Solidification of Porous Material under Natural Convection by Three Phases Modeling

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Abstract—**The performance of natural convective flow over a rectangular enclosure for coupled heat and mass transfer under freezing-thawing process is investigated experimentally and numerically. The enclosure filled with an unsaturated porous material. The two-dimensional model based on experimental observation for three phases' air, water and ice is sought and solved. Each phase in the porous media is assumed in local thermodynamic equilibrium (LTE) with each other. Comparison of the experiment and modeling is in good agreement.**

Index Terms— **porous media, coupled heat and mass, solidification, free convection**

I. INTRODUCTION

THIS Study looks to the freezing processes in porous media which is of great interest in environmental, energetic, biological and industrial systems, natural freezing and thawing in the cold regions, food preservation, agriculture, separation processes, thermal energy storage and freezing of biological tissues. Sorption of moisture in building materials can cause metal corrosion, structure deterioration and improper performance of building insulations. In addition, expansion and deformation of soil framework during freezing process is an important feature in the foundation and road engineering in the cold regions. In food chemistry, it involves the determination of the cooling rates the most suited to the food preservation. In biomechanics, a similar worry is the cryopreservation of organs in view of their further transplantation. Therefore, heat and moisture transfer in porous media during freezing process is important, but reported literature on this subject is not many. Luikov carried out heat and moisture transfer for the drying process in capillary porous bodies [1], His model is applicable for both hygroscopic and non-hygroscopic materials and accounts for all forms of water bonding. The physical and thermodynamic properties in Luikov's equations are functions of either temperature or moisture content or both. Therefore, this system of coupled equations is non-linear. Luikov and Mikhailov suggested that if calculations are carried out by zones, the transport coefficients can be taken as constant in each of these zones and Luikov's equations become linear [2]. Basirat Tabrizi and Hamdullahpur [3] introduced a source term due to the surface evaporation and used the energy and mass balances based on Luikov's model to investigate drying process in capillary porous bodies. Hamdami et al. [4]-[5] used scheme to simulate heat and mass transfer during freezing of humid porous media. They employed Lee's three-level model of the heat and mass transfer during freezing of par-baked bread. The model accommodates the effect of temperature dependent variables. They indicated cement pastes still expand if water replaced by benzene, which unlike water, contracted when solidifying. Few models were presented for partially frozen soil by using coupled heat and moisture transfer [6]-[9]. Bazant et al. [10] introduced a mathematical model for freeze–thaw durability of concrete as a coupled heat and moisture transfer problem. Nevertheless, due to the complexity of equilibrium relation of unfrozen water, they did not clearly show the closed form of the equations. Freezing (thawing) of soil around the buried pipes used for conveying various fluids [11], cryosurgery and cryo-preservation of biological tissue [12], and food processing [13]. The freezing of porous media has been studied extensively for low porosity media (porosity below 50% such as rock, sand, soil) whereas few are available for high porosity media [14]-[16] Studied on phase front propagation in soils. Chatterji investigated on the frost damage of concrete by freeze–thaw cycles [17]. The prediction of the temperature and moisture fields in a product needs to understand the physic of the phenomena and the importance of specific parameters. This could be of use for food freezing (i.e. bread) or soil freezing, concrete freezing. In most practical situations, the flow pattern in porous media is three-dimensional. However, two-dimensional flow does exist both under laboratory condition and some in nature. The understanding from two-dimensional analysis may facilitate the study on the three-dimensional situation. This paper investigates the natural convection in the saturated porous media by assuming two-dimensional pattern in freezing processes.

II. EXPERIMENTAL SETUP AND TEST PROCEDURES

A test rig is designed to measure the variation of temperature of a porous body during freezing. Figure 1 shows this apparatus and it consists of a loop channel, a compression refrigeration cycle, measuring instruments. The

