)^{\frac{1}{2}} F'(0).$$

The total volume flowing outward the z-axis,

$$Q = 2\pi R\int_0^{\infty} u\,dz = 2\pi R^2 \int_0^{\infty} \omega E(\alpha)\sqrt{\nu/\omega}\,d\alpha$$

$$= -\pi R^2\sqrt{\omega\nu}\,G(\infty) = 2.786094 R^2\sqrt{\omega\nu} = 2.786094 R^2 \nu\frac{\alpha}{z}$$

Hence, the total volume flowing outward the z-axis is proportional to the dimensionless parameter α. The fluid is taken to rotate at a large distance from the wall, the angle becomes

$$\tan\varphi = -\frac{E'(0)}{F'(0)} = \frac{0.54}{0.62} = 0.870967 \qquad \Rightarrow \quad \varphi = 41^0.$$

III. DISCUSSION

Figures 1 and 2 show the radial velocity profile with the variation of dimensionless parameter α (Karman's parameter) for different values of porosity ε at MFD viscosities $k = 1.1, 1.2$ and 1.3, respectively. The radial velocity at porosity $\varepsilon = 1.0$ for MFD viscosity $k = 1.0$ without rotation is the reduced case of viscous incompressible problem. For $\varepsilon = 0.01$, the radial component of velocity $E_1 = 0.001919$ is maximum at $\alpha = 0.009$, and E_2 and E_3 have the maximum values 0.002093 and 0.002267 at $\alpha = 0.01$ and 0.011, respectively. Whereas, in Ram et al. [16] case of MFD viscosity without porosity and without rotation, E_1, E_2 and E_3 have the maximum values 0.454648, 0.544554 and 0.590987 at $\alpha = 1.4$, 1.7 and 1.8, respectively. Thus, the convergence rate for radial component of velocity is faster for revolving ferrofluid flow with MFD viscosity along with porosity than the case reported in Ram et al. [16] for MFD viscosity only. However, from figures 1 and 2; we observe that for different increasing values of porosity with same set values of MFD viscosity k, the radial values of velocity lead to its slow convergence.

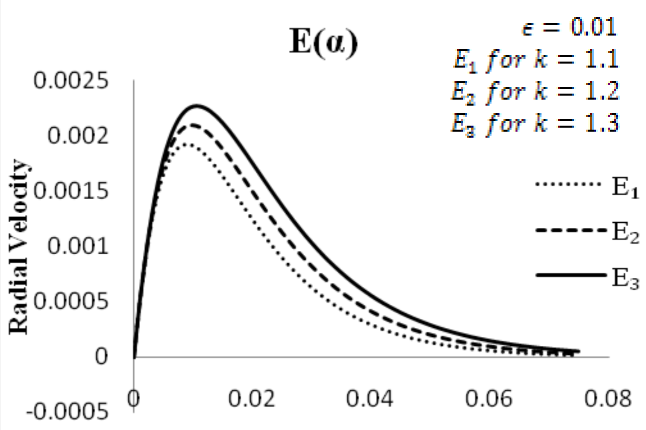

Figure 1: Effect of porosity $\varepsilon = 0.01$ with variation of MFD viscosity k on radial velocity

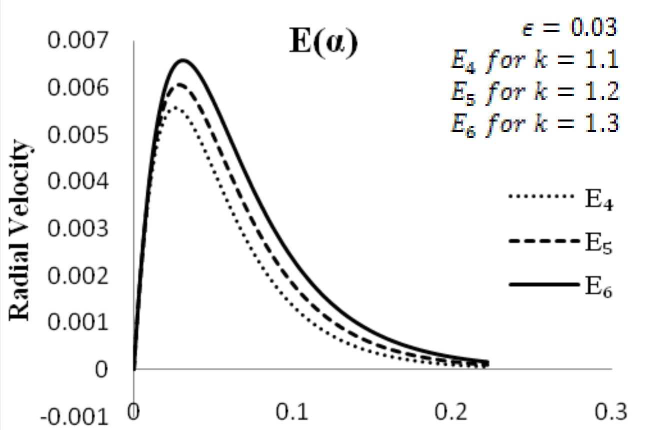

Figure 2: Effect of porosity $\varepsilon = 0.03$ with variation of MFD viscosity k on radial velocity

Figure 3 shows the tangential velocity profile for different values of porosity. As we have not considered the effect of magnetic field in tangential direction, there is no effect of MFD viscosity on tangential velocity component.

