On Fuzzy *µ - Irresolute Maps and Fuzzy *µ - Homeomorphism Mappings in Fuzzy Topological Spaces

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Abstract : **The aim of this paper is to introduce a new class of fuzzy sets, namely *µ - closed fuzzy sets for fuzzy topological spaces. This class is a super class of the closed fuzzy sets. We introduce and study new space namely fuzzy cTµ*-spaces and µTµ*-spaces.**

Further, the concept of fuzzy *µ -continuous, fuzzy *µ -irresolute mappings, fuzzy *µ -closed maps, fuzzy *µ -open maps and fuzzy *µ -homeomorphism in fuzzy topological spaces are also introduced, studied and some of their properties are obtained.

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I. Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L. A. Zadeh in his classical paper [25] in the year 1965. Subsequently several researchers have worked on topology using fuzzy sets and developed the theory of fuzzy topological spaces. The notion of fuzzy subsets naturally plays a very significant role in the study of fuzzy topology introduced by C.L.Chang [3].

N. Levine [4] introduced the concepts of generalized closed sets in general topology in the year 1970. G.Balasubramanian and P. Sundaram [2] introduced and studied generalized closed fuzzy sets in fuzzy topology. K.K.Azad [1] introduced semi-closed fuzzy sets in the year 1981. H.Maki, T. Fukutake, M.Kojima and H.Harada [5] introduced semi-generalized closed fuzzy sets (briefly fsg - closed) in fuzzy topological space in the year 1998.

In the year 2005, *µ - closed sets, *µ - continuous, *µ - irresolute, *µ -closed, *µ -open maps were introduced and studied by M.K.R.S.Veera Kumar [23] for general topology.

Recently author introduced and studied Ψ-closed fuzzy sets[10], pre-semi-closed fuzzy sets[11], g*-semi-closed fuzzy sets[6], g#- closed fuzzy sets[7], g#-semi-closed fuzzy sets[8], #g closed fuzzy sets[12], #g-semi-closed fuzzy sets[13], g*- closed fuzzy sets[9], µ- closed fuzzy sets[14], ĝ-closed fuzzy sets[9], *g- closed fuzzy sets[15], *g-semi- closed fuzzy sets[16], α-*g-closed fuzzy sets[17], µ-semi- closed fuzzy sets[18], µ-pre-closed fuzzy sets[19], semi-µ- closed fuzzy sets[20], gµ-closed fuzzy sets[22] and g*Ψ- closed fuzzy sets[21].

The class of Ψ-closed fuzzy sets is properly placed between the class of semi- closed fuzzy sets and the class of semi-pre-closed fuzzy sets. The class of pre-semi-closed fuzzy sets is placed properly between the class of semi-pre- closed fuzzy sets and the class of gsp- closed fuzzy sets. The class of g#- closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of g*- closed fuzzy sets. The class of g#-semi- closed fuzzy sets is properly placed between the class of semi-closed fuzzy sets and the class of gs- closed fuzzy sets. The class of g*- closed fuzzy sets is properly placed between the class of closed fuzzy sets and the class of g- closed fuzzy sets. The class of #g- closed fuzzy sets is placed properly between the class of g*- closed fuzzy sets and the class of g- closed fuzzy sets. The class of #g-semi- closed fuzzy sets is placed properly between the class of #g-closed fuzzy sets and the class of gs- closed fuzzy sets. This class also lies between the class of semi- closed fuzzy sets and the class of gs- closed fuzzy sets. The class of µ-closed fuzzy sets is a super class of the classes of g#-closed fuzzy sets, α- closed fuzzy sets and the class of closed fuzzy sets. The class of ĝ-closed fuzzy sets is placed properly between the class of closed fuzzy sets and the class of g- closed fuzzy sets. The class of *g- closed fuzzy sets is placed properly between the class of g*- closed fuzzy sets and the class of g- closed fuzzy sets. The class of *g-semi- closed fuzzy sets is placed properly between the class of g#s- closed fuzzy sets and the class of gs- closed fuzzy sets. The class of α-*g- closed fuzzy sets is properly placed between the class of α- closed fuzzy sets and the class of αg- closed fuzzy sets. The class of µs- closed fuzzy sets is a super class of the classes of semi- closed fuzzy sets, α- closed fuzzy sets, closed fuzzy sets, µ closed fuzzy sets, g#- closed fuzzy sets and the class of g#s- closed fuzzy sets. The class of µp- closed fuzzy sets is a super class of the classes of pre closed fuzzy sets, α- closed fuzzy sets, closed fuzzy sets, gα- closed fuzzy sets, ĝ -closed fuzzy sets, µ- closed fuzzy sets and the class of g#- closed fuzzy sets. The class of sµ- closed fuzzy sets properly contains the classes of semi- closed fuzzy sets, α- closed fuzzy sets and the class of closed fuzzy sets. The class of g*Ψ-closed fuzzy sets is properly placed between the class of Ψ-closed fuzzy sets and the class of gsp-closed fuzzy sets. The class of gµ- closed fuzzy sets is placed properly

