Revolving Ferrofluid Flow due to Rotating Disk

Paras Ram, Kushal Sharma

Abstract - **A revolving flow of ferrofluid over a rotating disk is investigated by solving the boundary layer equations with boundary conditions by using Neuringer-Rosensweig model. The components of velocity and pressure profile are calculated numerically. However, the solution for the flow profile and the change in boundary layer displacement thickness follows the lines for ordinary viscous flow. Expressions are obtained in the cylindrical co-ordinate system by considering the z-axis as axis of rotation. Here, we have solved non-linear differential equations numerically by using power series approximations.**

Index Terms - **Axi-symmetric, rotating disk, boundary layer, ferrofluid, magnetic-field.**

I. Introduction

FERROFLUIDS are stable suspensions of colloidal ferromagnetic particles of the order of 10nm in suitable non-magnetic carrier liquids. These colloidal particles are coated with surfactants to avoid their agglomeration. Because of the industrial applications of ferrofluids, the investigation on them fascinated the researchers and engineers vigorously since last five decades. One of the many fascinating features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice-versa [1, 2]. Sealing of the rotating shafts is the most known application of the magnetic fluid. Ferrofluid is widely used in sealing of hard disc drives, rotating x-ray tubes under engineering applications.

The major applications of ferrofluid in electrical field is that controlling of heat in loudspeakers. Control on heating makes the life of sound speakers longer and increases the acoustical power without any change in its geometrical shape. Magnetic fluids are used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery. Therefore, ferrofluids play an important role in the field of bio-medical science also.

There are rotationally symmetric flows of the incompressible ferrofluid in the field of fluid mechanics, having all three velocity components viz. radial, tangential and vertical, in space, different from zero. In such type of flows, the variables are independent of angular coordinates and the angular velocity is uniform at large distance from the disk. We consider this type of flow for an incompressible ferrofluid when the rotating disk is subjected to the magnetic field $(H_r, 0, H_z)$ by using Neuringer-Rosensweig model [3]. This model has been used by Verma [4] for solving paramagnetic Couette flow by taking into account the interactions of external magnetic field.

Rosensweig [5], in his monograph, has given an authoritative introduction to the research on magnetic liquids and revealed interesting information about the effect of magnetization. In general, magnetization is a function of magnetic field, temperature and density of the fluid. This leads to convection of ferrofluid in the presence of the magnetic field gradient. Karman [6] rotating disc problem is extended to the case of flow started impulsively from rest, and also the steady state is solved to a higher degree of accuracy than previously done by a simple analytical method which neglects the resembling difficulties in Cochran's [7] well known solution.

The pioneering study of ordinary viscous fluid flow, due to the infinite rotating disc was carried by Karman. He introduced the famous transformation, which reduced the governing partial differential equations into ordinary differential equations. Cochran obtained asymptotic solutions for the steady hydrodynamic problem formulated by Karman. Benton [8] improved Cochran's solutions and also, solved the unsteady case. Attia [9] studied the unsteady state in the presence of an applied uniform magnetic field. The effect of the steady flow of ordinary viscous fluid, due to the rotating disc with uniform solution was considered by Mithal [10]. Attia [11] discussed about flow due to an infinite disk rotating in the presence of an axial uniform magnetic field by taking Hall effect into consideration.

Sunil et al. [12] studied the effect of rotation on thermosolutal convection in a ferromagnetic fluid considering a horizontal layer of an incompressible ferromagnetic fluid. Venkatasubramanian and Kaloni [13] investigated the effect of rotation on the thermo-convective instability of a horizontal layer of ferrofluid heated from below in the presence of uniform vertical magnetic field. Das Gupta and Gupta [14] examined the onset convection in a horizontal layer of ferromagnetic fluid heated from below and rotating about a vertical axis in the presence of a uniform magnetic field. Ram et al. [15] solved the non-linear differential equations under Neuringer- Rosensweig model by using power series approximations and discussed the effect of magnetic field-dependent viscosity on velocity

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components and pressure profile. Further, the effect of porosity on velocity components and pressure profile has been studied by Ram et al. [16].

In the present case, we take cylindrical coordinates (r, θ, z) where z-axis is normal to the plane, and this axis is being considered the axis of rotation. The viscous effects are dominant over a region at a small distance from the disk, if Reynolds number is large which gives rise to a boundary layer over the surface of disk. We have presented the boundary layer equations together with boundary conditions. These equations along with Maxwell equations are solved theoretically as well as numerically. Also, it is found that there is a variation in the boundary layer displacement thickness as compared to the ordinary viscous flow case. Here, the effect of vertically applied magnetic field in a circular layer of ferrofluid on a rotating disk is studied within the framework of Neuringer-Rosensweig approach and various types of ferrofluid responses are considered. This problem, to the best of our knowledge, has not been investigated yet.

