# Buoyancy Driven Heat Transfer of Water-Based CuO Nanofluids in a Tilted Enclosure with a Heat Conducting Solid Cylinder on its Center

Ahmet Cihan, Kamil Kahveci, and Çiğdem Susantez

*Abstract*—**Buoyancy driven heat transfer of water-based CuO nanofluid in a tilted enclosure with a heat conducting solid circular cylinder on the center is studied numerically with Comsol Multiphysics modeling and simulation software. The upper and the bottom walls of the enclosure are kept in adiabatic conditions and the sidewalls of the enclosure are in isothermal conditions. The results show that nanoparticle usage enhances the heat transfer rate considerably. The results also show that average Nusselt number shows first an increase and then a decrease as the inclination angle is increased. The maximum heat transfer takes place at θ=45 deg for Ra=$10^4$ and at θ=30 deg for Ra=$10^5$ and $10^6$.**

*Index Terms*—**nanofluid, natural convection, enclosure, Nusselt number, Rayleigh number**

## I. Introduction

In many engineering applications, especially cooling of electronic equipment it is wanted to enhance the heat transfer rate. But conventional heat transfer fluids such as water, oil, ethylene glycol have lower thermal conductivities. One way to enhance heat transfer rate is using nanofluids in these types of application. Nanofluids are heat transfer fluids whose thermal conductivities are significantly high with respect to the conventional heat transfer fluids [1], [2], [7]. They are obtained by adding nanoparticles with high thermal conductivity to the base fluid. In the past because of technological difficulties, micron-sized particles have been used to enhance heat transfer rate. Micron-sized particles in the fluid cause sedimentation and clogging problems. These types of problem are eliminated by nanoparticle usage.

Thermal conductivity is the most important parameter for the heat transfer enhancement potential of nanofluids. The thermal conductivity of a nanofluid varies with the thermal conductivity of both the base fluid and the nanoparticle, the shape of the nanoparticles, the surface area, the distribution of the dispersed particles and the volume fraction. Because of the absence of the suitable theoretical formulas for the thermal conductivity of the nanofluid, some models for solid liquid mixtures with micron-sized particles are used to predict the thermal conductivity of the nanofluids. But in these models, the thermal conductivity of the nanofluids is not the function of the size and the distribution of the particles.

Maxwell model [3] is a well known thermal conductivity model for this type of solid-liquid mixtures. In this model, thermal conductivity is defined as follows:

$$k_{nf} / k_f = \frac{k_s + 2k_f + 2\left(k_s - k_f\right)\phi}{k_s + 2k_f - 2\left(k_s - k_f\right)\phi} \tag{1}$$

where $k_s$ and $k_f$ are the thermal conductivity of the solid particle and the base fluid, respectively, and $\phi$ is the solid particle volume fraction. Another model for the effective thermal conductivity has been proposed by Yu and Choi [4]. They claimed that a structural model of nanofluids might consist of a bulk liquid, solid nanoparticles, and solid-like nanolayers. The solid-like nanolayers act as a thermal bridge between a solid nanoparticle and a bulk liquid. If the thermal conductivity of the solid-like nanolayers is assumed to be equal to the thermal conductivity of solid particles, this model for spherical nanoparticle case takes the following form:

$$k_{nf} / k_f = \frac{k_s + 2k_f + 2\left(k_s - k_f\right)\left(1+\beta\right)^3 \phi}{k_s + 2k_f - 2\left(k_s - k_f\right)\left(1+\beta\right)^3 \phi} \tag{2}$$

where $\beta$ is defined as the ratio of the nanolayer thickness to the original particle radius. On the other hand after comparing the model results for $\beta$=0.1 with existing experimental results a reasonably good agreement is obtained in the related study. In the present study, this model was used for the thermal conductivity of the nanofluids.

