

On The Oscillatory Behavior of Transient Rayleigh Benard Convection of Air for 2D Channel Flow at Moderate Rayleigh Numbers

Atta Sojoudi, Farbod Vakilmoghaddam, Reihaneh Neishabouri, Suvash C. Saha

Abstract—Unsteady numerical simulation of Rayleigh Benard convection heat transfer from a 2D channel is performed. The oscillatory behavior is attributed to recirculation of ascending and descending flows towards the core of the channel producing organized rolled motions. Variation of the parameters such as Reynolds number, channel outlet flow area and inclination of the channel are considered. Increasing Reynolds number (for a fixed Rayleigh number), delays the generation of vortices. The reduction in the outflow area leads to the later and the less vortex generation. As the time progresses, more vortices are generated, but the reinforced mean velocity does not let the eddies to enter the core of the channel. Therefore, they attach to the wall and reduce the heat transfer area. The inclination of the channel (both positive and negative) induces the generated vortices to get closer to each other and make an enlarged vortex.

Index Terms—Numerical solution, Unsteady Rayleigh Benard convection, 2D channel flow, vortex generation

I. INTRODUCTION

THE onset of free convection in fluid contained between two horizontal plates and heated from below (The so called Rayleigh Benard problem) is of the great interest during past decades. The long cellular structure (convection cells) tends the fluid to behave oscillatory. Rayleigh Benard convection shows important aspects of many nonlinear processes such as dynamics of chemical reaction or diffusion systems. It is treated as the problems of transition mechanisms in hydrodynamics [1, 2]. A large number of research related to the Rayleigh Benard convection are devoted to steady state solutions. Arroyo et al. [3] conducted a three dimensional study of the flow velocity field to investigate the Rayleigh Benard convective flow using particle image velocity meter. During their experiment, four well-defined rolls separated by non-flow surfaces was found. Heat transfer enhancement in Rayleigh

Benard convection was investigated by Domaradzki [4]. The author employed direct 3D numerical simulation and found out that decreasing the size of convective rolls and the level of three dimensionality in connection with forcing, increases heat transfer by 15 - 20%. His experimental study at lower Rayleigh numbers conducted in mild unsteadiness. Recently, researchers simulate the turbulent Rayleigh Benard convection which occurs at high Ra . Yang and Zhu [5] used fourth order upwind scheme to validate their numerical simulation of turbulent Rayleigh Benard convection with experimental data. They declared that there might be simpler fundamentals for turbulent convection modeling. Ebert et al. [6] studied the mean temperature profiles and local heat flux in turbulent Rayleigh Benard convection for large rectangular enclosure. They found that the heat flux is uneven distributed over the plate surface and three different behaviors in the temperature profile were proved.

Prandtl number dependency of Rayleigh Benard convection has also been investigated during past decade. Lage et al. [7] summarized the results of numerical study to show the critical Rayleigh number increased substantially as the Pr became very small, near 10^{-4} , the natural shape of a single roll in this Pr range is square. Large Pr behavior of Rayleigh Benard convection was established by Wang [8]. The author demonstrated the validity of the infinite Pr model as an approximate of Boussinesq system at large Pr value on finite and infinite time interval.

Stability analysis of Rayleigh Benard problem have received relatively extensive numerical attention. Nicolas et al. [9] presented a study of the linear hydrodynamic stability of purely conductive Poiseuille flow in three-dimensional horizontal rectangular channels uniformly heated from below. They declared that, when the Ra is higher than the critical value, the instabilities were shown to be three-dimensional horseshoe-shaped transversal rolls for Reynolds numbers were smaller than the critical value Re^* . Mosta and Sibanda [10] investigated the linear viscous stability of thermally stratified plane Poiseuille channel flow over a compliant surface. The Chebyshev collocation spectral method is used to solve the eigenvalue system (Orr-Sommerfeld-like eigenvalue problem which is coupled to an energy equation). The critical Reynolds numbers, wave numbers, wave speeds, and curves of neutral stability are obtained for a wide range of compliant wall parameters and buoyancy parameters. Gage and Reid [11] showed that except very slow flow, the stability criteria are the same as those of Rayleigh Benard convection. In the thermal entrance region of plane Poiseuille flow heated isothermally

Manuscript received February 7, 2012; revised March 17, 2012.

