Fekete-Szego Inequalities for Generalized Sakaguchi Type Functions

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***Abstract*-- In this paper, we have investigated certain coefficient inequalities (Fekete-Szegö) and subordination properties for two subclasses of generalized Sakaguchi type functions defined recently by B. A. Frasin. We have obtained sharp upper bounds of $\left|a_3 - \mu a_2^2\right|$ for the functions $f(z) = z + a_2 z^2 + a_3 z^3 + \ldots$ belonging to a new subclass of generalized Sakaguchi type functions.**

***Keywords*--Fekete-Szegö inequality, Sakaguchi functions, Subordination.**
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I INTRODUCTION

LET A be the class of *analytic functions* of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \; ; \qquad (z \in \Delta := \{z \in \mathbb{C} : |z| < 1\}) \tag{1.1}$$

and S be the subclass of A consisting of univalent functions. For two functions f, $g \in A$, we say that the function $f(z)$ is *subordinate* to $g(z)$ in Δ and write $f \prec g$, or $f(z) \prec g(z); (z \in \Delta)$ if there exists an analytic function $w(z)$ with $w(0) = 0$ and $|w(z)| < 1$ $(z \in \Delta)$, such that $f(z) = g(w(z))$, $(z \in \Delta)$. In particular, if the function g is univalent in Δ, the above subordination is equivalent to $f(0) = g(0)$ and $f(\Delta) \subset g(\Delta)$.

Recently B. A. Frasin [1] introduced and studied a generalized Sakaguchi type class $S(\alpha, s, t)$ and $T(\alpha, s, t)$. A function $f(z) \in A$ is said to be in the class $S(\alpha, s, t)$ if it satisfies

$$\mathbf{Re}\left\{\frac{(s-t)zf'(z)}{f(sz) - f(tz)}\right\} > \alpha \tag{1.2}$$

for some $0 \le \alpha < 1$, $s, t \in \mathbb{C}$ with $s \neq t$ and for all $z \in \Delta$. We also denote by the subclass $T(\alpha, s, t)$ the subclass of A consisting of all functions $f(z)$ such that $zf'(z) \in S(\alpha, s, t)$. The class $S(\alpha, 1, t)$ was introduced and studied by Owa et al. [4,5], and by taking $t = -1$, the class $S(\alpha, 1, -1) \equiv S_s(\alpha)$ was introduced by Sakaguchi [2] and is called Sakaguchi function of order α (see [3], [4]), where as $S_s(0) = S_s$ is the class of starlike functions with respect to symmetrical points in Δ.

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Also, we note that $S(\alpha, 1, 0) \equiv S^*(\alpha)$ and $T(\alpha, 1, 0) \equiv C(\alpha)$ which are, respectively, the familiar classes of starlike functions of order $\alpha (0 \le \alpha < 1)$ and convex functions of order $\alpha (0 \le \alpha < 1)$. In this paper we define the following class, $S(\Phi, s, t)$ and $T(\Phi, s, t)$, which are generalization of the classes $S(\alpha, s, t)$ and $T(\alpha, s, t)$ respectively.

Definition 1.1 Let $\Phi(z) = 1 + B_1 z + B_2 z^2 + \ldots$ be univalent starlike function with respect to '1' which maps the unit disk Δ onto a region in the right half plane which is symmetric with respect to the real axis, and let $B_1 > 0$. The function $f \in A$ is in the class $S(\Phi, s, t)$ if

$$\left\{\frac{(s-t)zf'(z)}{f(sz) - f(tz)}\right\} \prec \Phi(z), \; s \neq t \tag{1.3}$$

Again $T(\Phi, s, t)$ denote the subclass of A consisting functions $f(z)$ such that $zf'(z) \in S(\Phi, s, t)$. Obviously $S(\Phi, 1, 0) \equiv S^*(\Phi)$ and $T(\Phi, 1, 0) \equiv C(\Phi)$, which are the classes introduced and studied by Ma and Minda [8]. The class $S(\Phi, 1, -1) \equiv S_S^*(\Phi)$, which is a known class studied by Shanmugam et al. [6].

When $\Phi(z) = (1 + Az)/(1 + Bz), (-1 \le B < A \le 1)$ we denote the subclasses $S(\Phi, s, t)$ and $T(\Phi, s, t)$ by $S[A, B, s, t]$ and $T[A, B, s, t]$ respectively. For $s = 1$, $t = 0$ and $\Phi(z)$ defined above the subclass $S(\Phi, 1, 0)$ reduces to the class $S^*[A,B]$ studied by Janowski [7].

