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Buckling Finite Element Analysis Of Beams and Frames

Messaoud Bourezane

***Abstract*—The classical method for buckling analysis of frames derives the geometric stiffness from the governing equation of the second order for bending with axial force, resulting in so-called stability functions that yield the exact solution for constant flexural stiffness and constant axial force. The approach to buckling used in the present work is one in which load is incremented and the determinant of the system matrix, is monitored. The nonlinear equilibrium equations are solved using Newton-Raphson method for which number of examples is given. Consequently fewer elements with the present formulation (with curvature correction) are needed to yield results of comparable accuracy. This is demonstrated with the analysis of several simple example structures by comparing the results of these examples and the so-called stability functions (the exact solution).**

***Index Terms* — Beam, frame, buckling, stability functions, Newton-Raphson method.**

I. INTRODUCTION

There are a number of practical phenomena in structures which simply cannot be described using a linear formulation. Among these are buckling, the behavior of cable nets and fabric structures, the formation of plastic hinges, and the nonlinear material behavior of concrete, structural collapse.

Elementary theory is a strange mix in which behavior is described in the " undeformed configuration " fig 2. That is, in elementary theory the effect of deformations even though they are computed – are neglected when writing the equations of equilibrium and motion. Biot [1] has used the structure of fig 1 to characterize these "geometric" difficulties which inherent in elementary theory. This structure is said to be "geometrically" unstable.

Real structures are in equilibrium in deformed configuration fig 3, not their undeformed configuration as implied by elementary theory. Using the deformed configuration implies nonlinear analysis in the present study which will typically involve applying Newton's method.

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Bourezane Messaoud, Civil Engineering Department, University of Biskra 07000 Biskra- Algeria, (e-mail: bourmess@yahoo.fr).

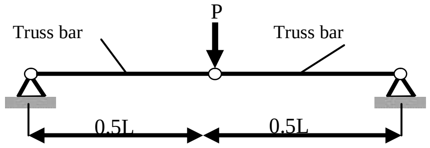

Fig 1 Geometrically Unstable Structure

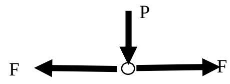

Fig 2 Undeformed joint Equilibrium

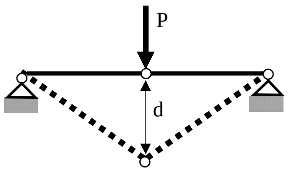

Fig 3 Deformed equilibrium

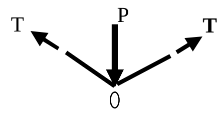

Fig 4 Joint free body diagram

In this study the formulation of frame elements for buckling analysis is based on appropriate interpolation function for the transverse and axial displacements of the member (displacement formulation) in which corrections for the effects of large displacements upon extensional and flexural stresses are taken into account.

II. SOLUTION TO NONLINEAR EQUILIBRIUM EQUATION

A structure must satisfy the conditions of equilibrium (deformed equilibrium in the nonlinear case).

A typical step of this analysis can be described as follows. Given a fixed joint load {P} matrix and a starting configuration which is not in equilibrium with this joint load matrix (if it were there would be no analysis to perform), the following sequence of actions must be taken:

- Compute the unbalanced load {P'}. Since the member forces {F} are not in equilibrium with the given load {P}, the unbalanced load can be computed as:

$$\{P'\} = \{P\}-[T]^T\{F\} \tag{1}$$

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Where [T] is the transformation matrix

▪ Solve for the incremental displacements {δ}. Under the unbalanced load {P'}. This computation involves solve the system (2):

$$[K_E + K_G]\{\delta\} = \{P'\} \qquad (2)$$

Where $[K_E]$ and $[K_G]$; are the usual element stiffness and element geometric stiffness respectively.

▪ Compute new member forces {F} as:

$$\{d\}= [T]^T\{\delta\} \qquad (3)$$

$$\{F\} = [ke]^{-1}\{d\} \qquad (4)$$

▪ Repeat this sequence of calculations. Computation stops when the unbalanced load {P'} satisfies a given convergence criteria.

For both users and developers of nonlinear finite element programs, an understanding of the fundamental concepts of nonlinear finite element analysis is essential, so that the reader should consult references [2, 3, and 4] for complete details.

III. NUMERICAL EXAMPLES

In this section, three numerical examples are given to demonstrate the accuracy, and effect of curvature correction. In all examples the details of the deflections are provided. The first one involves an analysis for large deflections on a cantilever beam whereas the last two involve frame buckling.

A. Cantilever Beam (figure 5)

The first problem considered is a cantilever beam; load and properties are shown in Figure 5.

E= $2x10^6$N/CM^2, I=5 cm^4
A=10 .CM2, L=100.CM, M=10 ^{5}N.CM

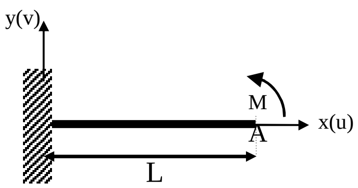

Fig 5 Cantilever beam with end moment, and beam properties

The total moment M of 10^5 KN.cm was applied in (n) equal increments as shown in the table I.