Brinkman [5] proposed an expression for the effective viscosity for a two-phased mixture. This model given below was used in the present study to determine the viscosity of the nanofluid.

$$\mu_{nf} = \mu_f / \left(1-\phi\right)^{2.5} \tag{3}$$

Xuan and Li [6] made an experimental study on the effective viscosity of the water-copper nanofluid and the transformer oil-water nanofluid in the temperature range 20-50$^o$C and they found that the obtained results were compatible with the Brinkman theory.

There are a number of studies in the literature on natural convection heat transfer of nanofluids. In one of these studies, Oztop and Abu-Nada [10] made a study on the

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buoyancy driven heat transfer and fluid flow in a partially heated enclosure filled with nanofluid. The results show that the heat transfer enhancement at low aspect ratios is higher than at high aspect ratios of the enclosure. Koo and Kleinstreuer [11] investigated the laminar nanofluid flow in microheatsinks and found that a high Prandtl number base fluid and a high aspect ratio channel give better heat transfer performance. Maiga et al. , 2005 [8] studied forced convection flow of nanofluids in a system consisting of uniformly heated tube and parallel, coaxial, and heated disks and observed that both the Reynolds number and the gap between disks have an insignificant effect on the heat transfer enhancement. Mirmasoumi and Behzadmehr [9] studied laminar mixed convection of $Al_2O_3$-based nanofluids in a horizontal tube and found that the secondary flow strength increases with the increase of the nanoparticle volume fraction and increasing solid volume fraction has no significant effect on the skin friction coefficient in the fully developed region, except for the entrance region.

In this study, buoyancy driven heat transfer and fluid flow of water based CuO nanofluids in a tilted enclosure with a heat conducting solid cylinder on its center is investigated numerically by the Comsol multiphysics modeling and simulation software.

## II. Analysis

The investigated geometry and the coordinate system are given in the Fig. 1. While the upper and the bottom walls of the enclosure are in adiabatic conditions, the left and the right walls are kept at constant temperatures.

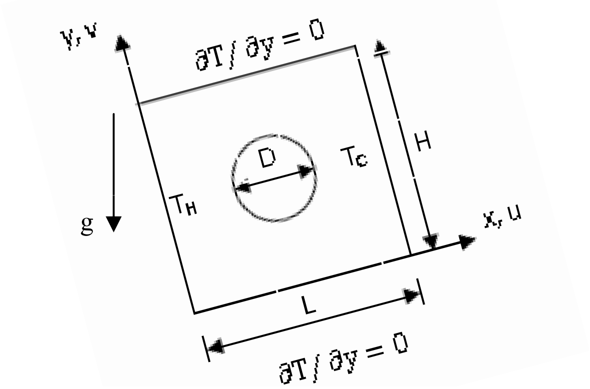


**Fig.1** Geometry and coordinate system

The flow is assumed to be Newtonian, two-dimensional, steady, incompressible and single phase. Depending on these assumptions, the governing equations can be expressed as follows:

Continuity equation

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{4}$$

Navier-Stokes equations

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial x^*}+\nu_{nf}\left[\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right]+\frac{(\rho\beta_T)_{nf}}{\rho_{nf}}g\left(T^*-T_C^*\right)\sin\theta \tag{5}$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial y^*}+\nu_{nf}\left[\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right]+\frac{(\rho\beta_T)_{nf}}{\rho_{nf}}g\left(T^*-T_C^*\right)\cos\theta \tag{6}$$

Energy equation for the nanofluid

$$u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\alpha_{nf}\left[\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right] \tag{7}$$

Energy equation for the circular cylinder

$$\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}=0 \tag{8}$$

where $u^*$ and $v^*$ are velocity components in x and y directions, respectively, g is gravitational acceleration, $\beta_T$ is coefficient of thermal expansion, $\nu$ is kinematic viscosity, $\alpha$ is the thermal diffusivity.