For $0 \le \alpha < 1$, let $S(\alpha) \equiv S[1-2\alpha, -1, 1, -1]$, which is a known class studied by Owa et al. [5]. Also, for $s = 1$, $t = -1$ and $\Phi(z) = \dfrac{1 + (1-2\alpha)z}{1-z}$, our class reduces to a known class $S(\alpha, -1)$ studied by Cho et al. ([3], see also [5]).

In the present paper, we obtain the Fekete-Szegö inequality for the functions in the subclass $S(\alpha, s, t)$. To prove our main results, we need the following lemma:

Lemma 1.2 [8] If $p(z) = 1 + c_1 z + c_2 z^2 + \ldots$ is a function with positive real part in Δ, then for any complex number μ

$$\left|c_2 - \mu c_1^2\right| \le 2 \max\{1, |2\mu - 1|\}$$

and the result is sharp for the functions given by

$$p(z) = \frac{1+z}{1-z}, \quad p(z) = \frac{1+z^2}{1-z^2}$$

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Lemma 1.3 [6] If $p(z)=1+c_1z+c_2z^2+...$ is an analytic function with positive real part in Δ, then for a real number v

$$\left|c_2-vc_1^2\right|\le\begin{cases}-4v+2 & if\ v\le 0\\ 2 & if\ 0\le v\le 1\\ 4v-2 & if\ v\ge 1\end{cases}$$

when $v < 0$ or $v > 1$, then the equality holds if and only if $p(z)$ is $(1 + z) / (1 - z)$ or one of its rotations. If $0 < v < 1$, then the equality holds if and only if $p(z)$ is $(1 + z^2) / (1 - z^2)$ or one of its rotations. If $v = 0$, the equality holds if and only if

$$p(z)=\left(\frac{1}{2}+\frac{1}{2}\lambda\right)\frac{1+z}{1-z}+\left(\frac{1}{2}-\frac{1}{2}\lambda\right)\frac{1-z}{1+z}$$

$$(0\le\lambda\le 1)$$

or one of its rotations. If $v = 1$, the equality holds if and only if $p(z)$ is the reciprocal of one of the functions such that the equality holds in the case of $v = 0$. Also the above upper bound is sharp, and it can be improved as follows when $0 < v < 1$:

$$\left|c_2-vc_1^2\right|+v\left|c_1\right|^2\le 2;\qquad (0<v\le 1/2)$$

and

$$\left|c_2-vc_1^2\right|+(1-v)\left|c_1\right|^2\le 2;\qquad (1/2<v<1)$$

II MAIN RESULTS

Theorem 2.1 If the function $f(z)$ given by (1.1) belongs to $S(\Phi, s, t)$, then

$$\left|a_3-\mu a_2^2\right|\le\beta\max\left\{B_1,B^*\right\}$$

provided $s+t\ne 2$.

Where $\beta=\dfrac{1}{\left|3-s^2-st-t^2\right|}$ **(2.1)**

and

$$B^*=\left|B_2+B_1^2\frac{s+t}{2-s-t}-\mu B_1^2\frac{3-s^2-st-t^2}{(2-s-t)^2}\right| \qquad \textbf{(2.2)}$$

The result is sharp.

Proof: Let $f\in S(\Phi, s, t)$, then there exists a *Schwarz* function $w(z)\in A$ such that

$$\left\{\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\right\}=\Phi(w(z)),\ (z\in\Delta, s\ne t) \qquad \textbf{(2.3)}$$

If $p_1(z)$ is analytic and has positive real part in Δ and $p_1(0)=1$, then

$$p_1(z)=\frac{1+w(z)}{1-w(z)}=1+c_1z+c_2z^2+...;\qquad (z\in\Delta) \qquad \textbf{(2.4)}$$

From (2.4) we obtain

$$w(z)=\frac{c_1}{2}z+\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)z^2+... \qquad \textbf{(2.5)}$$

Let $p(z)=\dfrac{(s-t)zf'(z)}{f(sz)-f(tz)}=1+b_1z+b_2z^2+...;(z\in\Delta)$ **(2.6)**

which gives

$$b_1=(2-s-t)a_2$$

and $b_2=(s+t)(s+t-2)a_2^2+(3-s^2-st-t^2)a_3$ **(2.7)**

Since $\Phi(z)$ is univalent and $p\prec\Phi$, therefore using (2.5), we obtain:

$$p(z)=\Phi(w(z))=1+\frac{B_1c_1}{2}z+\left\{\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)B_1+\frac{1}{4}c_1^2B_2\right\}z^2+...;\ (z\in\Delta) \qquad \textbf{(2.8)}$$