Atta Sojoudi is with Department of Mechanical Engineering Sharif University of Technology, Iran (corresponding author to provide phone: +989141100874; e-mail: meoiotu@yahoo.com).

Farbod V. Moghaddam is with Department of Mechanical Engineering, Tabriz University, Iran (e-mail: farbod.vakili@yahoo.com).

Reihaneh Neishabouri is with Department of Mechanical Engineering, Tabriz University, Iran (e-mail: rayni_m@yahoo.com).

Dr. Suvash C. Saha is Postdoctoral Research Fellow School of Chemistry, Physics and Mechanical Engineering of Queensland University Technology (QUT), Brisbane, Australia (e-mail: s_c_saha@yahoo.com)

from below, Hwang and Cheng [12] first conducted stability analysis. The onset position of secondary flow, which represents the starting point of mixed convection, is predicted as a function of the Prandtl number, Reynolds number and Rayleigh number. As expected, the critical position moves upstream as Ra increases and an increase in Re makes the system more stable.

Transient solution of Rayleigh Benard problem is limited in the literature. Non linear dynamics of Rayleigh Benard problem with time depended terms needs powerful scheme of discretization system. Kim et al. [13] analyzed the onset of buoyancy-driven instability in initially quiescent fluid layers having the various boundary conditions. The stability limits which are related to the onset time of instabilities were presented as a function of the Rayleigh number and the Prandtl number. The stability results predicted that the onset time of convective instability decreases with increasing Ra and Pr . Earlier, Neisen and Sabersky [14] studied the unsteady heat transfer in Rayleigh Benard convection experimentally. They designed the experiment to examine the effect of different heating rates on the onset of convection with time. Their experiment showed that as the heat flux at the lower surface is increased the temperature difference required for the initiation of convection increases while the time for the onset of motion decreases.

Extensive investigations were performed for the Rayleigh Benard problem, but few studies devoted to unsteady case for channel flow. It is important for a thermal system designer to know what happens when the setup starts from beginning or what the mechanism of the vortices generation and growth is. In the present study, we have performed unsteady Rayleigh Benard problem in a 2D channel. It is considered different cases including variations of Reynolds number ($Re = 50, 70, 90, 120$), decreasing outlet flow area and channel inclination ($-90^\circ < \phi < 90^\circ$) using air as working fluid.

II. PHYSICAL MODELING

A. Model assumptions and equations

The configuration of the two-dimensional channel, of length $L=1m$ and height $H=0.1m$, is sketched in Fig. 1. Both the upper and lower horizontal walls are assumed at constant temperature. The top and the bottom walls are at T_c and T_h , respectively ($T_h > T_c$). Air density is not affected by pressure changes, but it depends on temperature. Density on the buoyancy term is temperature dependent, so the Boussinesq approximation is used. All other remaining thermo physical properties of the fluid are assumed to be constant. The flow is also assumed to be laminar to ensure that mixed convection heat transfer presents in the channel. Thermal radiation heat transfer between the walls is negligible, and the energy term due to viscous dissipation is not taken into account.

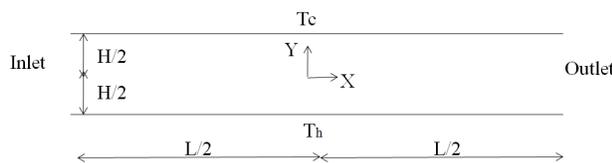


Fig. 1. Configuration of the two-dimensional channel

Mixed convection fluid flow inside the channel obeys the mass conservation equation that reads, in its dimensionless

form:

$$\frac{\partial \rho_*}{\partial \tau} + \frac{\partial(\rho_* u_*)}{\partial x_*} + \frac{\partial(\rho_* v_*)}{\partial y_*} = 0 \quad (1)$$

the momentum equations:

$$\frac{\partial(\rho_* u_*)}{\partial \tau} + \frac{\partial(\rho_* u_* u_*)}{\partial x_*} + \frac{\partial(\rho_* v_* u_*)}{\partial y_*} = -\frac{\partial p_*}{\partial x_*} + Pr \left(\frac{\partial^2 u_*}{\partial x_*^2} + \frac{\partial^2 u_*}{\partial y_*^2} \right) \quad (2)$$

$$\frac{\partial(\rho_* v_*)}{\partial \tau} + \frac{\partial(\rho_* u_* v_*)}{\partial x_*} + \frac{\partial(\rho_* v_* v_*)}{\partial y_*} = -\frac{\partial p_*}{\partial y_*} + Pr \left(\frac{\partial^2 v_*}{\partial x_*^2} + \frac{\partial^2 v_*}{\partial y_*^2} \right) + Ra Pr T_* \quad (3)$$

and the energy equation:

$$\frac{\partial T_*}{\partial \tau} + \frac{\partial(u_* T_*)}{\partial x_*} + \frac{\partial(v_* T_*)}{\partial y_*} = \frac{\partial^2 T_*}{\partial x_*^2} + \frac{\partial^2 T_*}{\partial y_*^2} \quad (4)$$

The dimensionless parameters appearing in the foregoing equations are the space coordinates, the velocity components, the temperature, the driving pressure and density, defined, respectively, as:

$$(x_*, y_*) = (x, y) / H \quad (5)$$

$$(u_*, v_*) = (u, v) / (\alpha_f / H) \quad (6)$$

$$T_* = (T - T_c) / (T_h - T_c) \quad (7)$$

$$p_* = (p + \rho_0 g y) / [\rho_0 f (\alpha_f / H)^2] \quad (8)$$

$$\rho_* = \rho / \rho_0 \quad (9)$$

$$Pr = \nu / \alpha_f \quad (10)$$

$$Re = \frac{uH}{\nu} \quad (11)$$

$$Ra = g\beta(T_h - T_c)H^3 / (\nu\alpha_f) \quad (12)$$

B. Heat transfer parameters

The total dimensionless heat transferred from the high temperature wall to the low temperature one is here represented by the overall Nusselt number, \overline{Nu} , defined as:

$$\overline{Nu} = \frac{\int_{-1/2}^{1/2} -k_f \left(\frac{\partial T}{\partial y} \right)_{y=-H/2} dy}{k_f [(T_h - T_c) / H] H} = \int_{-1/2}^{1/2} - \left(\frac{\partial T_*}{\partial y_*} \right)_{y_*=-1/2} dy_* \quad (13)$$

and the local Nusselt number is defined as:

$$Nu = - \left(\frac{\partial T_*}{\partial y_*} \right)_{y_*=-1/2} \quad (14)$$

III. NUMERICAL MODELING

The laminar model equations described above were solved using commercial flow simulation software FLUENT (version 6.3) [15]. Transient simulations have been carried out with a time step size of 0.001 s. For each time step, convergence criteria for sum of normalized residue were set to 10^{-3} for momentum and 10^{-6} for energy equation. Convergence has been ensured at every time step due to enough sub-iterations have been specified. It took 55 sub-iterations in order to reach the convergence. As the

solution proceeds transiently, it took less than 10 sub-iterations at every time step for convergence at every level. The numerical simulations were performed on a grid size of 200×80 along x and y directions respectively (see Fig. 2). A numerical experiment carried out for grid sensitivity verified that grid chosen in the present study was good enough for the accurate results and further increase in the grid size led to insignificant change (maximum 0.8% change in averaged velocity) and unnecessary increase in computational load. The momentum and energy equations have been discretized with the first order upwind scheme, and for the pressure equation, PRESTO scheme has been used. The two horizontal walls of the channel are assumed to hold the no-slip condition.