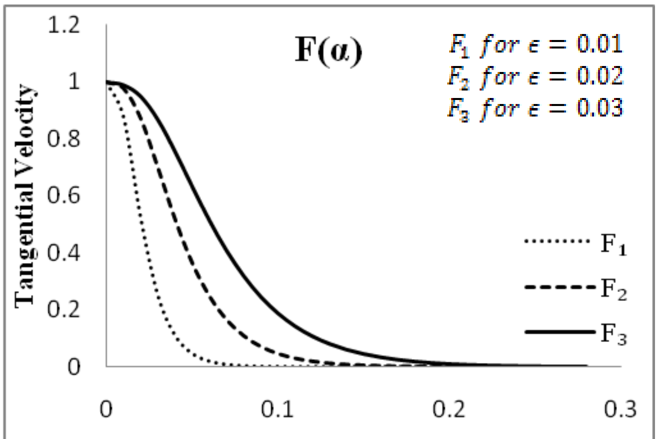

Figure 3: Effect of porosity $\varepsilon = 0.01$ with variation of MFD viscosity k on tangential velocity

Here, for $\alpha = 0.1$, the tangential velocity components are 0.000563 and 0.188683 for different values of porosity $\varepsilon = 0.01$ and 0.03, respectively. Also, it is observed that due to the effect of porosity, the tangential velocity component is increasing with the increase in Karman's parameter α. Whereas in Ram et al. [16] case, the tangential velocity component is free from the effect of porosity. Hence, if we increase the value of porosity, the convergence becomes slower.

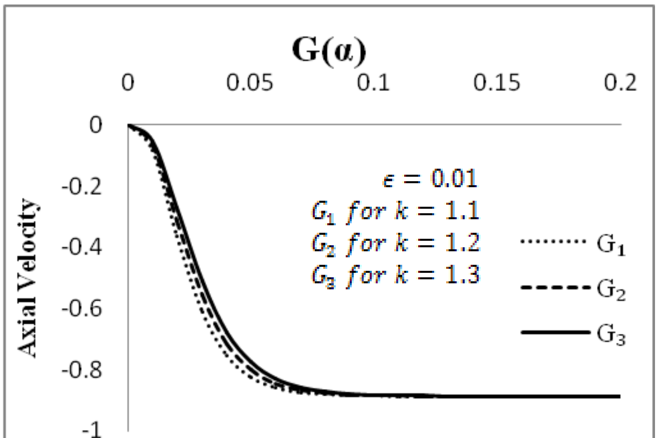

Figure 4: Effect of porosity $\varepsilon = 0.01$ with variation of MFD viscosity k on axial velocity

Figures 4 and 5, represent the axial velocity profiles which are zero in the beginning. It is clear that when we increase the magnetic field, the axial velocity goes to more negative region and the component G_3 tends to a negative finite value -0.886 little faster than G_1 and G_2. For $\varepsilon = 0.01$, the axial velocity G_1, G_2 and G_3 converges to finite negative value -0.886 at $\alpha = 0.17,\ 0.19$ and 0.20 at MFD viscosities $k = 1.1,\ 1.2$ and 1.3 respectively. Similarly, we can conclude for $\varepsilon = 0.03$, the late convergence is appeared for axial velocity components G_4, G_5 and G_6 at MFD viscosities $k = 1.1,\ 1.2$ and 1.3 respectively. In Ram et al. [16], the fluctuations in graph are negligible, but here fluctuations are prominent. There is a large variation in axial velocity components for different MFD viscosities along with the effect of porosity and revolution of ferrofluid. Also from the graphs 4 and 5, we can conclude that the axial velocity component is decreasing with increase in porosity and MFD viscosity, but for large values of α, it takes finite negative value -0.886.

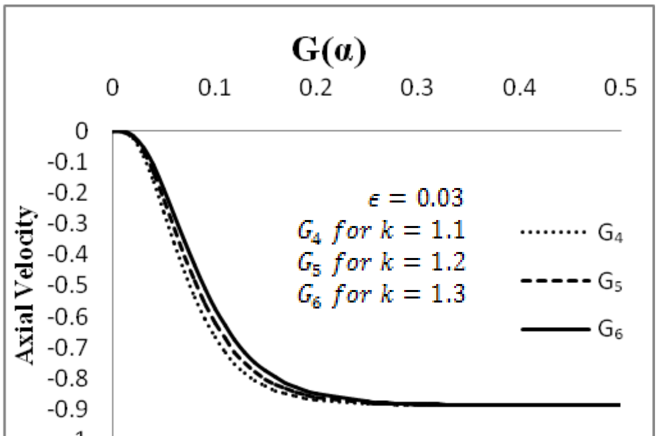

Figure 5: Effect of porosity $\varepsilon = 0.03$ with variation of MFD viscosity k on axial velocity

