

The Self-Excitation Damping Ratio in Variable Speed Milling

K. Saleh and N.D. Sims

Abstract— During High Speed Machining (HSM), unstable self-excited vibrations (known as regenerative chatter) can occur, leading to poor surface finish, excessive tool wear, and damage to the machine. Consequently there has been a great deal of research to understand the mechanisms of regenerative chatter so that productivity can be enhanced. A chatter suppression method that has received attention recently is the spindle speed variation technique. However, this approach results in stability boundaries that are defined by delay-differential equations with periodic coefficients and variable time delays. In the present study, a Simulink model which is designed for milling chatter simulation is modified to enable its use under variable spindle speed circumstances. This is achieved by rewriting the equations of motion in non-dimensional time. A novel signal processing technique is then adopted for analysing the chatter stability.

Keywords: Milling Speed Variation, Stability in HSM, Signal Processing, Time Domain Simulation.

I. INTRODUCTION

Milling with high speed machining is one of the common processes used in producing a variety of the industrial products. However, the productivity and quality is limited due the vibrations that arise during the cutting process. These vibrations cause poor surface finish, increase the rate of tool wear and reduce the tool lifetime. Self-excited vibration (chatter) induced by regeneration of surface waviness is the one of main reasons for such vibrations. One solution that has received some attention is the use of continuously varying spindle speeds during milling [1-6]. In general, models designed for determining and analyzing machining stability of Constant Speed Machining (CSM) are not directly applicable to Variable Speed Machining (VSM) due to the presence of the time varying delay in the differential equations [7]. Therefore some studies (for example [8-10]) have presented different mathematical techniques to be used for modeling the delayed differential equation with a time varying delay term.

In addition a number of the time domain simulations and analytical approaches have been developed for chatter detection. However the problem arises of how to ascertain, based upon the computed data, whether the simulated cut was stable or unstable. The Peak-To-Peak (PTP) technique

was used to identify the stability boundary during the CSM as in [9, 11- 12].

However for VSM case, it can be seen that the system displacement does not reach a steady state condition, making the PTP method difficult to use as a judgment of chatter stability. Therefore a signal processing method is considered as an effective method used to detect chatter and the stability boundary for this case. Sims [11], implemented this approach for CSM case. The structure of this paper as follows: It begins with the representation for time domain milling in CSM and VSM. A signal processing method is then described. Results are then presented starting with comparing the stability of the VSM versus the CSM case. Then results from changing the acceleration of the spindle speed linearly and by taking the triangular variation waveform are analysed. Finally the overall results of this study are concluded.

II. TIME DOMAIN MILLING WITH CSM

Consider the schematic representation of milling shown in Figure 1. Here, a milling tool is removing material from a workpiece. For simplicity the tool is shown with a single degree of freedom in the y-direction, although the following analysis can be readily extended to the multi-degree-of-freedom case. In order to predict the time response of this system, a discretised model is formulated in the Simulink modelling environment as shown in Figure 2. The model consists of three aspects, namely milling kinematics, milling forces, and system dynamics. These components will now be briefly summarised more detailed descriptions of the modelling procedure can be found in references [11, 13].

The kinematics model begins by dividing the tool into discrete axial slices (such as that shown in Figure 1, and calculating the tool and workpiece geometry within each slice. Two coordinate systems are used: a radial coordinate based upon the centre of the tool with angles taken relative to the feed direction, and a Cartesian coordinate system based upon the workpiece feed direction. The relative displacement of the workpiece and tool (accounting for feed rate and vibration effects) are provided as inputs to the calculation. The basis of the computation is the manipulation of a set of arrays of Cartesian coordinates (one array for each tooth on each axial slice), that define the surface of the workpiece that was produced by that tooth. The array length represents a complete revolution of the tooth, therefore with each tooth revolution the array values are overwritten, or updated. For each time step in the simulation, the following calculations are repeated for each tooth on each axial slice:

1) The position of the tooth is calculated based upon the current simulation time, and the spindle speed.