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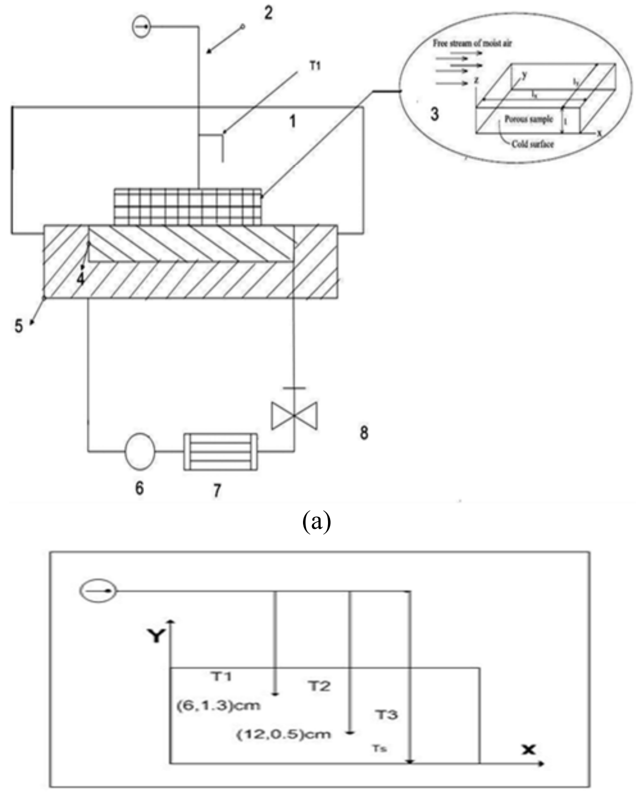
evaporator is embodied into a flat aluminum plate to provide a cold surface.



(a)

(b)

Fig.1. (a)Schematic diagram of apparatus: (1) Air channel, (2) Digital temperature recorder, (3) Cubic sample of porous media, (4) Cold surface (evaporator), (5) Insulation, (6) Compressor, (7) Condenser, (8) Expansion valve , (b) Location of thermocouples in porous medium

A cubic porous media is placed on the cold surface while air stream moved naturally over. Hence, the upper surface is exposed to the air. Sand is used as porous medium due to its extensive applications in building materials. This material is packed in a rectangular cubic perforated mold in dimensions of 20 × 20 × 2 cm in order to configure the porous media. Three thermocouples type K are used to measure the temperature inside the porous media. They are inserted at the various coordinates. The temperature measurement is carried out every minute during the freezing process. The estimated uncertainty due to experimental instruments was obtained ±1.5◦C for temperature measurement. The accuracy due to experimental instruments is shown in Table I.

TABLE I
ACCURACY OF MEASUREMENT

Process	Quantity
Air temperature	15±0.7 ºC
Wall temperature	-7 ±0.7 ºC
Sand weight	500±3g
Sand volume fr	0.4±0.005

III. MODELING

The physical problem involves investigation of heat and mass transfer during freezing of capillary porous material. During freezing, moisture in porous media exists in three phases: solid, liquid and gas. Governing equations are obtained by the conservation of mass and energy of each phase. Equations are based on the following assumptions:

a) each phase in the porous media is in local thermodynamic equilibrium
b) two-dimensional unsteady flow
c) for of lack of filtration motion and pressure gradient in porous matrix, momentum equations are neglected
d) all thermo physical parameters are assumed constant
e) no mass transfer from boundary of porous solids

In order to describe the simultaneous heat, moisture in saturated porous media with phase change following model is introduced. List of symbols are shown in Table II.

Mass balance of liquid phase:

$$\frac{\partial(\rho_l \varepsilon_l)}{\partial t} = -\dot{m} \quad (1)$$

Mass balance of solid phase:

$$\frac{\partial(\rho_s \varepsilon_s)}{\partial t} = \dot{m} \quad (2)$$

Mixture energy equation is:

$$(\rho c)_m \frac{\partial T}{\partial t} = k_m \nabla^2 T - \dot{m}h \quad (3)$$

Where

$$\varepsilon = \varepsilon_s + \varepsilon_l + \varepsilon_g + \varepsilon_p$$
$$(\rho c)_m = \varepsilon_s(\rho c)_s + \varepsilon_l(\rho c)_l + \varepsilon_g(\rho c)_g + \varepsilon_p(\rho c)_p \quad (4)$$
$$k_m = \varepsilon_s k_s + \varepsilon_l k_l + \varepsilon_g k_g + \varepsilon_p k_p$$