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between the class of closed fuzzy sets and the class of g-closed fuzzy sets.

The aim of this paper is to introduce a new class of fuzzy sets, namely *μ - closed fuzzy sets for fuzzy topological spaces. This class is a super class of class of the closed fuzzy sets. We introduce and study new space namely fuzzy cTμ*-spaces.

Further, the concept of fuzzy *μ -continuous, fuzzy *μ -irresolute mappings, fuzzy *μ -closed maps, fuzzy *μ -open maps and fuzzy *μ -homeomorphism in fuzzy topological spaces are also introduced, studied and some of their properties are obtained.

II. Preliminaries

Let X, Y and Z be sets. Throughout the present paper (X, T), (Y,σ) and (Z,η) and (or simply X, Y and Z) mean fuzzy topological spaces on which no separation axioms is assumed unless explicitly stated. Let A be a fuzzy set of X. We denote the closure, interior and complement of A by cl (A), int (A) and C (A) respectively. Before entering into our work we recall the following definitions, which are due to various authors.

Definition 2.01: A fuzzy set A in a fts (X, T) is called: 1) a semi - open fuzzy set, 2) a pre - open fuzzy set, 3) a α - open fuzzy set and 4) a semi pre-open fuzzy sets can be found in [4] and [9].

The semi - closure (resp. pre closure fuzzy, α - closure fuzzy and semipro closure fuzzy) of a fuzzy set A in a fts (X, T) is the intersection of all semi - closed (resp. pre closed fuzzy sets, α - closed fuzzy sets and sp-closed fuzzy sets) fuzzy sets containing A and is denoted by scl (A) (resp. pcl (A), α cl (A) and spcl(A)).

The following definitions are useful in the sequel.

Definition 2.02: A fuzzy set A of a fts (X, T) is called:
1) a generalized closed (g - closed fuzzy) fuzzy set, 2) a generalized – pre closed (gp -closed fuzzy) fuzzy set, 3) a α-generalized closed (αg-closed fuzzy) fuzzy set, [17] 4) a generalized α-closed (gα-closed fuzzy) fuzzy set, 5) a generalized semi - pre-closed (gsp-closed fuzzy) fuzzy set, 6) a generalized semi - closed (gs-closed fuzzy) fuzzy set, 7) a semi - generalized closed (sg-closed fuzzy) fuzzy set, 8) a g*- closed fuzzy set, 9) a Ψ-closed fuzzy set, 10) a $\hat{g}$ - closed fuzzy set, 11) a g#- closed fuzzy set, 12) a g#s-closed fuzzy set, 13) a g*Ψ- closed fuzzy set, 14) a μ - closed fuzzy set, 15) a *g- closed fuzzy set, 16) a *g-semi-closed fuzzy set, 17) a α-*g- closed fuzzy set and 18) a gμ - closed fuzzy sets can be found in [7], [8], [9], [10], [14], [15], [16], [17] and [22].