II. MATHEMATICAL FORMULATION AND SOLUTION

This model considers the liquid particle in magnetic fluid as a mathematical point with only three degrees of freedom. As a complete set of independent variables, the following functions are chosen: three scalars (density, pressure and temperature) and three vectors (velocity, magnetization and magnetic field). Thermodynamic coefficients of the magnetic fluid are considered as scalars. This model considers the magnetization $\vec{M}$ as being parallel to the applied magnetic field, thus implying that no interaction of magnetic fluid with external magnetic field through magnetic body couples, and kinetic processes are considered.

This model leads to governing equations which are considered from the Navier-stokes equation of magnetization. The system of equations consists of the following:

Equation of continuity $\qquad \vec{\nabla}.\vec{V} = 0 \qquad (1)$

Equation of motion (Momentum equation for an incompressible ferromagnetic fluid with constant viscosity in a frame of reference rotating with angular velocity $\vec{\Omega}$)

$$\rho\left[\frac{\partial \vec{V}}{\partial t} + (\vec{V}.\vec{\nabla})\vec{V}\right] = -\nabla p' + \mu_0(\vec{M}.\vec{\nabla})\vec{H} - \vec{\nabla}\psi + \mu_f \vec{\nabla}^2\vec{V} + 2\rho(\vec{\Omega}\times\vec{V}) + \frac{\rho}{2}\nabla\left|\vec{\Omega}\times\vec{r}\right|^2 \qquad (2)$$

The effect of rotation includes two terms:

(a) Centrifugal force $-\frac{1}{2}grad\left|\vec{\Omega}\times\vec{r}\right|^2$

(b) Coriolis acceleration $2(\vec{\Omega}\times\vec{V})$

In (2), $p' - \frac{\rho}{2}\left|\vec{\Omega}\times\vec{r}\right|^2 = p$ is the reduced pressure, where p' stands for fluid pressure. The velocity component v_z is less as compared to v_r and v_θ, and also the flow is steady and axi-symmetric i.e. $\frac{\partial}{\partial t}(\ ) = 0, \frac{\partial}{\partial \theta}(\ ) = 0$.

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu_0}{\rho}\left|\vec{M}\right|\frac{\partial}{\partial r}\left|\vec{H}\right| + \nu\left[\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_r}{r}\right) + \frac{\partial^2 v_r}{\partial z^2}\right] + 2\Omega v_\theta = v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} - \frac{{v_\theta}^2}{r} \qquad (3)$$

$$\nu\left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{\partial^2 v_\theta}{\partial z^2}\right] - 2\Omega v_r = v_r\frac{\partial v_\theta}{\partial r} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} \qquad (4)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\mu_0}{\rho}\left|\vec{M}\right|\frac{\partial}{\partial z}\left|\vec{H}\right| + \nu\left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2}\right] = v_r\frac{\partial v_z}{\partial r} + v_z\frac{\partial v_z}{\partial z} \qquad (5)$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \qquad (6)$$

The approximate initial and boundary conditions for the flow due to rotation of an infinitely long disk $(z = 0)$ with constant angular velocity ω are given by

$$\begin{aligned} &at \quad z = 0; \quad v_r = 0, \quad v_\theta = r\omega, \quad v_z = 0. \\ &at \quad z = \infty; \quad v_r = 0, \quad v_\theta = 0. \end{aligned} \qquad (7)$$

Here, v_z does not vanish at $z = \infty$; but tends to a finite negative value.

From boundary layer approximation, we get for (3) that

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu_0}{\rho}\left|\vec{M}\right|\frac{\partial}{\partial r}\left|\vec{H}\right| = -r\omega^2 \qquad (8)$$