Now from (2.6), (2.7) and (2.8), we have

$$(2-s-t)a_2=\frac{B_1c_1}{2},$$

$$\frac{1}{2}\left(c_2-\frac{c_1^2}{2}\right)B_1+\frac{1}{4}c_1^2B_2$$

$$=(s+t)(s+t-2)a_2^2+(3-s^2-st-t^2)a_3$$

Therefore we have

$$a_3-\mu a_2^2=\frac{B_1}{2(3-s^2-st-t^2)}\left\{c_2-vc_1^2\right\},\qquad \left\{s+t\ne 2,\ \ s\ne t\right\} \qquad \textbf{(2.9)}$$

where

$$v:=\frac{1}{2}\left[1-\frac{B_2}{B_1}-\left(\frac{s+t}{2-s-t}\right)B_1+\frac{\left(3-s^2-st-t^2\right)}{(2-s-t)^2}\mu B_1\right],$$

$$\left\{s+t\ne 2,\ \ s\ne t\right\} \qquad \textbf{(2.10)}$$

Our result now follows by an application of Lemma 1.2. The result is sharp for the function defined by

$$\left\{\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\right\}=\Phi(z),\ s\ne t \qquad \textbf{(2.11)}$$

and

$$\left\{\frac{(s-t)zf'(z)}{f(sz)-f(tz)}\right\}=\Phi(z^2),\ s\ne t \qquad \textbf{(2.12)}$$

If we take parameters s and t to be real numbers then by using Lemma 1.3 we obtain following result:

Corollary 2.2 If the function $f(z)$ given by (1.1) belongs to $S(\Phi, s, t)$, for real parameters s and t such that $s+t\ne 2$ and $s\ne t$, then

$$\left|a_3-\mu a_2^2\right|\le\beta\begin{cases}B^*, & \mu\le\sigma_1\\ B_1, & \sigma_1\le\mu\le\sigma_2\\ -B^*, & \mu\ge\sigma_2\end{cases}$$

where β, B^* are defined by equation (2.1), (2.2) respectively and

$$\sigma_1=\frac{(2-s-t)^2}{B_1(3-s^2-st-t^2)}\left[-1+\frac{B_2}{B_1}+B_1\left(\frac{s+t}{2-s-t}\right)\right] \qquad \textbf{(2.13)}$$

$$\sigma_2=\frac{(2-s-t)^2}{B_1(3-s^2-st-t^2)}\left[1+\frac{B_2}{B_1}+B_1\left(\frac{s+t}{2-s-t}\right)\right] \qquad \textbf{(2.14)}$$

The result is sharp.

Remark 1: To show that these bounds are sharp for real parameters s and t, we define the functions K_Φ^n $(n=2,3,...)$ by

$$\left\{\frac{(s-t)z\left(K_\Phi^n\right)'(z)}{K_\Phi^n(sz)-K_\Phi^n(tz)}\right\}=\Phi(z^{n-1}), \tag{2.15}$$

$K_\Phi^n(0)=0=\left(K_\Phi^n\right)'(0)$ - 1

and the functions F_λ and G_λ $(0\le\lambda\le 1)$ by

$$\left\{\frac{(s-t)zF_\lambda'(z)}{F_\lambda(sz)-F_\lambda(tz)}\right\}=\Phi\left(\frac{z(z+\lambda)}{1+\lambda z}\right), \tag{2.16}$$

$F_\lambda(0)=0=\left[F_\lambda\right]'(0)-1$

and

$$\left\{\frac{(s-t)zG_\lambda'(z)}{G_\lambda(sz)-G_\lambda(tz)}\right\}=\Phi\left(\frac{-z(z+\lambda)}{1+\lambda z}\right), \tag{2.17}$$

$G_\lambda(0)=0=\left[G_\lambda\right]'(0)-1$

Obviously the functions $K_\Phi^n, F_\lambda, G_\lambda$ are in $S(\Phi, s, t)$. If $\mu<\sigma_1$ or $\mu>\sigma_2$, then equality holds if and only if f is K_Φ^2 or one of its rotations. When $\sigma_1<\mu<\sigma_2$ then equality holds if and only if f is K_Φ^3 or one of its rotations. $\mu=\sigma_1$ then equality holds if and only if f is F_λ or one of its rotations. $\mu=\sigma_2$ then equality holds if and only if f is G_λ or one of its rotations.

If $\sigma_1\le\mu\le\sigma_2$, in view of Lemma 1.3, Corollary 2.2 can be improved.