The nondimensional variables used for the nondimensionalization of the governing equations can be defined as follows:

$$x=\frac{x^*}{L},\ y=\frac{y^*}{L},\ u=\frac{u^*}{\alpha_f/L},\ v=\frac{v^*}{\alpha_f/L},\quad P=\frac{L^2}{\rho_f\alpha_f^2}P^*,\ T=\frac{T^*-T_C^*}{T_H^*-T_C^*} \tag{9}$$

Nondimensional governing equations can therefore be obtained as follows:

Nondimensional continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{10}$$

Nondimensional Navier Stokes equations

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial x}+\frac{\nu_{nf}}{\nu_f}Pr\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]+\frac{(\rho\beta_T)_{nf}}{\rho_{nf}\beta_{T,f}}RaPrT\sin\theta \tag{11}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial y}+\frac{\nu_{nf}}{\nu_f}Pr\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]+\frac{(\rho\beta_T)_{nf}}{\rho_{nf}\beta_{T,f}}RaPrT\cos\theta \tag{12}$$

Nondimensional energy equation for the nanofluid

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\alpha_{nf}}{\alpha_f}\left[\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right] \tag{13}$$

Nondimensional energy equation for the heat conducting circular cylinder

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0 \tag{14}$$

Rayleigh and Prandtl numbers are defined as follows:

$$Pr=\frac{\nu_f}{\alpha_f},\quad Ra=\frac{g\beta_{T,f}L^3\left(T_H^*-T_C^*\right)}{\nu_f\alpha_f} \tag{15}$$

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Thermophysical properties of nanofluid are defined as follows:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{16}$$

$$\left(\rho c_p\right)_{nf} = (1-\phi)\rho_f c_{p,f} + \phi\rho_s c_{p,s} \tag{17}$$

$$\left(\rho\beta_T\right)_{nf} = (1-\phi)\rho_f \beta_{T,f} + \phi\rho_s \beta_{T,s} \tag{18}$$

The appropriate boundary conditions for the nondimensional governing equations could be given as:

$$T(0,y)=1\ ,\ T(1,y)=0\ ,\ \left.\frac{\partial T}{\partial y}\right|_{x,0}=0\ ,$$
$$\left.\frac{\partial T}{\partial y}\right|_{x,1}=0\ ,\ u\big|_s=0\ ,\ v\big|_s=0 \tag{19}$$

The boundary conditions on the surface of the solid cylinder could be given as:

$$\vec{n}\cdot\left(\vec{q}_{s,1}-\vec{q}_{s,2}\right)=0\ ,\ T_{s,1}=T_{s,2} \tag{20}$$

where 1 and 2 stand for fluid and cylinder. $\vec{n}$ is unit normal vector. From the continuity condition given above, an extra parameter $k_r$ emerges. The parameter $k_r$ is defined as the ratio of the thermal conductivity of heat conducting circular cylinder to the thermal conductivity of the base fluid.

$$k_r = k_{cyl} / k_f \tag{21}$$

Local Nusselt number can be defined as:

$$Nu = -\frac{k_{nf}}{k_f}\left.\frac{\partial T}{\partial \eta}\right|_{\eta=0} \tag{22}$$

where η is the outer direction normal to the surface, k is the thermal conductivity.

## III. Results and Discussion

In this study, natural convection of water-based CuO nanofluids in a tilted enlosure with a heat conducting solid circular cylinder is studied numerically. Computational results are obtained by Comsol Multiphysics modelling and simulation software. A direct solver is used for all the examined values of inclination angle, solid volume fraction and Rayleigh number. The ratio of the cylinder diameter to the enclosure is taken as 0.25. The ratio of the nanolayer thickness to the original particle radius is taken as β=0.1. The thermal conductivity ratio is taken as $k_r$=1. The effects of inclination angle, solid volume fraction and Rayleigh number on the velocity and temperature fields are investigated for the values of inclination angle, 0, 30, 45 and 60; for the values of solid volume fraction, 0, 0.04, 0.08 and for the values of Rayleigh number, $10^4$, $10^5$, $10^6$. Computational results are obtained for water based CuO nanofluid. The thermophysical properties of the fluid and nanoparticle are given in Table I.