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- 2) The workpiece surface array for the present tooth is updated. This surface array is depicted by the 'o' markers on the workpiece surface in Figure 1.
- 3) The instantaneous chip thickness h_i for the present tooth is calculated, based upon the current tooth position, and the surface array for the preceding tooth.

This kinematic model was programmed in the C programming language, and interfaced with the Simulink model via a mex-function. The milling forces computation takes the instantaneous chip thicknesses that have been calculated for each tooth N_i and axial slice l of the kinematics computation. The corresponding cutting forces are determined as a radial component F_r and a tangential component F_t opposing the rotation of the tool, and are assumed proportional to the instantaneous chip thickness [7]. The forces are then transformed into the workpiece Cartesian coordinates, and numerically integrated to give the resultant cutting forces in the x and y -directions. The system dynamics are then modeled in standard transfer-function formulations, so as to calculate values for the vibration in the x and y directions (as appropriate).

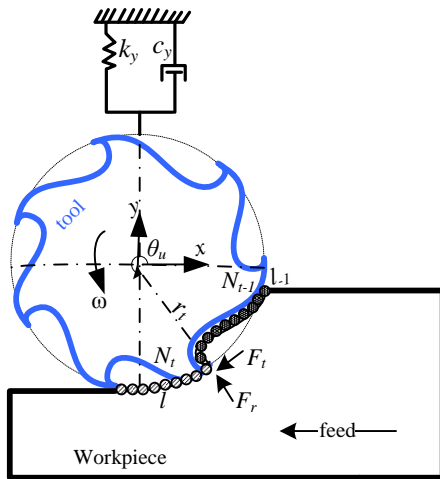


Figure 1 One-Dimensional Model of Milling Process

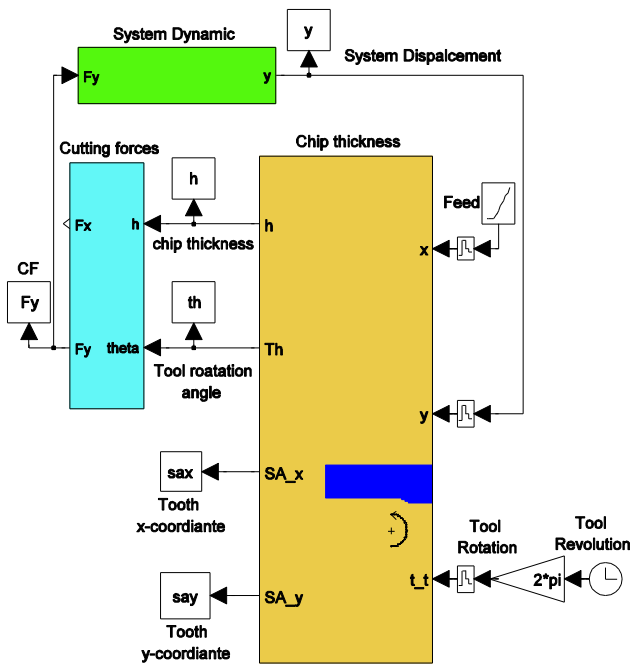


Figure 2 Simulink model of milling vibrations.

III. TIME DOMAIN MILLIN WITH VSM

In the present work, the time-domain model shown in Figure 2 will be modified for use in variable speed simulations. A detailed description of this model can be found in [11, 13]. Traditionally real-time t is used as the independent variable for writing the solution of the equations of motion. This is particularly the case when simulating dynamic systems using the Simulink modelling environment. Here, physical time (with units of seconds) is assumed to be the independent variable, and a wide range of numerical integration routines are available for solving the equations of motion (e.g. the 4th order Runge Kutta method, etc). However, a close inspection of the Simulink formulation described above reveals that the milling kinematics model requires a fixed number of time steps for each revolution of the tool. This means that a fixed-step solver (e.g. the Runge Kutta method) must be used, and the spindle speed must be fixed. In this section, this problem is overcome by using tool revolution as the independent variable in rewriting the system equation of motion. Consequently, 'Simulink time' is no longer equal to physical time, but rather the number of tool revolutions. With this approach, a fixed step solver will always involve a fixed number of time steps per tool revolution, even if the spindle speed is changed. This concept will now be derived more formally. With reference to Figure 1, the single degree of freedom system representing the tool vibration can be described by:

$$m_y \frac{d^2 y}{dt^2} + c_y \frac{dy}{dt} + k_y y(t) = F_y \quad (1)$$

It should be re-iterated that the force F_y is a function of the current and delayed vibration y , along with the instantaneous angle θ_u of the tool. Consequently, equation (1) is a delay differential equation with nonlinear and time periodic coefficients. Loss of stability could be associated with secondary Hopf bifurcations, cyclic fold bifurcations, or period doubling bifurcations, which in practice are collectively referred to as regenerative chatter.

The relationship between physical time t and instantaneous spindle speed ω is:

$$\frac{d\tau}{dt} = \omega \text{ (rev/sec)} \quad (2)$$

Where τ is the number of tool revolutions, the velocity can then be rewritten using the chain rule as:

$$\frac{dy}{dt} = \frac{dy}{d\tau} \frac{d\tau}{dt} = \frac{dy}{d\tau} \omega \quad (3)$$

Moreover the system acceleration can be rewritten as:

$$\frac{d^2 y}{dt^2} = \frac{d^2 y}{d\tau^2} \omega^2 + \frac{dy}{d\tau} \frac{d\omega}{dt} \quad (4)$$

Here, the rate of change of spindle speed ω can be rewritten to give:

$$\frac{d^2 y}{dt^2} = \frac{d^2 y}{d\tau^2} \omega^2 + \frac{dy}{d\tau} \frac{d\omega}{d\tau} \omega \quad (5)$$

The equation of motion (1) can then be written in terms of the derivative expressions on the right-hand-side of equations (3) and (5).

$$F_y - k_y y(\tau) - c_y \frac{dy}{d\tau} \omega = m_y \left(\frac{d^2 y}{d\tau^2} \omega^2 + \frac{dy}{d\tau} \frac{d\omega}{d\tau} \omega \right) \quad (6)$$

This can be rearranged to give:

$$\frac{d^2 y}{d\tau^2} = \frac{F_y}{m_y \omega^2} - \frac{k_y y(\tau)}{m_y \omega^2} - \frac{c_y}{m_y \omega} \frac{dy}{d\tau} - \frac{1}{\omega} \frac{dy}{d\tau} \frac{d\omega}{d\tau} \quad (7)$$

The system dynamics which are represented by the equation of motion (7) give rise to the Simulink system shown in Figure 3.