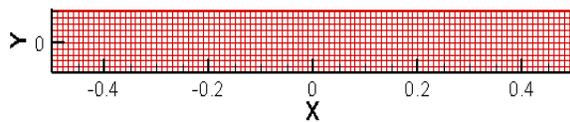


Fig. 2. The sample grid distribution

IV. RESULTS AND DISCUSSIONS

For the validation purpose, transient numerical simulations are performed for $Pr = 0.025$ and different values of the Ra in the range of 1000 to 3000. In comparison with the diffusive heat transport, convection enhances heat transfer rate. Initially, Nu , which clearly indicates the onset of convection, has been determined from the CFD simulations and validated with the available experimental data. Figure 3 shows the comparison of predicted Nu versus Ra with the experimental data conducted by Rossby [16] (Inlet and out let of the channel have been insulated during this simulation to create the Rossby's experimental conditions). It can be seen that the model predictions are in good agreement with the experimental data (Fig. 3).

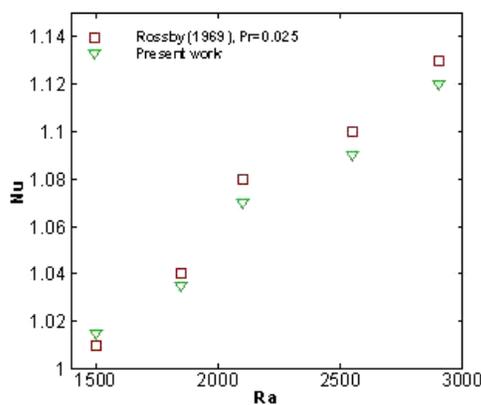


Fig. 3. Comparison of predictions of Nu with the experimental data

A. Variation of Reynolds number

In this section, we have studied all cases at a fixed Rayleigh number ($Ra = 10^6$) to diagnose the effects of Reynolds number variation. We have used four different Reynolds numbers, $Re = 50, 70, 90, 120$. y velocity [17] is a suitable parameter to realize the streamline's oscillations which can influence heat transfer

and pressure drop parameters. If a straight channel flow does not include any eddy, there will exist no oscillation in y velocity plots. Based on this, it enables us to respond some questions related to the size and number of vortices through the channel. Fig. 4 (a, b and c) shows the variation of dimensionless y velocity through the channel.

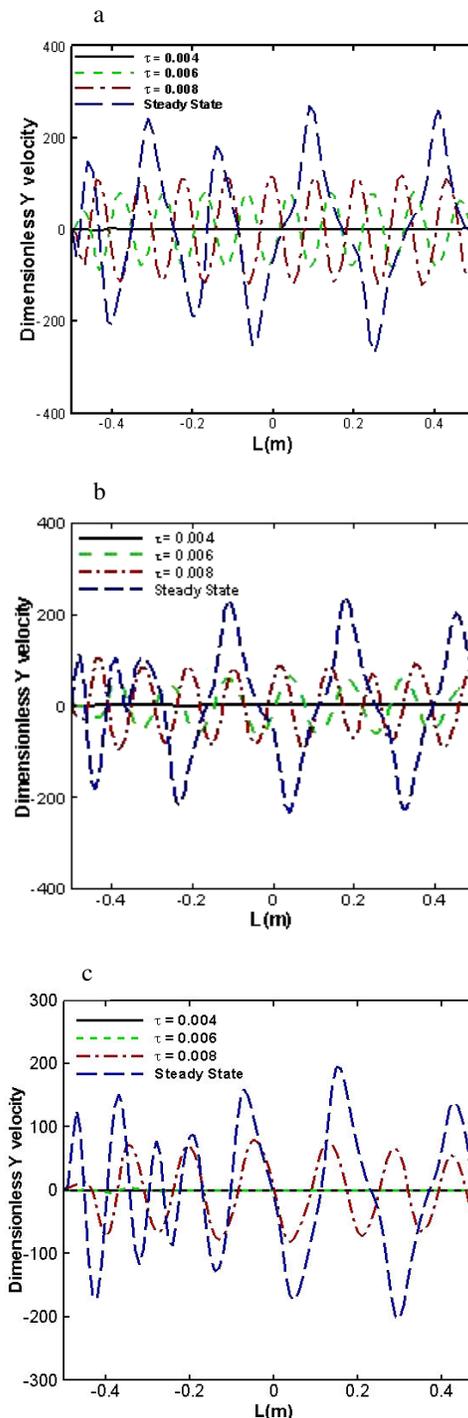


Fig. 4. Variation of y velocity for $Ra=10^6$ at different τ a) $Re=50$ b) $Re=70$ c) $Re=90$