Temperature and velocity fields are shown in Figs. 2-5 for various values of inclination angle, solid volume fraction, and the Rayleigh numbers. As it is seen from the figures, the heated fluid particles rise along the left wall as a result of buoyancy forces until they reach near the top wall, where

TABLE I
THERMOPHYSICAL PROPERTIES OF THE BASE FLUID AND THE NANOPARTICLE

| Property | Water | CuO |
|---|---|---|
| $\rho$ (kg/m$^3$) | 997.1 | 6500 |
| $c_p$ (J/kgK) | 4179 | 536 |
| $k$ (W/mK) | 0.613 | 20 |
| $\alpha$x$10^7$ (m$^2$/s) | 1.47 | 57.5 |
| $\beta_T$x$10^6$ (1/K) | 210 | 51 |

they turn rightward, towards the sidewalls. Then they turn downward near those walls and move towards the bottom wall while they are cooled. Finally, the restriction imposed by the bottom wall forces the fluid particles to turn leftward. The flow path is completed as the colder fluid is entrained to the ascending flow along the heated wall. As it can also be seen from the figures that the flow structure evolves toward the boundary layer regime with increasing Rayleigh numbers. This is clear by the increasing steepness of the velocity and temperature profiles near the walls. The strength of convective circulation increases considerably with an increase of Rayleigh number due to higher buoyancy forces. The circulation strength also increases with an increase of solid volume fraction. This is due to the increase in thermal energy transport from the hot wall to the fluid particles. Note that an increase in the viscous forces with increasing solid volume fraction is also a fact, although it does not compensate for the increase in the thermal energy from the hot wall to the nanofluid due to increased thermal conductivity. Hence, convective circulation strengthens with increasing solid volume fraction. With an increase of inclination angle, circulation intensity shows first an increase then a decrease. As the inclination angle is increased, fluid particles is directed towards to right sidewall in a more smooth way. This creates a positive effect on the circulation intensity. On the other hand, the fluid particles begin to move away from the hot wall without heated enough as the inclination angle is increased. This creates a negative effect on the circulation intensity. Depending on the dominancy of these two effects, circulation intensity shows first an increase then a decrease.

The variation of the average Nusselt number on the left wall with the inclination angle, solid volume fraction and Rayleigh number are given in Table 2. It is obviously seen from the results that as Rayleigh number increases, a considerable increase is seen at the average Nusselt number depending on the strengthening convective circulation. There is also a remarkable increase in the average heat transfer rate with increasing solid volume fraction due to the increase in thermal energy transport from the hot wall to the fluid particles. The average heat transfer rate shows an increasing trend and then a decreasing trend as the inclination angle is increased. The maximum heat transfer takes place at θ=45 deg for Ra=$10^4$ and at θ=30 deg for Ra=$10^5$ and $10^6$.

## IV. CONGLUSION

Natural convection in an inclined enclosure which includes a heat conducting circular cylinder on its center is studied. Numerical results are obtained with Comsol Multiphysics modeling and simulation software. The results show that adding nanoparticles to the base heat transfer fluid enhances heat transfer rate significantly. The results also show that average Nusselt number shows first an increase and then a decrease as the inclination angle is increased. The maximum heat transfer takes place at θ=45 deg for $Ra=10^4$ and at θ=30 deg for $Ra=10^5$ and $10^6$.