Definition 2.03: Let X, Y be two fuzzy topological spaces. A function f: X→ Y is called:
1) fuzzy continuous (f-continuous), 2) fuzzy α-continuous (fα-continuous), 3) fuzzy semi- continuous function (fs-continuous), 4) fuzzy pre-continuous (fp-continuous) function, 5) fg-continuous function, 6) fgp-continuous function, 7) fgs-continuous function, 8) fsg- continuous function, 9) fgα-continuous function, 10) fαg-continuous function, 11) fgsp- continuous functions, 12) fg*-continuous function, 13) fΨ-continuous function, 14) fg*Ψ- continuous function, 15) fμ-continuous function, 16) f*gs-continuous function, 17) f#g-continuous function, 18) f#gs-continuous function, 19) fΨ-irresolute, 20) gc-irresolute and 21) g*-irresolute functions can be found in [7], [8], [9], [10], [14], [15], [16], [17] and [22].

Definition 2.04: Let X, Y be two fuzzy topological spaces. A function f: X→ Y is called:
1) fuzzy T ½ - space can be found in [9].

Definition 2.05: Let X, Y be two fuzzy topological spaces. A function f: X→ Y is called:
1) fuzzy-homeomorphisms, 2) fuzzy g*s-homeomorphisms, 3) fuzzy g#-homeomorphisms, 4) fuzzy g#s-homeomorphisms and 5) fuzzy g#α-homeomorphisms can be found in [6], [7], [8] and [9].

III. *μ -Closed fuzzy sets in FTS

Definition 3.01: A fuzzy set A of a fuzzy topological space (X, T) is called *μ -closed fuzzy set if cl(A)≤U whenever A≤U and U is *gs - open fuzzy set in (X, T).

Theorem 3.02: Every closed (resp: μ -closed fuzzy set, α-closed fuzzy set, pre-closed fuzzy set, gμ - closed fuzzy set, g- closed fuzzy set, sg- closed fuzzy set, g#-s- closed fuzzy set and g#- closed fuzzy set) fuzzy set is *μ -closed fuzzy set in any fts X.

Proof: Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 3.03: Let X = {a, b, c} and the fuzzy sets A and B be defined as follows: A = {(a, 0.4), (b, 0.5), (c, 0.6)}, B = {(a, 1), (b, 0.9), (c, 0.7)}. Consider the fts (X, T), where T = {0, 1, A}. Note that the fuzzy subset B is *μ -closed fuzzy set but not a closed (resp: not a μ -closed fuzzy set, not a α-closed fuzzy set, not a pre-closed fuzzy set, not a gμ - closed fuzzy set, not a g- closed fuzzy set, not a sg- closed fuzzy set, not a g#-s- closed fuzzy set and not a g#- closed fuzzy set) fuzzy set in (X, T).

Theorem 3.04: In a fts X, if a fuzzy set A is both *gs - open fuzzy set and *μ -closed fuzzy set, then A is closed fuzzy set.

Theorem 3.05: If A is *μ -closed fuzzy set and cl (A) ∧ (1−cl (A)) = 0. Then there is no non - zero g - closed fuzzy set F such that F ≤ cl (A) ∧ (1−A).

Theorem 3.06: If a fuzzy set A is *μ -closed fuzzy set in X such that A≤B≤cl (A), then B is also a *μ -closed fuzzy set in X.

Definition 3.07: A fuzzy set A of a fts (X, T) is called *μ - open fuzzy (*μ -open fuzzy set) set if its complement 1−A is *μ -closed fuzzy set.

Theorem 3.08: A fuzzy set A of a fts is *μ -open iff F≤int (A), whenever F is g-closed fuzzy set and F ≤ A.

Proof: The following proof omitted.

Theorem 3.09: Every open (resp: μ - open fuzzy set, α-open fuzzy set, pre- open fuzzy set, gμ - open fuzzy set, g-open fuzzy set, sg- open fuzzy set, g#-s- open fuzzy set and g#- open fuzzy set) fuzzy set is a *μ -open fuzzy set but not conversely.