As the time passes, reduction in number of vortices happens owing to sticking of the vortices and making a big one. Whenever the air flow reaches to a vortex, the flow is deviated from main way. Therefore the y velocity increases to pass it and the larger vortices in size, the higher y velocity

in value. A reduction in y velocity amplitude reveals that the size of eddies decreases by increasing Reynolds number. Figure 5 shows the variation of local Nusselt number for three Reynolds number. It can be understood that as the time passes, large variation of Nu is seen due to variation in y velocity. When the flow has larger negative y velocity, the local Nusselt number increases, because the negative y velocity shows that the flow is going toward the hot wall, which happens between two adjacent vortices (noting that the Nu has been defined for hot wall). So the higher negative y velocity results in higher Nusselt number. For instance, among the 0.2 m and 0.4 m of $Re=50$ for all

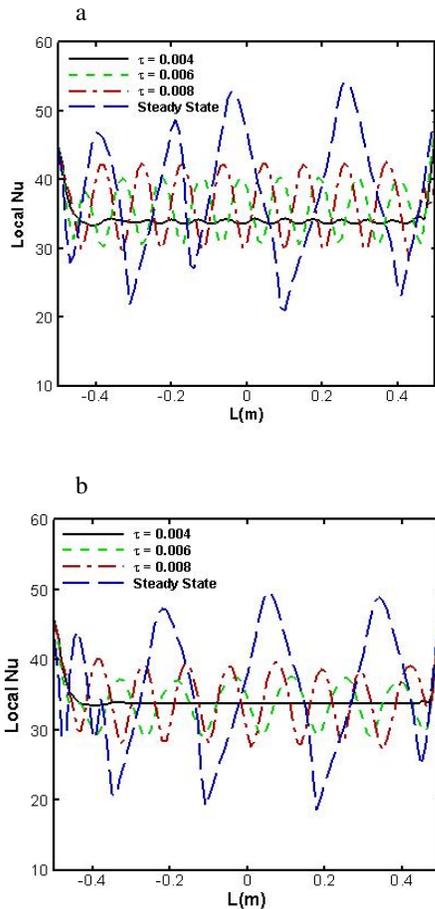


Fig. 5. Variation of local Nusselt number of bottom wall at a) $Re=50$ and b) $Re=70$ for $Ra=10^6$ at different τ

dimensionless times, an increase in local Nusselt number is seen. When the local Nu is ascending, it refers to the y velocity hitting the bottom wall and when the local Nu is descending, it is due to the heat removal of eddy (the flow is passing through top of this eddy). This phenomena is regarded as the oscillatory behavior of local Nu .

B. Decreasing outflow area

In this section, we have studied the effect of decreasing the outflow area. We define the ratio $A = A_{out}/A_{in}$. Fig. 6 shows the variation of average Nusselt number. As the time passes, more vortices are generated, but the reinforced mean velocity does not let the eddies to enter the core of the channel. They attach to the wall and reduce the heat transfer area. Also a slight

decrease in Nusselt number is seen according to Fig. 6. For steady state consideration, $A=0.6$ has more attached vortices and considerable region of wall is not participated in heat transfer, so it will have minimum Nu (Fig. 6 -a).

C. Inclination of channel

This section is devoted to the effect of the inclination of the channel. To investigate the heat transfer coefficient, Fig. 7 shows that:

- As the time passes, local Nusselt number is increased for positive inclinations but for negative inclination Nu is decreased.
- Increasing or decreasing ϕ from horizon results in higher Nu .
- For positive inclination, Nu decreases through the channel, because the gravity contradicts the velocity profile and reinforces the free convection heat transfer (Fig. 7a, b).