On considering very less variation of magnetic field along z-direction and using Karman transformations,

$$v_r = r\omega E(\alpha),\ v_\theta = r\omega F(\alpha),\ v_z = \sqrt{\nu\omega}\,G(\alpha),$$
$$p = \rho\omega\nu P(\alpha);\ \text{where}\ \alpha = z\sqrt{\frac{\omega}{\nu}} \qquad (9)$$

in equations (3) - (6) with the help of (8), we get a system of non-linear differential equations in *E*, *F*, *G* and *P* as follows:

$$E'' - GE' - E^2 + F^2 + 2F - 1 = 0 \qquad (10)$$
$$F'' - GF' - 2EF - 2E = 0 \qquad (11)$$
$$P' - G'' + GG' = 0 \qquad (12)$$
$$G' + 2E = 0 \qquad (13)$$
$$\left.\begin{aligned} &E(0) = 0,\ F(0) = 1,\ G(0) = 0,\ P(0) = P_0 \\ &E(\infty) = F(\infty) = 0 \end{aligned}\right\} \qquad (14)$$

The values of *E*, *F*, *G* and *P* are compared graphically with their corresponding values in classical case. G must tend to a finite limit, say $-c$, as α tends to infinity. Cochran indicated that formal asymptotic expansion (for large α) of the system of equations (10) - (13) is a power series in $exp(-c\alpha)$, i.e.

$$E(\alpha) \approx \sum_{i=1}^{\infty} A_i e^{-ic\alpha} \tag{15}$$

$$F(\alpha) \approx \sum_{i=1}^{\infty} B_i e^{-ic\alpha} \tag{16}$$

$$G(\alpha) \approx -c + \sum_{i=1}^{\infty} C_i e^{-ic\alpha} \tag{17}$$

$$(P - P_0)(\alpha) \approx \sum_{i=1}^{\infty} D_i e^{-ic\alpha} \tag{18}$$

Let $E'(0) = a$ and $F'(0) = b$. Using this supposition and equations (10) - (14), we get some additional boundary conditions for the approximate solution.

III. RESULTS

First four coefficients in the equations (15) - (18) are calculated with the help of (14) and additional boundary conditions, which are as follows:

$$A_1 = \left(-\frac{2b}{3c^3} - \frac{3}{c^2} + \frac{13a}{3c}\right),\quad A_2 = \left(\frac{2b}{c^3} + \frac{8}{c^2} - \frac{19a}{2c}\right)$$

$$A_3 = \left(-\frac{2b}{c^3} - \frac{7}{c^2} + \frac{7a}{c}\right),\quad A_4 = \left(\frac{2b}{3c^3} + \frac{2}{c^2} - \frac{11a}{6c}\right)$$

$$B_1 = \left(\frac{2a}{3c^3} + \frac{13b}{3c} + 4\right),\quad B_2 = \left(-\frac{2a}{c^3} - \frac{19b}{2c} - 6\right)$$

$$B_3 = \left(\frac{2a}{c^3} + \frac{7b}{c} + 4\right),\quad B_4 = \left(-\frac{2a}{3c^3} - \frac{11b}{6c} - 1\right)$$

$$C_1 = \left(\frac{2}{3c^3} - \frac{3a}{c^2} + 4c\right),\quad C_2 = \left(-\frac{2}{c^3} + \frac{8a}{c^2} - 6c\right)$$

$$C_3 = \left(\frac{2}{c^3} - \frac{7a}{c^2} + 4c\right),\quad C_4 = \left(-\frac{2}{3c^3} + \frac{2a}{c^2} - c\right)$$

$$D_1 = \left(\frac{4b}{3c^3} + \frac{6}{c^2} - \frac{26a}{3c}\right),\quad D_2 = \left(-\frac{4b}{c^3} - \frac{16}{c^2} + \frac{19a}{c}\right)$$

$$D_3 = \left(\frac{4b}{c^3} + \frac{14}{c^2} - \frac{14a}{c}\right),\quad D_4 = \left(-\frac{4b}{3c^3} - \frac{4}{c^2} + \frac{11a}{3c}\right)$$

Using the values $a = 0.54, b = -0.62$, and $c = 0.886$ from Cochran [7], we calculate the values of the coefficients A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1, C_2, C_3, C_4, D_1, D_2, D_3, and D_4. We draw the graphs of velocity components and asymptotic pressure with the dimensionless parameter α.

The present results give the good approximate solution of the above system of non-linear differential equations.

The boundary layer displacement thickness is calculated as

$$d = \int_0^{\infty} F(\alpha) d\alpha = 1.3456145$$

The fluid is taken to rotate at a large distance from the wall, the angle becomes

$$\tan\varphi_0 = -\left(\frac{\partial v_r}{\partial z} / \frac{\partial v_\theta}{\partial \theta}\right) = -\frac{E'(0)}{F'(0)} = 0.870967 = 41^0$$

IV. DISCUSSION

The problem considered here involves a number of parameters, on the basis of which, a wide range of numerical results have been derived. Of these results, a small section is presented here for brevity. The numerical results for the velocity profiles, for r, θ, z components of the velocity, commonly known as radial, tangential, vertical (axial) velocities, are shown in figure 1.1, 1.2, 1.3 respectively.