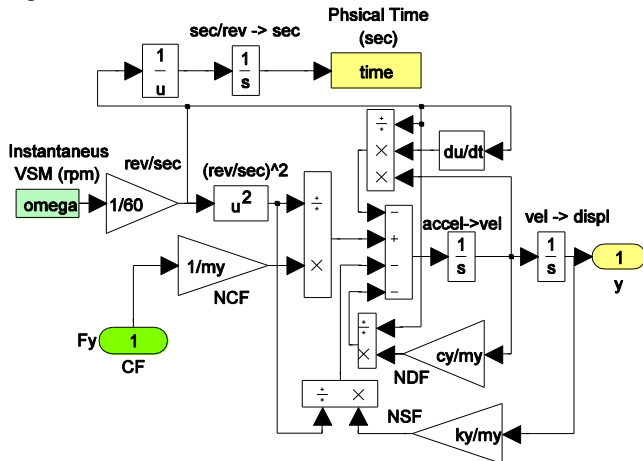


Figure 3 Simulink model structure in non-dimensional time

IV. SIGNAL PROCESSING METHOD

In metal cutting, self-excited vibrations that cause chatter are not clearly evident, since the forced vibrations (due to the rotation of the tool) are dominating the response of the system. This is particularly the case in the region of marginal chatter instability, where the self-excited vibration will grow very slowly. Therefore, many chatter criteria require the simulation to run for a long period of time before it is possible to determine if the system is stable or unstable. Consequently the technique of isolating the self-excited vibration from the forced vibration was adopted particularly in milling machining. The procedure of this technique is now briefly summarized in Figure 4 and more description can be found in [11]. The first step (block A) is a time-domain simulation of the cutting process under given cutting parameters. These data signals are then divided into number of frames, each frame represents one complete tool cycle (block B). A Fourier analysis approach is then applied to each of these data sets (block C). Isolating the self-excited vibration from the forced vibration is then executed (block D). Maximum values of the spectral lines are then determined (block E). These lines can then be plotted on a natural logarithmic scale, thereby indicating the self-excited vibration behavior per revolution of the tool (Block F). Finally (block G) is the step of calculating the self-excited damping ratio ζ by fitting a first order polynomial and selecting the spectral line ($k_{p,max}$) with the steepest gradient.

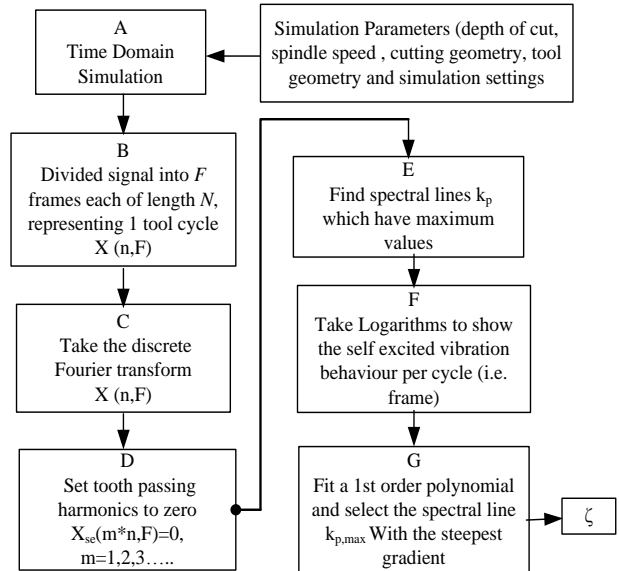


Figure 4 Flow chart to illustrate evaluation of the chatter criterion

V. STABILITY ANALYSIS OF VSM

In the present contribution, the time domain modeling approach is combined with the signal processing method in order to illustrate the stability of the VSM. A simple milling scenario with one degree of freedom is considered. The techniques and the procedures that are used in the current model can be applied to systems with multiple modes of vibration, and involving interrupted cutting with multiple teeth engaged in the cut simultaneously. It is also straightforward to include a nonlinear relationship between cutting force and chip thickness, and the periodic excitation force is inherently modeled.

Model validation is then presented by repeating some of the analysis presented in [4, 9]. The milling parameters were also chosen to closely match those used in previous work by [4, 9] and are summarized in Table 1. A variable spindle speed was then used in the time-domain model, with a triangular speed variation around a mean value. This was defined in accordance with reference [4, 9], with fixed parameters of RVA and RVF. To begin with demonstrating the difference in the chatter behavior for CSM and VSM, a small selection of the results is considered. In addition the stability lobe diagram for the constant spindle speed case is produced. The self-excited damping ratio technique is then used for stability investigation. Here chatter was analysed by considering a two cases of the milling speed variation. In the first case, and at a constant depth of cut of 1mm, milling speed was accelerated from 7500 to 10000 rpm. This procedure was performed for different scenarios of the spindle accelerations. The second case, milling speed was assumed to be a variable, with a periodic triangular speed variation around a mean value $\omega_m=9100$ rpm. For the time-domain simulations, the signal processing technique is used to analyze the data signal of the simulated chatter and to determine so called self-excited damping ratio of each case of the simulation. It should be pointed out that the parameters are chosen in the present study may not completely match those used in reference [4, 9] since the tool helix angle; tool radial immersion; number of tool cycles, and number of simulation axial layers were not given.