For negative inclinations, Nu increases through the channel, because the gravity helps velocity profile to weaken the free convection (Fig 7c, d).

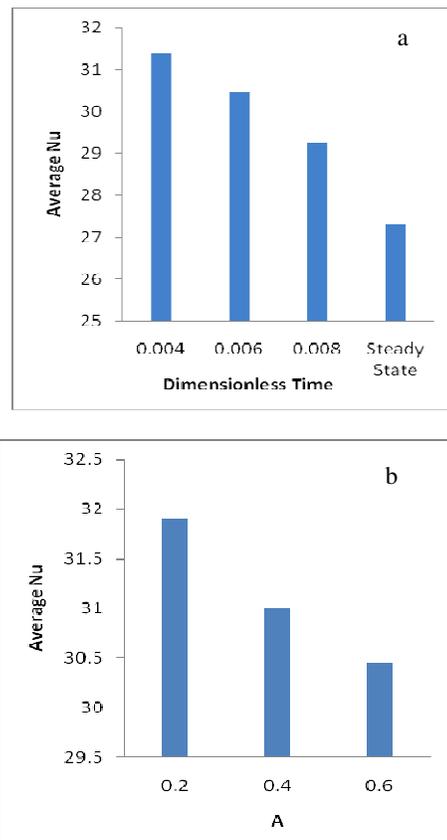


Fig. 6. Variation Of average Nusselt number a) at different dimensionless time b) for steady state consideration at different A

V. CONCLUSION

A parametric study of transient Rayleigh Benard convection heat transfer from a 2D channel is presented numerically. Variation of parameters such as Reynolds number, channel outlet flow area and the inclination of

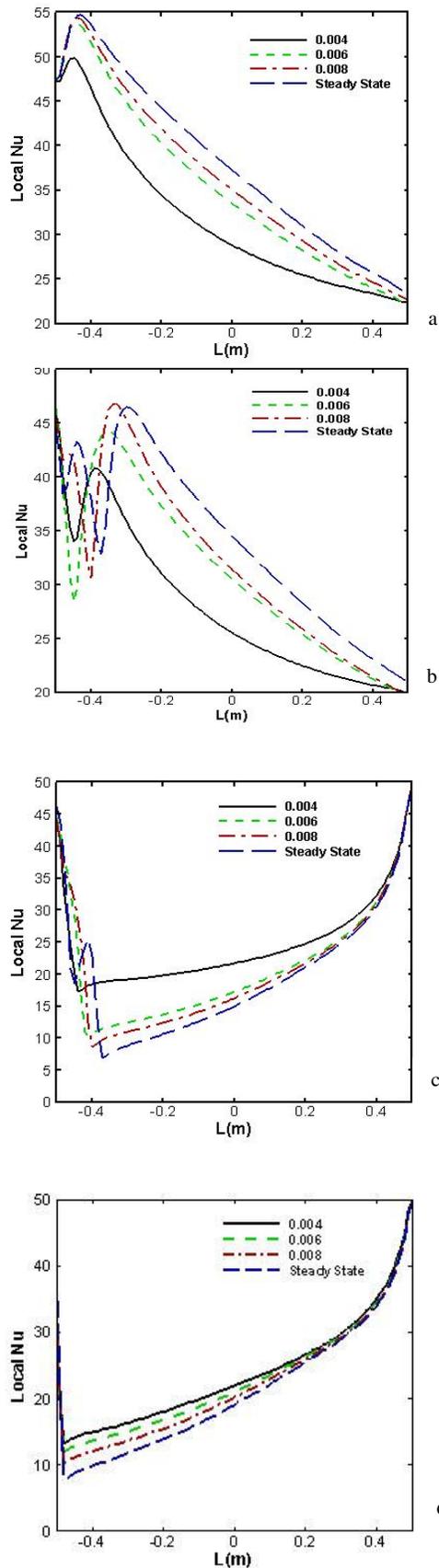


Fig. 7. Variation of local Nu for inclinations of various τ at $Re=90$ and $Ra=10^6$ a)30 deg b) 90 deg c)-30 deg d) -90 deg

channel are considered. It is found for the numerical simulations that:

- Increasing the Reynolds number, delays the generation of vortices and reduces the Nu for steady state consideration preserving the mixed convection conditions.
- Decreasing the out flow area delays the generation of vortices and the average Nu for steady state consideration is decreased. As the time passes, more vortices are generated, but the reinforced mean velocity don't let the eddies to enter the core of the channel, so they attach to the wall and reduce the heat transfer area.
- Variation of angle (the both positive and negative inclination) induces the generated vortices to get closer to each other and make an enlarged vortex then it affects on the variation of heat transfer coefficient. As the time passes, local Nusselt number is increased for positive inclinations but is decreased for negative inclinations.

REFERENCES

- [1] S. Chandrasekhar, Hydrodynamic and Hydro magnetic Stability, Clarendon Press, Oxford, 1961.
- [2] R.M. Clever, F.H. Busse, Transition to time-dependent convection, *J. Fluid Mech.* 65 (4) (1974) 625–645.
- [3] M. P. Arroyo, M. Quintaniila, J. M. Saviron, Three-Dimensional Study of Rayleigh Benard Convection by Particle Image Velocimetry, *Experimental Thermal and Fluid Science* 5(1992)216-224.
- [4] J. Andrzej Domaradzki, Heat transfer enhancement in Rayleigh Benard convection, *I. J. Heat Mass Transfer* 32(1989)2475-2483.
- [5] Hongxing Yang, Zuojin Zhu, Numerical simulation of turbulent Rayleigh Benard convection, *International Communications in Heat and Mass Transfer* 33 (2006) 184–190.
- [6] Anna Ebert, Christian Resagk, Andre Thess, Experimental study of temperature distribution and local heat flux for turbulent Rayleigh Benard convection of air in a long rectangular enclosure, *International Journal of Heat and Mass Transfer* 51 (2008) 4238–4248.
- [7] J. L. Lage, A. Bejan and J. Georgiadis, On the effect of the Prandtl number on the onset of Benard convection, *Int. J. Heat and Fluid Flow* 12, No. 2, June 1991.
- [8] Xiaoming Wang, Large Prandtl Number Behaviour of the Boussinesq System of Rayleigh Benard Convection, *Applied Mathematics Letters* 17 (2004) 821-825.
- [9] X. Nicolas, J.M.Luijckx, J.K. Platten, Linear stability of mixed convection flows in horizontal rectangular channels of finite transversal extension heated from below, *International. J. Heat Mass Transfer* 43(2000) 589-610.
- [10] Sandile S.Mosta,P. Sibanda, On the stability analysis of thermally stratified channel flow with a compliant boundary, *Internat. J. Heat Mass Transfer* 46 (2003) 939-948.
- [11] K.S. Gage, W.H. Reid, The stability of thermally stratified plane Poiseuille flow, *J. Fluid Mech.* 33 (1968) 21–32.
- [12] G.J. Hwang, K.C. Cheng, Convective instability in the thermal entrance region of a horizontal parallel-plate channel heated from below, *J. Heat Transfer* 95 (1973) 72–77.
- [13] Min Chan Kim, Chang Kyun Choi, Do-Young Yoon, Relaxation on the energy method for the transient Rayleigh Benard convection, *Physics Letters A* 372 (2008) 4709–4713.
- [14] R. C. Neisen , R. H. Sabersky, Transient heat transfer in Benard convection, *Internat. J. Heat Mass Transfer* 16 (1973) 2407-2420.
- [15] Fluent User's Guide, release 6.3 Fluent Incorporated, 2006.
- [16] Rossby, H.T., 1969, A study of Benard convection with and without rotation, *J Fluid Mech.* 36(2) 309–355.
- [17] P.-H. Kao, R.-J. Yang, Simulating oscillatory flows in Rayleigh Benard convection using the lattice Boltzmann method, *Internat. J. Heat Mass Transfer* 50 (2007) 3315-3328.

The author name, "Farbod V.moghaddam" was modified to "Farbod Vakilmoghaddam" on 23 May 2012.