Here, the field variables include temperature T, liquid content ε_l, solid content ε_s, gas content ε_g and porous content ε_p. The phase change rate of condensation which stands for the source term due to the moisture freezing on the particle surface and negative sign of m in (1) means that the amount of liquid content decreases during experiment and therefore in (2) the positive sign means the amount of solid content increases. The term can be expressed as [18]:

$$\dot{m} = \sigma_c(\dot{x}_{cl} - \dot{x}_l) \quad (5)$$

The evaporation coefficient, σ_c, is:

$$\sigma_c = \frac{Nu*\rho_l*D_l}{d} \quad (6)$$

It can be assumed the relation for sphere particle as [19]:

$$Nu = 2 + \frac{0.589 Ra_D^{\frac{1}{4}}}{\left(1+\left(\frac{0.5}{\mathrm{Pr}}\right)^{\frac{9}{16}}\right)^{\frac{4}{9}}} \quad (7)$$

$$Pr = \nu/\alpha \quad (8)$$

$$Ra_D = g\beta\left(T_{inf} - T_s\right)d^3/\alpha\upsilon \quad (9)$$

The moisture content of the saturated wetting porous medium at the surface of the solid particle x_{cl} as a function of the temperature and moisture content of the particle is [18]:

$$\dot{x}_{cl} = \varphi_1(T)\varphi_2(x) \quad (10)$$

The above functions can be computed from the tension curve of the moisture and the sorption characteristic of the solid moisture in the system. The approximations are [18]:

$$\varphi_1(T) = 0.622 * \left(\frac{P_w}{760-P_w}\right) \quad (11)$$

$$\varphi_2(x) = x^n(x_{sc}^n + l)/(x_{sc}^n(x^n + l)) \quad (12)$$

$$P_w = 10\hat{}(0.622 + \frac{7.5*T_{inf}}{238+T_{inf}}) \quad (13)$$

Where n and l are constants (n = 3; l = 0.01).

In order to convert ε_l to x, we have:

$$\varepsilon_l = V_l/V = m_l/\rho_l V \tag{14}$$

Also using Eq. (1) follows:

$$\dot{x}_l = \partial\left(\frac{V}{m_o} \times \rho_l\, \varepsilon_l\right)/\partial t \tag{15}$$

The initial and boundary conditions are employed according to the stated experiment and follows:

$$\varepsilon_s = 0,\ \varepsilon_l = 0.3,\ \varepsilon_g = 0.01 \text{ at } t=0 \tag{16}$$

$$T = T_s \quad \text{for lower surface} \tag{17}$$

$$k\frac{\partial T}{\partial n} + h\left(T - T_{inf}\right) = 0 \quad \text{for upper surface} \tag{18}$$

TABLE II
UNITS FOR SOLIDIFICATION PROPERTIES

Symbol	Quantity	Unit
c	specific heat	J/ (kg K)
d	diameter of particle	m
D_l	diffusivity of liquid in porous medium	m^2/s
h	specific enthalpy	J/kg
H	Height of porous layer	M
k	conduction heat transfer coefficient	W/mK
$\dot{m}$	mass rate of phase change	Kg/(m^2 s)
m_0	dry porous medium weight	Kg
n	constant value	
Nu	local Nusselt number	
P_w	partial pressure of vapor in gas	Pa
Ra	Rayleigh number	
t	Time	s
T	Temperature	°C
W	width of the porous layer	m
x , y	spatial coordinates	m
Φ1	Palanz Temperature function	
Φ2	Palanz Moisture function	
x	moisture content	kg/kg
β	thermal expansion coefficient	1/k
Δ	difference	
ε	porosity or voidage	m^3/m^3
μ	viscosity	kg/ (ms)
ρ	density	kg/m^3
σ_c	particle freezing coefficient	kg/(m^2)