Table I

Input Simulation Parameters		
Sample	Parameter	Value
D	Tool diameter	25 mm
N_t	Number of teeth	3
λ_θ	Flute helix	0^0
r_{imm}	Radial immersion	2 mm
f_{pt}	Feed per tooth	1 mm
m	Mass	1.637 kg
f_n	Natural Frequency	222.5 Hz
ζ	Damping ratio	0.005
K_t and K_r	Tangential and Radial cutting stiffness	700 and 140 MPa
RVA	Amplitude ratio	0.28
RVF	Frequency ratio	0.003

mm=millimetre, kg=kilogram, Hz=Hertz, MPa= Mega-Pascal

VI. RESULTS

A. Stability Analysis for CSM versus VSM

Before presenting an analysis of the chatter stability using the self-excited damping ratio method, it is worth illustrating the time response of the system, and demonstrating how continuously varying the spindle speed can suppress the chatter level more than the constant case. Here the CSM and VSM simulation results for last 30 cycles are compared at the same simulation conditions. Figure 5 presents the simulation results of the system displacement for both CSM and VSM with the depth of cut ($b=1\text{mm}$) and the constant and mean value of the spindle speed ($\omega_m=9100\text{ rpm}$). With reference to [4, 9], this is a small selection just to show the effectiveness of the VSS technique in suppressing the chatter levels.

A typical result of the case of constant speed machining CSM ($\omega=9100\text{ rpm}$) is shown in plot (a), here the chatter level is clearly growing, and consequently the system is unstable. However plots (b) and (d) demonstrate the case of variable speed machining. Here, the triangular waveform of the spindle speed is varying periodically about the mean value of the speed ($\omega_m=9100\text{ rpm}$) shown in plot (d). Plot (b) illustrates the overall chatter level which is clearly stabilized at a level significantly lower compared to the CSM case (plot (a)). Thereby the system here is considered to be stable. The comparison between the CSM and VSM is further illustrated in plot (c). From this plot it can be clearly noticed that VSM approach has lower and almost constant vibration which indicates significantly greater stability than for the constant spindle speed condition. Therefore these results agree reasonably well with those given in [4, 9], and the small differences could be attributed to the different parameters used in the simulations. Consequently these outcomes have validated the proposed variable spindle speed modeling procedure. However, close inspection of the displacement signal shown in Figure 5 reveals that the response of the transient behavior of the signal is periodically arising which can be attributed to the spindle speed alterations. This makes it difficult to apply standard approaches for fairly analyzing the stability. Here the self-excited damping ratio approach is considered to be the more appropriate method for analyzing milling stability behavior.