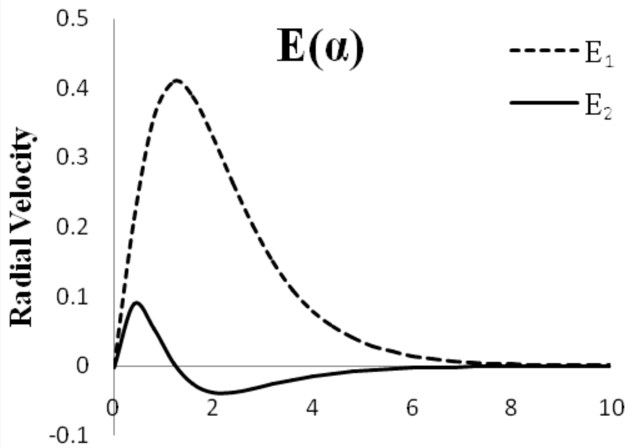

Figure 1.1

In figure 1.1, E_2 shows the radial velocity profile with the variation of dimensionless parameter α, known as Karman parameter. Here, the radial velocity E_2 = 0.08894 is maximum at $\alpha = 0.4$, after that it decreases smoothly and for large values of α, it converges to zero. E_1 shows the radial velocity component for ordinary viscous fluid case with peak value is 0.4101159 as $\alpha = 1.2$. It is noticeable that the radial velocity E_2 has very less peak value in comparison to E_1 because of thickening of the ferrofluid layer due to the rotation of the whole system. On the other hand, the magnetic force reduces the pressure on fluid and increases the radial velocity. The effect of rotation is more pronounced than the force of magnetization in the sense of fluid thickening. In other words, figures 1.1 and 2, with the variation of dimensionless parameter α, have converse behavior to each other.

In figure 1.2, F_2 shows the tangential velocity profile comparison with Newtonian case, i.e. F_1. In our case, if we increase the value of α, the tangential velocity F_2 decreases continuously and goes to zero for large value of α. It is observed from the table, the value of tangential velocity is 0.49762 at $\alpha = 1$, whereas in Benton's case for the ordinary viscous fluid, the tangential velocity is 0.468 for the same value of α. Therefore, at $\alpha = 1$, our value increases 6.7% approximately in comparison to the Benton's value. From figure, it is clear that F_1 converges to zero little faster than F_2. Here the tangential component of velocity follows almost the same trend as that of the ordinary viscous flow.

ISBN: 978-988-19252-2-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

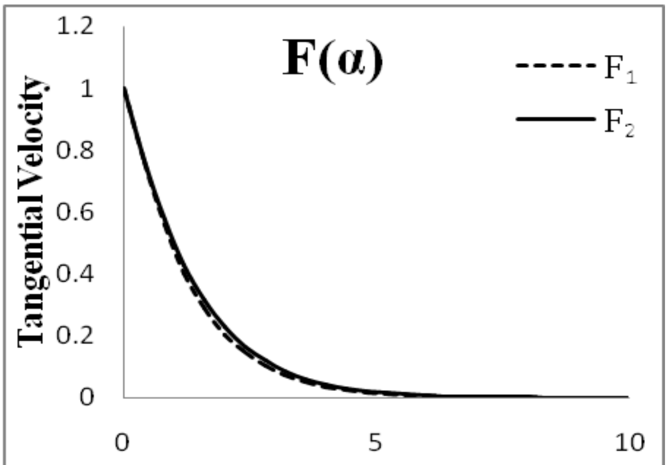

Figure 1.2

Figure 1.3, shows the axial velocity profile, which is zero in the beginning and tends to a finite value in the last. When we increase the value of α , it decreases continuously in the negative region. Here the axial velocity converges to -0.886 at $\alpha = 14.8$ onwards.