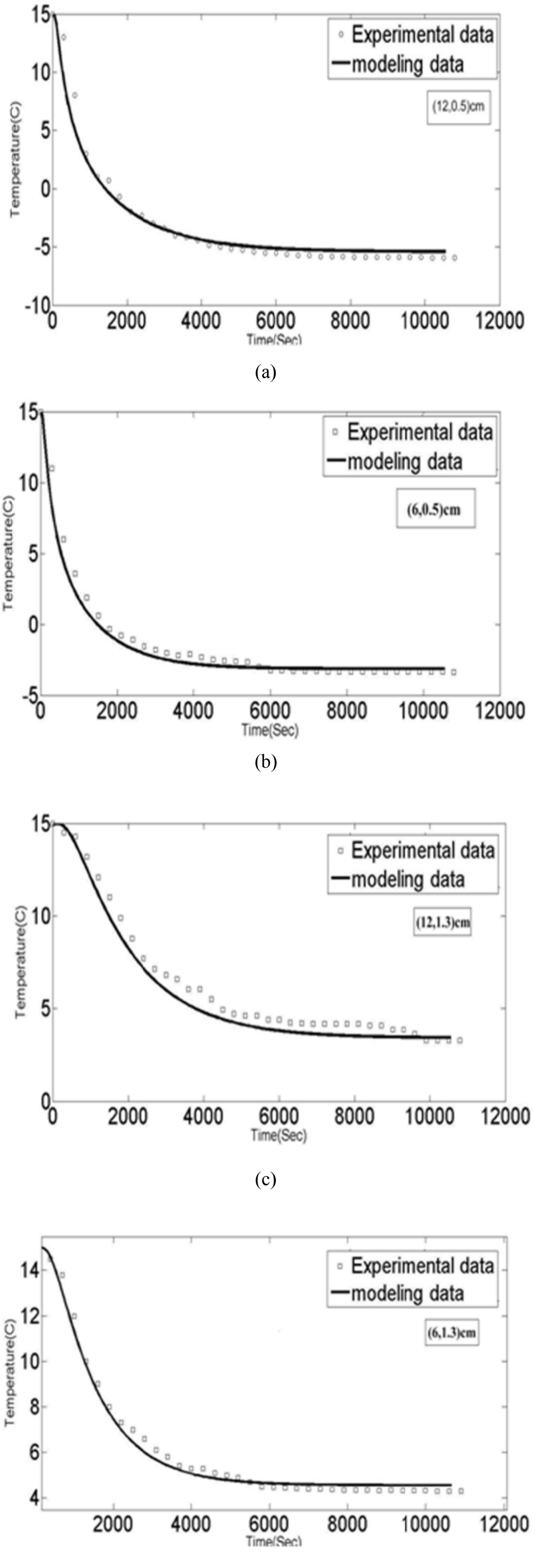
IV. RESULTS AND DISCUSSION

Variation of different porosity, thickness, material, initial temperature, lower surface temperature is examined and the source term effect is investigated.

Fig. 2 (a, b, c, and d) shows the simulation results of temperature with experimental results at different locations of porous cube (12, 0.5), (6, 0.5), (12, 13), and (6, 13) cm. Because of thin porous layer there is large temperature gradient not only in y direction but also in x direction. Numerical results are about 3% lower than experiment results. Absolute errors except in earlier time of experiment are lower than uncertainty of thermocouples. By considering the limit of uncertainty, it can be seen that the model based on mass rate of phase change could predict the variation of temperature and moisture content in porous media more closely.

Fig. 3 illustrates the introduced source term effect with the experimental results. The introduced model predicts much better than without source term (or pure conduction).

Fig. 4 (a, b) indicates the effect of different initial and surface temperature. The higher initial temperature needs higher time for freezing. With lower surface temperature, then final temperature is at lower temperature.



(a)

(b)

(c)

(d)

Fig. 2 Comparison of simulated temperature with the measured temperature of porous sample at pts (12, 0.5), (6, 0.5), (12, 13), and (6, 13) cm

Fig. 5 shows the effect of thickness. Since all parameters are constant and just the thickness of sand is varied here, then the equilibrium time is shorter and the final equilibrium temperature is higher. In short times, the variation of thickness had important role in mass rate and temperatures, because the cold air doesn’t have time to influence inside the porous sample. Finally Fig. 6 illustrates the effect of different porosity; it shows that the porosity doesn’t have main effect in final equilibrium temperature.