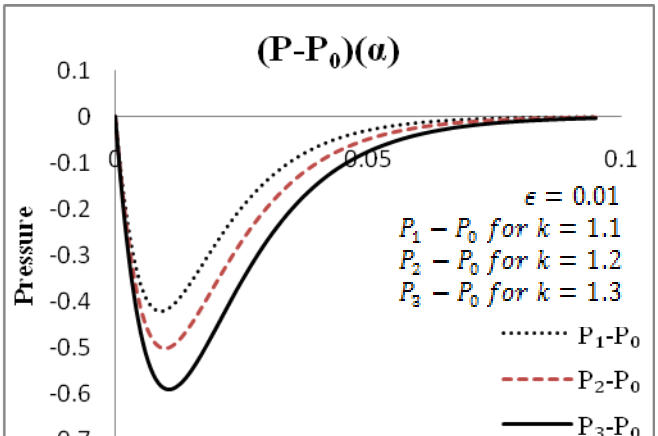

Figure 6: Effect of porosity $\varepsilon = 0.01$ with variation of MFD viscosity k on pressure

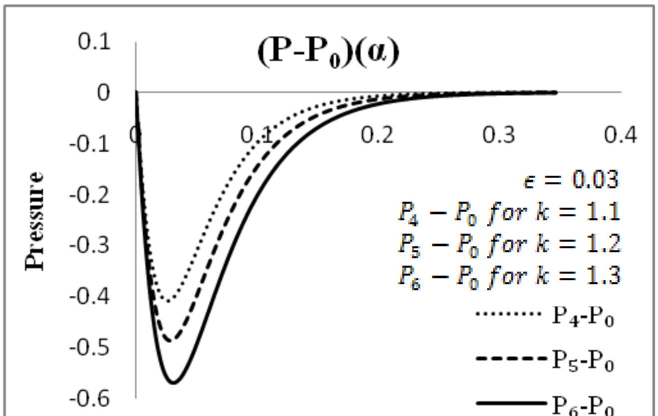

Figure 7: Effect of porosity $\varepsilon = 0.03$ with variation of MFD viscosity k on pressure

Figures 6 and 7 show the pressure profile with initial pressure P_0, for different values of porosity ε at MFD viscosities $k = 1.1,\ 1.2$ and 1.3 respectively. In figure 6, the pressure $(P_1 - P_0)$ reaches to maximum negative value -0.42214 at $\alpha = 0.009$ for $k = 1.1$ and $\varepsilon = 0.01$, whereas, for the same value of ε, the pressures $(P_2 - P_0)$ and $(P_3 - P_0)$ take the maximum negative values -0.50232 and

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-0.58932 at $\alpha = 0.01$ and 0.011 for $k = 1.2$ and 1.3, respectively. We conclude from these graphs that due to the increment in magnetic field and rotation of ferrofluid, the convergence rate is going slow and slow with the increment in porosity ε along with the variation in Karman's parameter α. Also, in comparison to Ram et al. [16], the pressure decreases due to the effect of porosity and rotation of ferrofluid.

IV. CONCLUSION

Under the consideration of effect of MFD viscosity, porosity and rotation, the radial and tangential components of velocity converge to zero faster than in comparison to the case of MFD viscosity alone. The radial velocity increases with increase in MFD viscosity and porosity both, whereas the tangential component of velocity increases with the increase in porosity only. The effect of MFD viscosity is more pronounced in radial direction whereas moderate in axial direction. Also, the axial velocity converges to finite negative value faster than the case of MFD viscosity alone. Numerical value of the axial component decreases with increase in MFD viscosity and porosity, both. Here, we can observe that the radial velocity and pressure are converse in behavior to each other. The boundary layer displacement thickness becomes very small as comparison to the case of MFD viscosity alone as well as the case of ordinary viscous flow reported in [16] and [6], respectively.

The present study projects certain practical applications in many areas such as rotating machinery, lubrication, oceanography, computer storage devices, and viscometry and crystal growth processes. In nut shell, this problem is a theoretical motivation explaining physical effects of variable field dependent viscosity, porosity and rotation on various flow characteristics of ferrofluids.

APPENDIX

$\vec{B}$ Magnetic induction
d Thickness of the ferrofluid layer
$\vec{H}$ Magnetic field intensity
$\vec{M}$ Magnetization
p' Fluid pressure
p Reduced Pressure
P_0 Initial pressure (absolute value)
$\vec{V}$ Velocity of ferrofluid
ν Kinematic viscosity
ν_1 Kinematic variable MFD viscosity
μ_f Reference viscosity of fluid
μ_0 Magnetic permeability of free space
ρ Fluid density
$\vec{\nabla}$ Gradient operator
α Karman's parameter
r Radial direction
θ Tangential direction
z Axial direction
ω Angular velocity of the disk
u Radial velocity
v Tangential velocity
w Axial velocity
$\vec{\Omega}$ Angular velocity of whole system
φ Angle of rotation
$\vec{\delta}$ Linear measure of the viscosity variations with the applied magnetic field
k Ratio of kinematic variable MFD viscosity and kinematic viscosity
M Moment

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