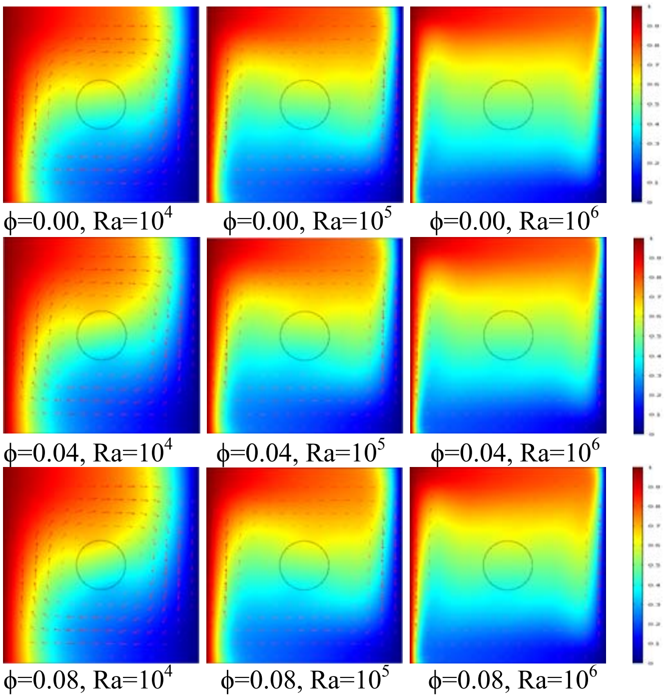


**Fig. 2** Temperature and velocity fields for inclination angle θ=0.

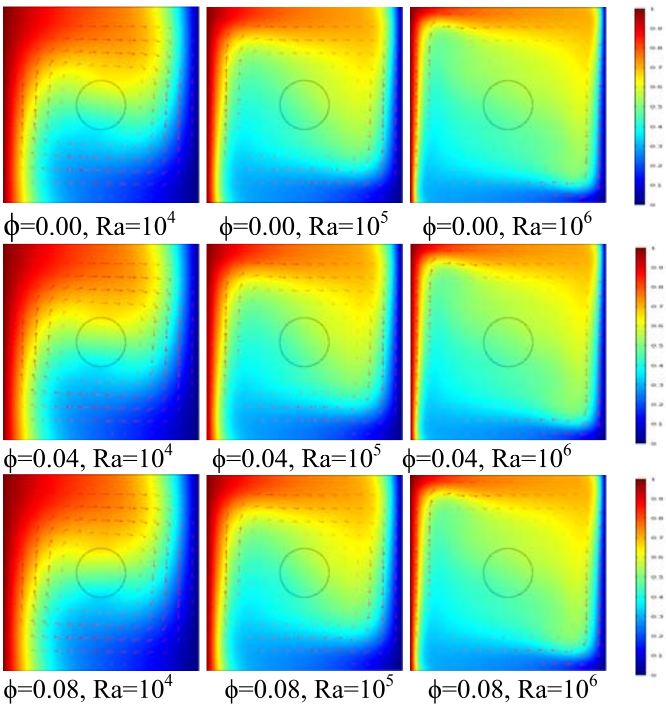


**Fig. 3** Temperature and velocity fields for inclination angle θ=30.

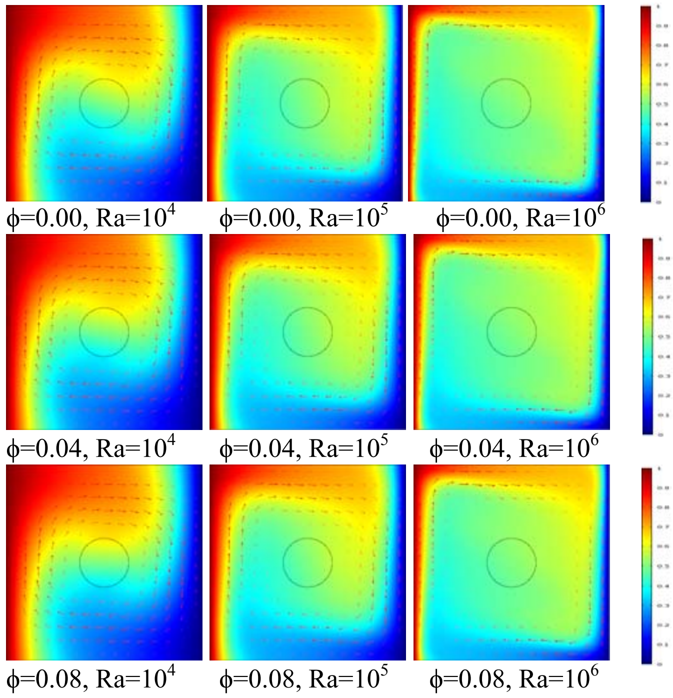


**Fig. 4** Temperature and velocity fields for inclination angle θ=45.

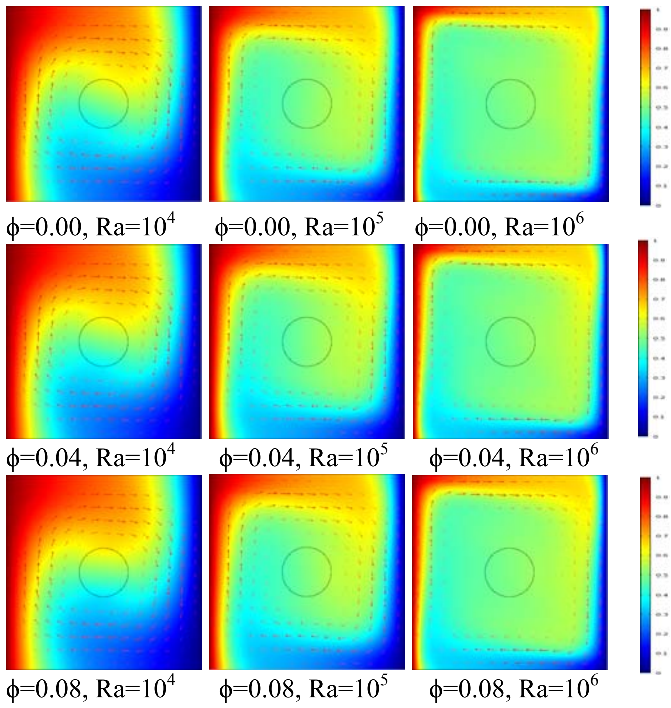


**Fig. 5** Temperature and velocity fields for inclination angle θ=60.

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TABLE I
THE AVERAGE NUSSELT NUMBER

| $\theta$ | Ra | $\phi$ | $Nu_a$ |
|---|---|---|---|
| 0 | $10^4$ | 0.00 | 2.26 |
| | | 0.04 | 2.41 |
| | | 0.08 | 2.55 |
| | $10^5$ | 0.00 | 4.73 |
| | | 0.04 | 5.09 |
| | | 0.08 | 5.44 |