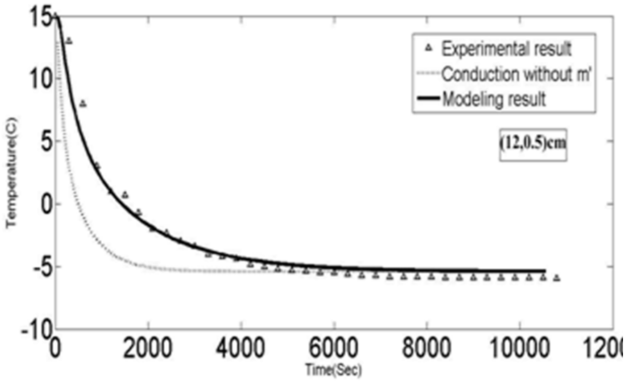
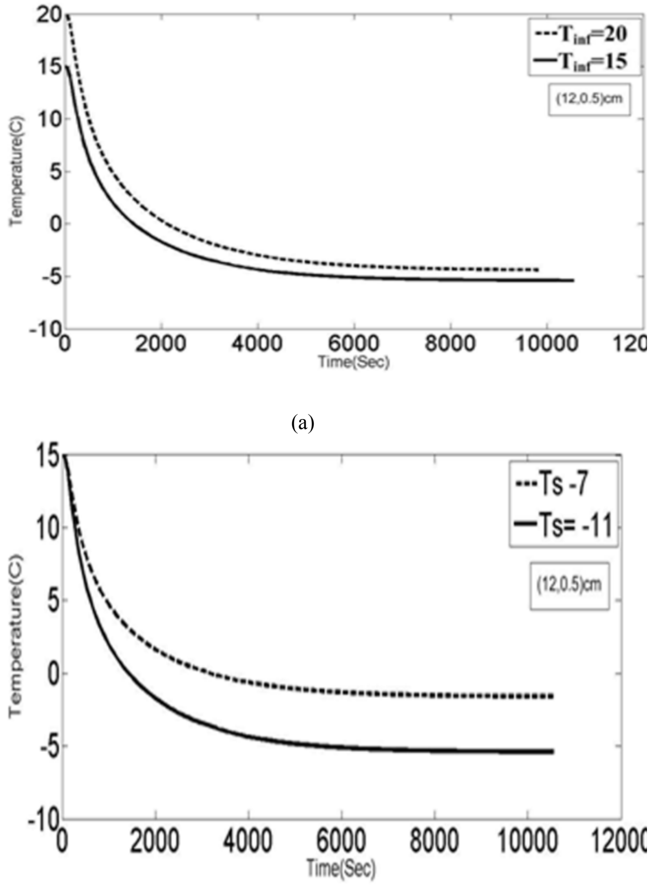


Fig. 3 Comparison of simulated with the measured temperature of porous sample and modeling without mass rate of phase change at pt. (12, 0.5)



(a)

(b)

Fig. 4 Simulation of different temperature a. initial and b. surface temperature at pt. (12, 0.5)

V. CONCLUSION

Numerical study of two-dimensional heat and mass transfer in capillary porous media by introducing a source term was proposed. In order to validate the model, an experimental setup was built to measure the temperature of a cubic porous media during freezing process. The experimental data was obtained and the uncertainty analysis was performed. A relatively good agreement was achieved between experimental and model results. The mass rate of phase change predicts the behavior of heat and mass transfer characteristics much better than without source term (or pure conduction).

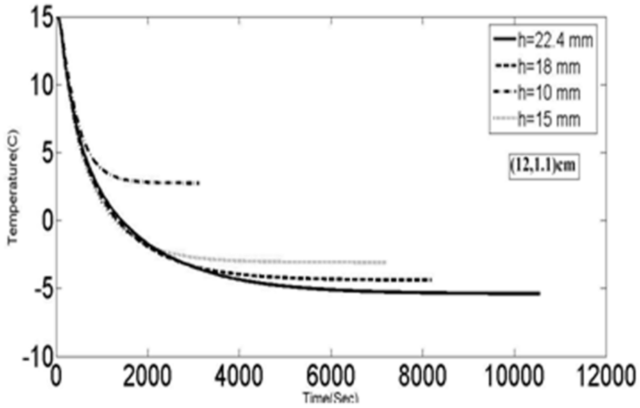


Fig. 5 Simulated temperature data for different thickness vs. time (sec)

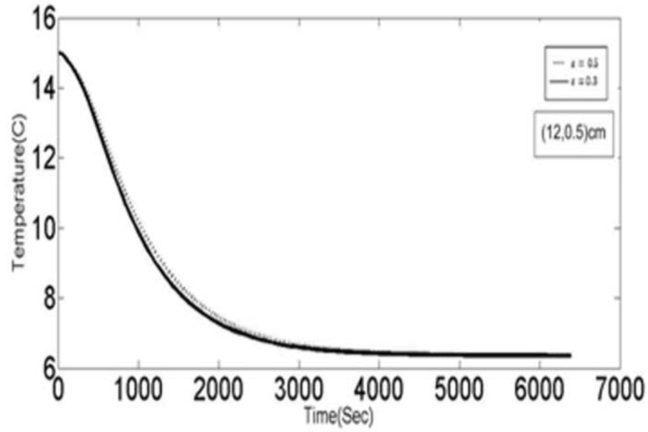


Fig. 6 Simulated temperature data for different porosity, dot line ε=0.5, black line ε=0.5

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