Proof: Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 3.10: In the example 3.03, the fuzzy subset 1−B={(a, 0), (b, 0.1), (c, 0.3)} is *μ -open fuzzy set but not

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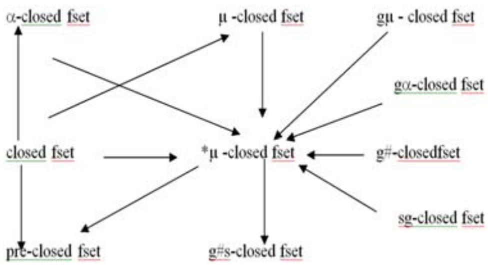
a open (resp: not a μ - open fuzzy set, not a α- open fuzzy set, not a pre- open fuzzy set, not a gμ - open fuzzy set, not a g- open fuzzy set, not a sg- open fuzzy set, not a g#-s- open fuzzy set and not a g#- open fuzzy set) fuzzy set in (X, T).

Remark 3.11: The following diagram 1 shows the relationships of *μ -closed fuzzy sets with some other fuzzy sets.



where A $\longrightarrow$ B (A $\longleftrightarrow$ B) represents A implies B but not conversely. (A and B are independent).

Theorem 3.12: If int (A) ≤ B ≤ A and if A is *μ -open fuzzy set, then B is *μ -open fuzzy set in a fts (X, T).

Theorem 3.13: If A ≤ B ≤ X where A is *μ -open fuzzy relative to B and B is *μ -open fuzzy relative to X, then A is *μ -open fuzzy relative to fts X.

Theorem 3.14: Finite intersection (Union) of *μ -open fuzzy set is a *μ - open fuzzy set.

IV. Fuzzy *μ -Closure and Fuzzy *μ -Interior Fuzzy Sets in FTS

In this section we introduce the concepts of fuzzy *μ - closure (f *μ - cl) and fuzzy *μ -interior (f *μ -int), and investigate their properties.

Definition 4.01: For any fuzzy set A in any fts is said to be fuzzy *μ -closure and is denoted by f *μ -cl (A), defined by f *μ -cl (A) = ∧ {U:U is *μ -closed fuzzy set and A ≤U}.

Definition 4.02: For any fuzzy set A in any fts is said to be fuzzy *μ -interior and is denoted by f *μ -int (A), defined by f *μ -int (A) = ∨{V:V is *μ -interior fuzzy set and A ≥ V}.

Theorem 4.03: Let A be any fuzzy set in a fts (X, T).

Then f *μ -cl (A) = f *μ -cl (1– A) = 1– f *μ -int (A).

= f *μ -int (1–A) = 1– f *μ -cl (A).

Proof: Omitted.

Theorem 4.04: In a fts (X, T), a fuzzy set A is *μ -closed iff A = f *μ -cl (A).

Proof: Omitted.

Theorem 4.05: In a fts X the following results hold for fuzzy *μ -closure.

1) *μ -cl(0)=0.
2) *μ -cl(A) is *μ -closed fuzzy set in X .
3) *μ -cl(A) ≤ *μ -cl(B) if A ≤ B.
4) *μ -cl (*μ -cl(A)) = *μ -cl(A).
5) *μ -cl(A ∨ B) ≥ *μ -cl(A) ∨ *μ -cl(B).
6) *μ -cl (A ∧ B) ≤ *μ -cl(A) ∧ *μ -cl(B).

Proof: The easy verification is omitted.

Theorem 4.06: In a fts X, a fuzzy set A is *μ -open iff A=f *μ -int(A).

Proof: omitted.

Theorem 4.07: In a fts X, the following results hold for *μ -interior.

1) *μ -int(0)=0.
2) *μ -int(A) is *μ -open fuzzy set in X.
3) *μ -int(A) ≤ *μ -int(B) if A ≤ B.
4) *μ -int (*μ -int(A)) = *μ -int(A).
5) *μ -int(A ∨ B) ≥ *μ -int(A) ∨ *μ -int(B).
6) *μ -int(A ∧ B) ≤ *μ -int(A) ∧ *μ -int(B).