Corollary 2.3 Let $f(z)$ given by (1.1) belongs to $S(\Phi, s, t)$, for real parameters s and t such that $s+t\ne 2$ and $s\ne t$ and σ_3 is given by

$$\sigma_3=\frac{(2-s-t)^2}{\left(3-s^2-st-t^2\right)B_1}\left\{\frac{B_2}{B_1}+\frac{B_1(s+t)}{(2-s-t)}\right\} \tag{2.18}$$

If $\sigma_1<\mu\le\sigma_3$, then

$$\left|a_3-\mu a_2^2\right|+\frac{1}{B_1^2}\left[\begin{array}{l}(B_1-B_2)\dfrac{(2-s-t)^2}{\left(3-s^2-st-t^2\right)}\\ -B_1^2\dfrac{(2-s-t)(s+t)}{\left(3-s^2-st-t^2\right)}+\mu B_1^2\end{array}\right]\left|a_2\right|^2$$

$$\le\frac{B_1}{(3-s^2-st-t^2)} \tag{2.19}$$

If $\sigma_3<\mu\le\sigma_2$, then

$$\left|a_3-\mu a_2^2\right|+\frac{1}{B_1^2}\left[\begin{array}{l}(B_1+B_2)\dfrac{(2-s-t)^2}{\left(3-s^2-st-t^2\right)}\\ +B_1^2\dfrac{(2-s-t)(s+t)}{\left(3-s^2-st-t^2\right)}-\mu B_1^2\end{array}\right]\left|a_2\right|^2$$

$$\le\frac{B_1}{(3-s^2-st-t^2)} \tag{2.20}$$

where σ_1 and σ_2 are same as defined in Corollary 2.2.

Example 2.4 Let $(-1\le B<A\le 1)$. If $f(z)$ given by (1.1) belongs to $S[A, B, s, t]$, for real parameters s and t, then

$$\left|a_3-\mu a_2^2\right|\le\beta(A-B)\begin{cases}C^*, & \mu\le\sigma_1^*\\ 1, & \sigma_1^*\le\mu\le\sigma_2^*\\ -C^*, & \mu\ge\sigma_2^*\end{cases}$$

where β is defined in (2.1) and

$$C^*=-B+(A-B)\left\{\left(\frac{s+t}{2-s-t}\right)-\mu\frac{\left(3-s^2-st-t^2\right)}{(2-s-t)^2}\right\} \tag{2.21}$$

$$\sigma_1^*=\frac{(2-s-t)^2}{(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B+1}{A-B}\right)\right] \tag{2.22}$$

$$\sigma_2^*=\frac{(2-s-t)^2}{(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right)-\left(\frac{B-1}{A-B}\right)\right] \tag{2.23}$$

Since $f(z)\in T(\alpha, s, t)$ if and only if $zf'(z)\in S(\alpha, s, t)$, Theorem 2.1 with an obvious change of the parameter μ, leads to the following Corollary:

Corollary 2.5 If the function $f(z)$ given by (1.1) belongs to $T(\Phi, s, t)$, then

$$\left|a_3-\mu a_2^2\right|\le\frac{\beta}{3}\max\left\{B_1, B^{**}\right\}$$

provided $s+t\ne 2$.

where

$$B^{**}=\left|B_2+B_1^2\frac{(s+t)}{(2-s-t)}-\frac{3\mu B_1^2}{4}\frac{\left(3-s^2-st-t^2\right)}{(2-s-t)^2}\right| \tag{2.24}$$

The result is sharp.

If we take parameter s and t to be real numbers, then we have following result:

Corollary 2.6 If the function $f(z)$ given by (1.1) belongs to $T(\Phi, s, t)$, for real parameters s and t such that $s+t\ne 2$ and $s\ne t$, then

$$\left|a_3-\mu a_2^2\right|\le\frac{\beta}{3}\begin{cases}B^{**}, & \mu\le\sigma_1^*\\ B_1, & \sigma_1^*\le\mu\le\sigma_2^*\\ -B^{**}, & \mu\ge\sigma_2^*\end{cases}$$

where

$$\sigma_1^*=\frac{4(2-s-t)^2}{3B_1(3-s^2-st-t^2)}\left[-1+\frac{B_2}{B_1}+B_1\left(\frac{s+t}{2-s-t}\right)\right] \tag{2.25}$$

$$\sigma_2^*=\frac{4(2-s-t)^2}{3B_1(3-s^2-st-t^2)}\left[1+\frac{B_2}{B_1}+B_1\left(\frac{s+t}{2-s-t}\right)\right] \tag{2.26}$$

The result is sharp.