For each increment, equilibrium iteration where performed until the convergence tolerances were satisfied. Table I shows the horizontal deflection (u), vertical deflection (v), and rotation (θz). From the results presented in the table I , we observe that Lagrangian formulation produces satisfactory results. We also note that Eulerian and Lagrangian with curvature corrections produce quite accurate results for different load increments.

TABLE I DIPLACEMENTS AT A

Type of Method	Load steps	Total cycles	N. elements	V(cm)	U(cm)	ϕ (RAD)
Eulerian *	10	40	10	40.22	-11.11	0.77
Eulerian **	10	10	50	45.80	-15.86	0.98
Lagrangian **	1	6	1	46.05	-15.73	1.0
Exact solution	1	6	1	45.97	-15.85	1.0
Linear solution	1	1	1	50.0	0.0	1.0

* Without curvature correction, ** with curvature correction

B. Plane Frame (figure 6)

The second problem is a plane frame subjected to concentrated load (P) at nodes 1, and 2, plane frame properties are shown in Figure 6.

E=29x106KN/cm^2, I=800cm^2 A=100cm^2, L=144cm

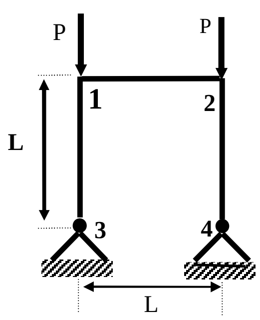

Fig 6 Plane frame buckling A

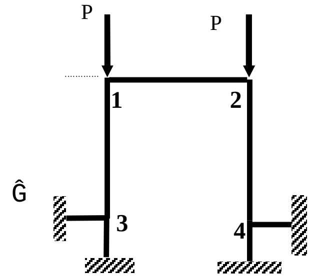

Fig 7 Plane frame buckling B

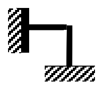

Fig 8 Fictitious members

The first load step is in this case specified to be 220,000 KN. Note that fictitious bars with I=0. And 1

insh length has been added to model the pinned supports. Also large areas have been used in some members to simulate Timoshenko's neglect of member length change. By scanning the output file it's found that negative term appears on the diagonal of the system matrix during the 9th load step. The buckling load lies, therefore between the 8th and 9th load step. This implies that the buckling load occurs between $\left(\frac{9}{10}22000 \rightarrow \frac{10}{10}22000\right)$ means between (198000 and 220000) which agree with Timoshenko's result [5]:

$$P_{cr} = \frac{1.82}{L^2}EI \;\; =2.04 \times 10^6 KN$$

TABLE II DISPLACEMENTS (LOAD STEP 8 – ITERATION 4 AND BUCKLING LOAD)

Nodes	U(cm)	V(cm)	ϕ (RAD)
1	0.594D-2	-.990D-1	-0.127D-4
2	0.594D-2	-.990D-1	-0.127D-4
3	-0.152D-6	-.683D-3	-0.568D-4
4	-0.152D-6	-.683D-3	-0.568D-4
Buckling Load (present study) =2.07 10^6KN			
Buckling Load – Timoshenko [5] = 2.04 10^6KN			

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C. Plane Frame (figure 9)

The third problem is a plane frame whose details are given in figure 9 has been used in the finite element literature.

TABLE III PLANE FRAME PROPERTIES

Element	Length(cm)	Area (cm^2)	Inertia(cm^4)
2-1	144	0.387 E+01	0.125 E+01
3-2	120	0.424 E+01	0.150 E+01
2-4	96	0.335 E+01	0.937 E+01

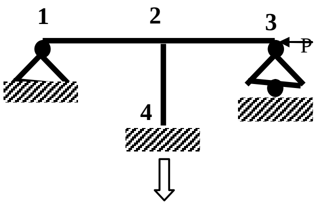

Fig 9 Plane frame A

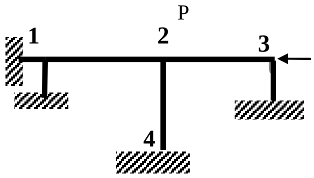

Fig 10 Plane frame B

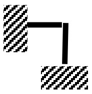

Fig 11 Fictitious members

This example which is described in Figure 9 is taken from the test problems that appear in the ANSYS (1987, example 12) manual. Ten load steps with four iterations per load are used to solve the problem, with starting load of 30000KN. The results of the last iteration of the last converged load step (load step 8, iteration 4). The buckling load is bounded between ($\frac{9}{10}\times 30000 \rightarrow \frac{10}{10}\times 30000$) that means between (27000 and 30000). This result agrees with the result of [6]

TABLE IV DISPLACEMENTS (LOAD STEP 8 - ITERATION 4) AND BUKLING LOAD

Nodes	U(cm)	V(cm)	ϕ(rad)
1	-0.900E-7	-0.337E-12	-0.317E-3
2	-0.335E-1	-0.740E-05	0.321E-3
3	-0.590E-1	-0.173E-04	-0.217E-3
Buckling Load (present study) =28000 KN			
Buckling Load [6]=27789 KN			

IV. CONCLUSION

The main objective of this study is to make use of the correction for the effect of large displacement upon extensional and a flexural stresses. The residual load method with only the extensional correction gives poor results. Note that for this Lagrangian method only a single element and a single load step was used and yet the results are satisfactory. Much of this greater economy is due to the use of four freedoms and cubic interpolation to interpolate the extensional displacement u.

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