Proof: Omitted.

Now we introduce the following.

Definition 4.08: A fts (X, T) is called a fuzzy – cTμ* space if every *μ - closed fuzzy set is a closed fuzzy set.

Theorem 4.09: A fts (X, T) is called a fuzzy – cTμ* space iff every *μ - open fuzzy set is a open fuzzy set in X.

Theorem 4.10: Every fuzzy – T ½ space is fuzzy – cTμ*- space.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.11: Let X= {a, b, c}.The fuzzy sets A, B and C defined as follows: A= {(a, 1), (b, 0), (c, 0)}, B= {(a, 0), (b, 1), (c, 1)} and C= {(a, 0), (b, 1), (c, 0). Then (X, T) is a fts space with T= {0, 1, A}. Then (X, T) is fuzzy – cTμ* space as *μ closed fuzzy set B is closed in X. But (X, T) is not fuzzy –T ½ space since g-closed fuzzy set C is not closed fuzzy set in X.

Definition 4.12: A fts (X, T) is called a fuzzy - μTμ space if every *μ - closed fuzzy set is a μ-closed fuzzy set.

Theorem 4.13: Every fuzzy - T ½ space is fuzzy - μTμ space.

Proof: Omitted.

The converse of the above theorem need not be true as seen from the following example.

Example 4.14: Let X= {a, b, c}.The fuzzy sets A, B and C defined as follows: A= {(a, 1), (b, 0), (c, 0)}, B= {(a, 1), (b, 0), (c, 1)} and C= {(a, 1), (b, 1), (c, 0)}. Let (X, T) be fts with T= {0, 1, A, B}. Then X is fuzzy – μTμ space but not fuzzy –T ½ space as the fuzzy set C is g-closed fuzzy set and is *μ - closed fuzzy set but not closed fuzzy set.

Theorem 4.15: A fts X fuzzy – T ½ space iff it is fuzzy – cTμ* space and fuzzy – μTμ space.

Theorem 4.16: A fts X is called a fuzzy – μTμ space iff every open fuzzy set in X is a *μ - open fuzzy set in X.

V. Fuzzy *μ -Continuous and Fuzzy *μ -Irresolute Mappings in FTS

Definition 5.01: A function f: X→ Y is said to be fuzzy *μ -continuous (f *μ -continuous) if the inverse image of every open fuzzy set in Y is *μ -open fuzzy set in X.

Theorem 5.02: A function f: X→ Y is f *μ -continuous if the inverse image of every closed fuzzy set in Y is *μ - closed fuzzy set in X.

Proof: Omitted.

Theorem 5.03: Every fuzzy continuous (resp: fμ - continuous, fα- continuous, fpre- continuous, fgμ - continuous, fg- continuous, fsg- continuous, fg#-s- continuous and fg#- continuous) function is fuzzy *μ - continuous.

Proof: Omitted.

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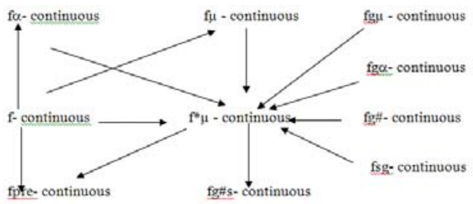
The converse of the above theorem need not be true as seen from the following example.

Example 5.04: Let X = Y = {a, b, c} and the fuzzy sets A, B, and C defined as follows. A= {(a, 0), (b, 0.1), (c, 0.3)}, B= {(a, 0.4), (b, 0.5), (c, 0.6)}, C = {(a, 1), (b, 0.9), (c, 0.7)}. Consider T = {0, 1, B} and σ = {0, 1, A}. Then (X, T) and (Y,σ) are fts. Define f: X→ Y by f(a)=a, f(b)=b and f(c)=c. Then f is f*μ-continuous but not f-continuous (resp: not a fμ-continuous, not a fα-continuous, not a fpre-continuous, not a fgμ-continuous, not a fg- continuous, not a fsg-continuous, not a fg#-s-continuous and not a fg#-continuous). As the fuzzy set C is closed fuzzy set in Y and $f^{-1}(C)$ = C is not closed fuzzy set in X but *μ -closed (resp: μ-closed fuzzy set, α-closed fuzzy set, pre-closed fuzzy set, gμ-closed fuzzy set, g- closed fuzzy set, sg-closed fuzzy set, g#-s-closed fuzzy set and g#-closed fuzzy set) fuzzy set in X. Hence f is fuzzy *μ-continuous.