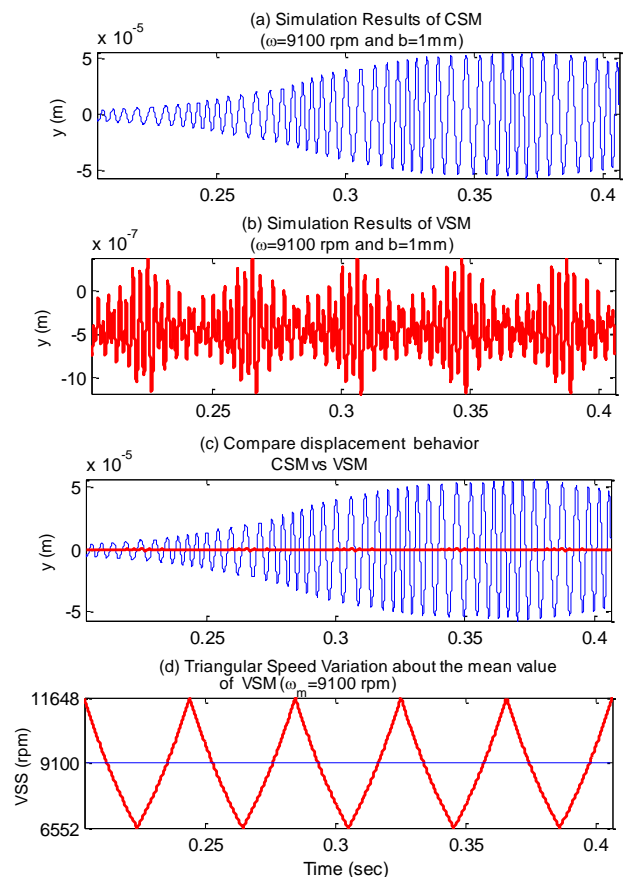


Figure 5 Simulation results for CSM and VSM

B. Stability Analysis for Accelerating Spindle Speed

As can be seen in section A chatter levels are significantly reduced when the spindle speed varies periodically with triangular waveform. Now in this section, effects of linearly varying the spindle speed with different accelerations behaviors will be investigated. Figure 6 shows the overall results of changing the spindle acceleration on the milling stability behavior. Plot (a) shows the stability lobe with markers (*) A and (Δ) B are corresponding to the start and end of the milling speed variation. In this case, simulation starts from an unstable region at 7500 rpm (associated with a secondary Hopf bifurcation) and ends by the stable region at 10000 rpm, while markers at (\circ) C, (\square) D and (\times) E correspond to the boundary of the stability at that defined milling speeds at the same depth of cut 1mm.

Plot (b) shows the variation of the milling speed from 7500 to 10000 rpm with accelerations 12.5, 25, 50 and 125 rpm/rev, while plots (c), (d), (e) and (f) of (Figure 6) show the evaluation of the chatter obtained by the time domain simulation, at these variable milling speeds and the depth of cut 1mm. As can be seen from the plot (c) the stages of the cutting stability are changing along the milling speed variation with a low acceleration of 12.5 rpm/rev. Here milling machining starts from unstable region where the chatter grows steadily between points A and C, this region is associated with a secondary Hopf bifurcation.

Then the signal of the self-excited vibration is decaying between the points C to D and beyond the point E, these are corresponding to stable cutting regions. However chatter vibrations are steeply growing between points D' (star) and E. This region corresponds to the unstable cut along with the flip bifurcation region.