| | $10^6$ | 0.00 | 9.22 |
| | | 0.04 | 9.97 |
| | | 0.08 | 10.72 |
| 30 | $10^4$ | 0.00 | 2.54 |
| | | 0.04 | 2.71 |
| | | 0.08 | 2.87 |
| | $10^5$ | 0.00 | 5.07 |
| | | 0.04 | 5.46 |
| | | 0.08 | 5.85 |
| | $10^6$ | 0.00 | 9.61 |
| | | 0.04 | 10.38 |
| | | 0.08 | 11.15 |
| 45 | $10^4$ | 0.00 | 2.56 |
| | | 0.04 | 2.74 |
| | | 0.08 | 2.91 |
| | $10^5$ | 0.00 | 4.94 |
| | | 0.04 | 5.34 |
| | | 0.08 | 5.74 |
| | $10^6$ | 0.00 | 9.08 |
| | | 0.04 | 9.81 |
| | | 0.08 | 10.55 |
| 60 | $10^4$ | 0.00 | 2.52 |
| | | 0.04 | 2.70 |
| | | 0.08 | 2.87 |
| | $10^5$ | 0.00 | 4.72 |
| | | 0.04 | 5.11 |
| | | 0.08 | 5.51 |
| | $10^6$ | 0.00 | 8.41 |
| | | 0.04 | 9.12 |
| | | 0.08 | 9.84 |

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