Remark 5.05: The following diagram 2 shows the relationships of f-*μ-continuous maps with some other fuzzy maps.



where A ⟶ B (A ←|→ B) represents A implies B but not conversely. (A and B are independent).

WE INTRODUCE THE FOLLOWING DEFINITIONS.

Definition 5.06: A function f: X→ Y is said to be fuzzy *μ -irresolute (f *μ-irresolute) if the inverse image of every *μ-closed fuzzy set in Y is *μ-closed fuzzy set in X.

Theorem 5.07: A function f: X→ Y is f *μ-irresolute function iff the inverse image of every *μ-open fuzzy set in Y is *μ-open fuzzy set in X.

Theorem 5.08: Every f *μ -irresolute function is f *μ -continuous.

Proof: Follows from the definitions.

The converse of the above theorem need not be true as seen from the following example.

Example 5.09: Let X = Y = {a, b, c} and the fuzzy sets A, B, C, D and E be defined as follows. A={(a, 1), (b, 0), (c, 0)}, B = {(a, 0), (b, 1), (c, 0)}, C = {(a, 1), (b, 1), (c, 0)}, D={(a, 1), (b, 0), (c, 1)}, E={(a, 0), (b, 1), (c, 1)}. Consider T = {0, 1, A, B, C, D} and σ= {0, 1, C}. Then (X, T) and (Y,σ) are fts. Define f: X→ Y by f(a)=b, f(b)=c and f(c)=a. Then f is f *μ -continuous but not f *μ -irresolute as the fuzzy set E is *μ-closed fuzzy set in Y but $f^{-1}(E)$ = C is not *μ-closed fuzzy set in X.

Theorem 5.10: Let X, Y, Z be three fuzzy topological spaces. Let f: X→ Y and g: Y→ Z be any two fuzzy functions. Then

(1) gof is *μ -continuous if g is continuous and f is *μ -continuous.
(2) gof is *μ -irresolute if g is *μ -irresolute and f is *μ -irresolute.
(3) gof is *μ -continuous if g is *μ -continuous and f is *μ -irresolute.
(4) gof is *μ -continuous if g is *μ -continuous and f is gc -irresolute.

Proof: Omitted.

VI. Fuzzy *μ -Open Maps and Fuzzy *μ -Closed Maps in FTS

This study was further carried out by Sadanand N. Patil [9]. We introduced the following concepts.

Definition 6.01: A function f: X→ Y is said to be fuzzy *μ -open (briefly f *μ -open) if the image of every open fuzzy set in X is *μ -open fuzzy set in Y.

Definition 6.02: A function f: X→ Y is said to be fuzzy *μ -closed (briefly f *μ -closed) if the image of every closed fuzzy set in X is *μ -closed fuzzy set in Y.

Theorem 6.03: Every fuzzy - open map (resp: fμ - open map, fα- open map, fpre- open map, fgμ - open map, fg- open map, fsg- open map, fg#-s- open map and fg#- open map) is fuzzy *μ -open map.

Proof: The proof is follows from the definition 6.01.

The converse of the above theorem need not be true as seen from the following example.