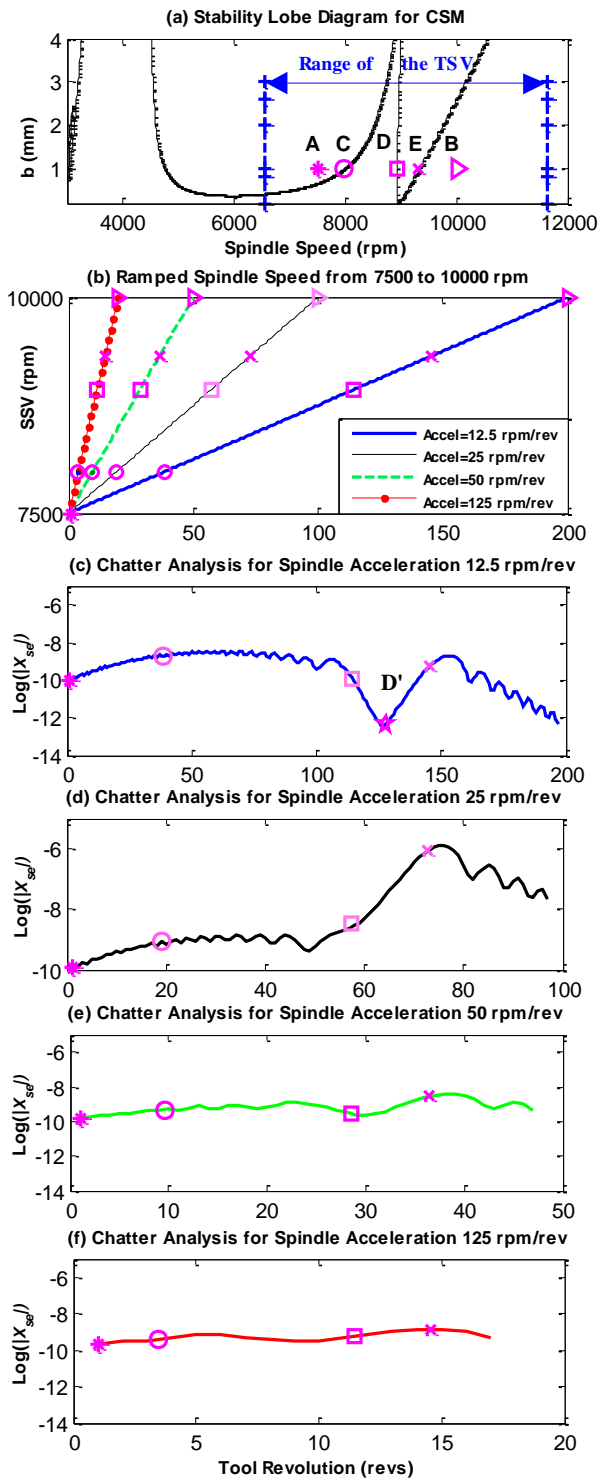


Figure 6 Effect of the changing speed acceleration in milling stability.

However the stable region between points D and D' in plot (c) is inspected and can be attributed to the acceleration of the spindle. Plots (d), (e) and (f) demonstrate the stages of the chatter behavior obtained from accelerating the spindle speed faster at 25, 50 and 125 rpm/rev respectively. Here the result obtained from the time domain simulation is clearly demonstrating that, stability region (C to D) was reduced by increasing the milling speed acceleration through all the cases.

However there is a positive point of this approach which is that the sharp growth of the chatter (D' to E) through the flip bifurcation region is reduced by increasing the spindle acceleration. This result is considered as worthy of more investigations for high speed machining.

C. Stability Analysis for Triangular Speed Variation

For the same range of the spindle speed variation that is highlighted on plot (a) of (Figure 6), the depth of cut was now changed, and the chatter is analyzed as shown in Figure 7. Plot (a) shows the triangular speed variation around the mean value $\omega_m=9100$ rpm in a periodic fashion, with a fundamental period 6 tool revolutions per one tool cycle, while plot (b) shows the results of the chatter analysis obtained from simulating this speed with the given frequency ratio RVF and amplitude ratio RVA.

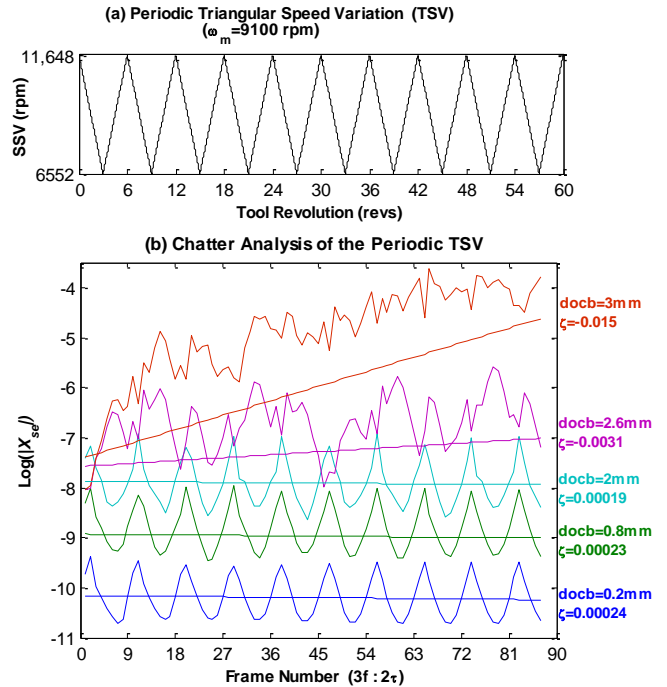


Figure 7 Model simulation results, (a) Triangular milling speed variation, (b) chatter analysis due to the periodic milling speed.