Figure 2, shows the pressure profile with the initial pressure, p_0 at $\alpha = 0$. The pressure goes to negative region for first few values of α . At $\alpha = 0.4$, it goes to maximum negative

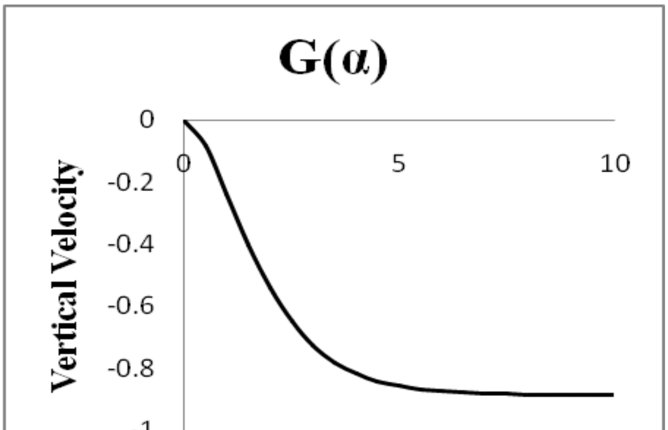

Figure 1.3

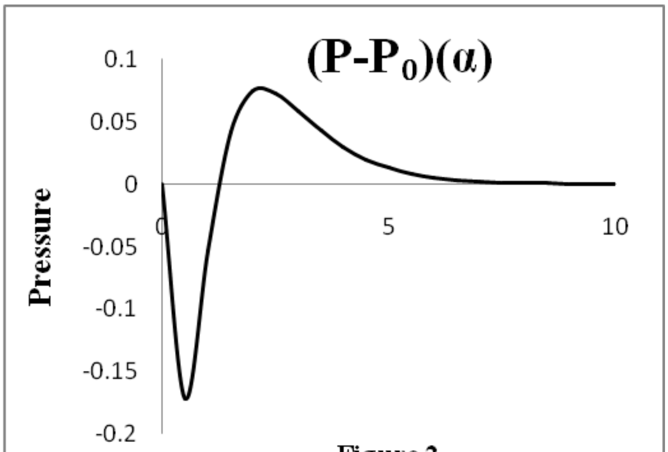

Figure 2

value, which is -0.17909. After continuously increasing the value of α , pressure also increases and at $\alpha = 1.3$, it comes out to 0.01194. When we increase the value of α continuously from $\alpha = 1.3$ onwards, pressure increases in the positive direction and it takes peak value as 0.07667 at $\alpha = 2.2$. After that, for large value of α , pressure becomes p_0 .

Comparing figures 1.1 and 2, we conclude that when radial velocity increases, the pressure of the ferrofluid decreases and when radial velocity decreases, ferrofluid pressure increases. These figures reveal converse behavior to each other. Here the tangential velocity diminishes slower than axial velocity components. The change in the curve of radial velocity is faster due to effect of external magnetic field, resulting in reducing the time required for velocity profile to reach their convergence level.

In our problem, we have calculated the displacement thickness, numerically. Here, the disk is rotating along with rotation of the ferrofluid, due to which thickness is increasing. The displacement thickness in our case is 1.34562, whereas in Benton’s case is 1.27144. In nut shell, we conclude that if we rotate the plate with rotation of the ferrofluid, there is an increment in the thickness of boundary layer. Here, we have also calculated angle between wall and ferrofluid, which is 41^0

V. CONCLUSION

From these results, we conclude that magnetization force i.e., $\mu_0 \vec{M}.\vec{\nabla}\vec{H}$ reduces the pressure. Also, it has been observed that magnetic field intensity increases the radial velocity; whereas, the fluid rotation has reverse effect. The effect of rotation is more pronounced than the force of magnetization due to which the radial velocity takes very less peak value in comparison to the ordinary viscous flow case. Due to the rotation, retardation of the radial velocity increases the thickness of the magnetic fluid layer. Conducting ferrofluids flow with rotating disk have the practical applications in many areas such as rotating machinery, lubrication, oceanography, computer storage devices, and viscometry and crystal growth processes.

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ISBN: 978-988-19252-2-0
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

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NOMENCLATURE

$\vec{H}$ Magnetic field intensity
$\vec{M}$ Magnetization
p' Fluid pressure
p Reduced pressure
P Karman pressure
P_0 Initial pressure (absolute value)
$\vec{V}$ Velocity of ferrofluid
ν Kinematic viscosity
μ_f Reference viscosity of fluid
μ_0 Magnetic permeability of free space
ρ Fluid density
$\vec{\nabla}$ Gradient operator
α Dimensionless Karman's parameter
d Displacement thickness of the ferrofluid layer
φ_0 Angle of rotation
ω Angular velocity of the disk
v_r Radial velocity
v_θ Tangential velocity
v_z Axial velocity
$\vec{\Omega}$ Angular velocity of whole system

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ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)