Example 6.04: Let X = Y = {a, b, c}. Fuzzy sets A, B and C be defined as follows. A= {(a, 0), (b, 0.1), (c, 0.3)}, B= {(a, 0.4), (b, 0.5), (c, 0.6)} and C = {(a, 1), (b, 0.9), (c, 0.7)}. Consider T = {0, 1, A} and σ= {0, 1, B}. Then (X, T) and (Y,σ) are fts. Define f: X→ Y by f(a)=a, f(b)=b and f(c)=c. Then f is f *μ -open map but not a f-open map (resp: not a fμ - open map, not a fα- open map, not a fpre-open map, not a fgμ - open map, not a fg- open map, not a fsg- open map, not a fg#-s- open map and not a fg#- open map) as the fuzzy set A is fuzzy open in X, its image f(A) = A is not open fuzzy set in Y which is *μ -open fuzzy set in Y.

Theorem 6.05: Every fuzzy - closed map is fuzzy *μ - closed map.

Proof: The proof is follows from the definition 6.02.

The converse of the above theorem need not be true as seen from the following example.

Example 6.06: In the example 6.04, the function f is fuzzy *μ -closed map but not closed fuzzy map as the fuzzy set C is closed in X and its image f(C) =C is *μ -closed fuzzy set in Y but not closed in Y.

Theorem 6.07: A function f: X→ Y is f *μ -closed iff for each fuzzy set S of Y and for each open fuzzy set U such that $f^{-1}(S) \leq U$, there is a *μ -open fuzzy set V of Y such that $S \leq V$ and $f^{-1}(V) \leq U$.

Theorem 6.08: If a map f: X→ Y is fuzzy gc - irresolute and f *μ -closed and A is *μ -closed fuzzy set of X, then f(A) is *μ -closed fuzzy set in Y.

Theorem 6.09: Let f: X→ Y, g: Y→Z be two maps such that gof: X →Z is f *μ -closed map.

i) If f is f - continuous and surjective, then g is f *μ -closed map.

ii) If g is f *μ -irresolute and injective, then f is f *μ -closed map.

Proof: omitted.

Definition 6.10 [17]: Let X and Y be two fts. A bijective map f: X→ Y is called fuzzy - homeomorphism (briefly f - homeomorphism) if f and f^{-1} are fuzzy - continuous.

We introduce the following.

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Definition 6.11: A function f: X→ Y is called fuzzy f *μ -homeomorphism (briefly f *μ -homeomorphism) if f and f^{-1} are f *μ -continuous.

Theorem 6.12: Every f - homeomorphism is fg-homeomorphism, fg# -homeomorphism, fg#s-homeomorphism, fg#α-homeomorphism, fg*s-homeomorphism, fĝ-homeomorphism and f *μ -homeomorphism.

Proof : The proof is follows from the definition 6.02.

The converse of the above theorem need not be true as seen from the following example.

Example 6.13: Let X = Y = {a, b, c} and the fuzzy sets A, B and C be defined as follows. A = {(a, 1), (b, 0), (c, 0)}, B = {(a, 1), (b, 1), (c, 0)}, C = {(a, 1), (b, 0), (c, 1)}. Consider T = {0, 1, A} and σ= {0, 1, B}. Then (X, T) and (Y,σ) are fts. Define f: X→ Y by f(a)=a, f(b)=c and f(c)=b. Then f is fg-homeomorphism, fg# -homeomorphism, fg#s-homeomorphism, fg#α-homeomorphism, fg*s-homeomorphism, fĝ-homeomorphism and f *μ -homeomorphism but not f - homeomorphism as A is open in X f (A) = A is not open in Y. f^{-1} : Y→ X is not f-continuous.

Theorem 6.14: Let f: X→ Y be a bijective function. Then the following are equivalent:

a) f is f *μ -homeomorphism.

b) f is f *μ -continuous and f *μ -open maps.

c) f is f *μ -continuous and f *μ -closed maps.

Proof: Omitted

Definition 6.15: Let X and Y be two fts. A bijective map f: X→ Y is called fuzzy *μ -c - homeomorphism (briefly f *μ -c - homeomorphism) if f and f^{-1} are fuzzy *μ -irresolute.

Theorem 6.16: Every f *μ -c- homeomorphism is f *μ -homeomorphism.

Proof: Omitted.

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