Example 2.7 Let $(-1\le B<A\le 1)$. If $f(z)$ given by (1.1) belongs to $T[A, B, s, t]$, for real parameters s and t, then

$$\left|a_3-\mu a_2^2\right|\le\frac{\beta}{3}(A-B)\begin{cases}C^{**}, & \mu\le\sigma_1^{**}\\ 1, & \sigma_1^{**}\le\mu\le\sigma_2^{**}\\ -C^{**}, & \mu\ge\sigma_2^{**}\end{cases}$$

where

$$C^{**} = -B + (A-B)\left\{\left(\frac{s+t}{2-s-t}\right) - \frac{3\mu}{4}\frac{\left(3-s^2-st-t^2\right)}{(2-s-t)^2}\right\} \quad \textbf{(2.27)}$$

$$\sigma_1^{**} = \frac{4(2-s-t)^2}{3\times(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right) - \left(\frac{B+1}{A-B}\right)\right] \quad \textbf{(2.28)}$$

$$\sigma_2^{**} = \frac{4(2-s-t)^2}{3\times(3-s^2-st-t^2)}\left[\left(\frac{s+t}{2-s-t}\right) - \left(\frac{B-1}{A-B}\right)\right] \quad \textbf{(2.29)}$$

If $\sigma_1^{**} \le \mu \le \sigma_2^{**}$, in view of Lemma 1.3, Corollary 2.6 can be improved.

Corollary 2.8 Let $f(z)$ given by (1.1) belongs to $T(\Phi, s, t)$, for real parameters s and t such that $s+t \ne 2$ and $s \ne t$ and σ_3^{**} is given by

$$\sigma_3^{**} = \frac{4(2-s-t)^2}{3\times\left(3-s^2-st-t^2\right)B_1}\left\{\frac{B_2}{B_1} + \frac{B_1(s+t)}{(2-s-t)}\right\} \quad \textbf{(2.30)}$$

If $\sigma_1^{**} < \mu \le \sigma_3^{**}$, then

$$\left|a_3 - \mu a_2^2\right| + \frac{1}{3B_1^2}\left[\begin{array}{l}(B_1 - B_2)\dfrac{(2-s-t)^2}{\left(3-s^2-st-t^2\right)} \\ -B_1^2\dfrac{(2-s-t)(s+t)}{\left(3-s^2-st-t^2\right)} + \dfrac{3\mu B_1^2}{4}\end{array}\right]\left|a_2\right|^2 \quad \textbf{(2.31)}$$

$$\le \frac{B_1}{3\times(3-s^2-st-t^2)}$$

If $\sigma_3^{**} < \mu \le \sigma_2^{**}$, then

$$\left|a_3 - \mu a_2^2\right| + \frac{1}{3B_1^2}\left[\begin{array}{l}(B_1 + B_2)\dfrac{(2-s-t)^2}{\left(3-s^2-st-t^2\right)} \\ +B_1^2\dfrac{(2-s-t)(s+t)}{\left(3-s^2-st-t^2\right)} - \dfrac{3\mu B_1^2}{4}\end{array}\right]\left|a_2\right|^2 \quad \textbf{(2.32)}$$

$$\le \frac{B_1}{3\times(3-s^2-st-t^2)}$$

where σ_1^{**} and σ_2^{**} are same as defined in Corollary 2.6.

Remark 2: For s = 1 and t = -1 in aforementioned Theorem 2.1, Corollaries 2.2, 2.3, 2.5, 2.6, 2.8 and Example 2.4, 2.7, we arrive at the results obtained recently by Shanmugham et al. [6].

III APPLICATIONS

The results obtained in Theorem 2.1 and in Corollaries 2.2, 2.3, 2.5, 2.6, 2.8 are further applicable in certain functions defined through convolution (or Hadamard product) and in particular for certain classes of functions defined through fractional derivatives.

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Brief description of the changes:

i) **Corollary 2.5** equation (**2.24**) the second term of B^{**} is changed from $\frac{B_1^2}{4}\frac{(s+t)}{(2-s-t)}$ to $B_1^2\frac{(s+t)}{(2-s-t)}$

ii) Due to this change there are changes in **Equations (2.25) to (2.30)** and in **Corollaries 2.6 and 2.8**, also in **Example 2.7**

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Brief description of the changes:

i) **Corollary 2.3** inequalities (**2.19**) and (**2.20**) the second term is multiplied by $\left|a_2\right|^2$

ii)) **Corollary 2.8** inequalities (**2.31**) and (**2.32**) the second term is multiplied by $\left|a_2\right|^2$