In this section, the analysis frames have been overlapped to three frames per two cycles of the tool revolutions ($3f:2\tau$), so as to increase the number of data points available. As can be seen from the plot (b) for depth of cuts 0.2 and 0.8 mm the data of the self-excited vibrations (fitted by straight lines) declining with each tool rotation, and the patterns of the signal are in a periodic fashion, giving a positive damping ratios of $\zeta=0.00024$ and $.00023$ respectively. Increasing depth of cut results is decreasing of the damping ratio values ($\zeta \rightarrow 0$). This indicates that the system is approaching the stability boundary where the value of the $\zeta=0$. This can be clearly realized from the case of the depth of cut 2 mm, here the signal of the chatter is almost periodic with less slope inclination of the vibration data, giving a positive damping ratio value $\zeta=0.00019$. This means that at this depth of cut the system is approximately reaching the stability boundary, which this case gives a more stability than the CSM case. However increasing the depth of cut to 2.6 mm the system becomes unstable. The self-excited vibrations now grow with each cycle, and the signal is start losing its periodicity, giving a negative $\zeta=-0.0031$. Increasing depth of cut 3mm causes the instability to worsen, the vibrations grow rapidly at a higher rate, and the signal is clearly losing the periodicity behavior, giving $\zeta=-0.015$.

Similar results were acquired by using PTP method in [4, 9], and the small differences could be attributed to the different parameters used in the simulations. Consequently

these outcomes have validated the proposed variable spindle speed modeling procedure.

VII. CONCLUSION

In this study, a comprehensive milling simulation model has been modified to account for milling scenarios that involve a variable spindle speed. The model is formulated in a Simulink environment, and in order to accommodate variable spindle speeds the system equation of motion has been reformulated in non-dimensional time. As a result, the simulation time used as the independent variable in Simulink becomes the tool revolution, rather than the simulated time in seconds.

The model results have been validated by comparing a small selection of simulation results with the work presented in reference [4, 9]. Reasonable agreement was observed, but the present study has re-enforced the issue of analyzing chatter stability for variable spindle speed simulations.

Chatter amplitude can be significantly reduced by using a periodic triangular speed variation approach.

In addition, accelerating the spindle speed linearly shows some positive results of reducing the sharp growth of the chatter through the flip bifurcation region. This promise is considered as worthy of more investigations for high speed machining.

It is clear that the signal processing approach is able to provide a formal interpretation of chatter stability for variable spindle speed simulations.

ACKNOWLEDGMENT

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NOMENCLATURE

b	Depth of cut
f	Frame numbers
CF	Cutting forces
c_v	System damping coefficient
F_r	Radial cutting forces
F_t	Tangential cutting forces
F_y	Total cutting forces in y-direction
h_t	Instantaneous dynamic chip thickness
k_v	System stiffness coefficient
$k_{p,max}$	Maximum spectral line
l	Number of axial layers
$\text{Log}(X_{sc})$	Natural logarithmic for maximum spectral line per frame, see [11]
NCF	Normalized cutting forces
NDF	Normalised damping forces
NSF	Normalized stiffness forces
TSV	Triangular speed variation
SSV	Spindle Speed Values
y	System displacement in y-direction
ω	Constant value of the spindle speed
ω_m	Mean value of the spindle speed
θ_u	Tool position angle
τ	Number of tool revolutions

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