

# Application of Simulated Annealing Method on Aircrew Assignment Problems in Garuda Indonesia

N. Sumarti, R. N. Rakhman, R. Hadianti, and S. Uttunggadewa

**Abstract**— Problems of flight scheduling consist of the flight rotation pairings and the crew assignment problems. In this paper, we are dealing with the aircrew-assignment problem and its computational aspects that are implemented on data of Garuda Indonesia, a national airline company in Indonesia, serving of at least 42 domestic and international destinations. In this problem, the crew pairing or formally named Crew Rotation Pattern (CROPA) is given as an input to the method of the assignment of aircrews to each of the flights that the company has to cover. There are two issues which will be answered: the minimum number of the aircrew needed for operating the flights for all CROPAs and the allocation of crews to CROPAs with balancing of the flying and duty hours for each crew. The latter problem will be solved by the application of the simulated annealing method, which is a random search method for solving optimization problems. This method use an analog simulation of the annealing of solids, where the objective function to be minimized corresponds to temperature of the solid. This method allows the occasional acceptance of a new inferior solution in order to avoid being trapped in a local optimum. The lower the temperature, the smaller the chance of this new solution to be accepted. It is shown that the results are satisfactory in finding the solution of the aircrew assignment problem.

**Index Terms**— Simulated annealing method, optimization methods, aircrew-assignment problem, computational mathematics.

## I. INTRODUCTION

The aircrew scheduling problem is the optimization problem dealing with allocating the optimal rotations of the flight and the assignment of the crews to each rotation. Research on this problem has been conducted since the last decade of 20th century [1,2,3,4,5,6,7]. This paper uses the simulated annealing method [8] and improvements of the technique in [4] and applied it for solving a large

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scale problem in Garuda Indonesia (GI), the flag carrier of Indonesia. In 2012 GI has 105 aircrafts, and in line with its ongoing Quantum Leap program, by 2015 GI will operate as many as 194 aircraft, consisting of B737-800NGs, A330-300/200s, B777-300ERs - and the A320s where all aircraft's age is an average of 5 years [9]. There are 42 domestic and international destinations with the frequency is 230 flights per day. Recently more destinations are added so it will reach 62 by 2014 [10]. This huge and complicated company is definitely required an efficient and systematic procedure to operate its flight schedules. GI has two flight schedules with duration respectively of six months, which are Winter Schedule and Summer Schedule. This research is one of initial studies concerning potential improvement efforts in order to achieve GI's Quantum Leap program.

The Crew Rotation Patterns (CROPAs) for the input of this research, are resulted from the heuristic approach with minimum number of pairings that serve 702 flights by A330, which consist of 279 flights from CGK, 192 from DPS, and 231 from 11 other airports. Notes that it is assumed that the airport bases for the airway crews are Jakarta or CGK and Denpasar or DPS. Another result with quite different flight schedules [7,8] with CGK as the only airport base yields less number of required crews and larger number of CROPAs with the existence of additional deadhead trips, where there are off-flying pilots for later flights as passengers in the previous flights. In the further research, the resulted CROPAs used in this paper will be optimised by implementing simulated annealing approach with the existence of deadhead trips.

On managing the schedule, the Basic Operations Manual (BOM) containing general guidance for flight crew members regarding policies, procedures, and aspects of flight operations which are applicable to all aircraft types which the company may operate must be fulfilled. In Section 2 we formulate the optimization problem and its constraints based on BOM and local regulation. The minimization problem of number of the crew needed for all CROPAs will be solved in Section 3. The crew assignment problem using simulated annealing method will be discussed in Section 4. An improvement of the method to speed up the process is explained in Section 5. Finally the results on the data are shown in Section 6.

## II. FORMULATION OF THE PROBLEM AND ITS CONSTRAINTS

We are dealing with the aircrew-assignment problem in the long and short haul markets with monthly bases. The flight rotation or CROPA is given as an input to the algorithm for solving the problem. One CROPA is a flights sequence where the first flight departs from the crew's base;

the next flights have departure airports as the same as the arrival airports of previous flights, and the last flight goes to in the crew's base.

A cockpit crew has at least one licence to fly a type of aircrafts. It is unusual for a crew to fly several types of aircrafts during the month due to the safety consideration. It is safer for the crew to adapt as much as possible to a certain type of aircraft in order to fulfill at least one month assignments. There are two different types of crews assigned to the aircraft depending on the distance and the time length, those are the regular crew consisting of 2 pilots and the enlarged crews consisting of 3 to 4 pilots. In this paper, a crew assigned to a CROPA means a group of crews classified as regular or enlarged crew.

The first optimisation problem to be solved is the minimum number of the aircrew needed to operate all CROPAs. If  $K$  is the fixed number of CROPAs for one schedule's period,  $M$  is the number of existing crews,

$$x_{ij} = \begin{cases} 1 & \text{if crew } -i \text{ is assigned to CROPA } -j \\ 0 & \text{if other} \end{cases}$$

and  $\bar{x}_j$  are vectors of  $M$  dimension, where

$$\bar{x}_j = (x_{ij})_{i=1}^M,$$

then the objective function is

$$\min_{x_{ij}} \text{rank} (\bar{x}_1 \bar{x}_2 \bar{x}_3 \dots \bar{x}_K). \quad (1)$$

Having fulfilled all the given constraints, this function gives the minimum value that makes all vectors  $\bar{x}_j$  be linearly independent with fixed value of  $K$ . This value must be less than or equal to  $M$ .

The second optimisation problem is the allocation of crews to CROPAs with balancing of the flying time for each crew. If  $d_j$  is the flying time to run CROPA- $j$ , and  $D$  is the average flying time for all crews,

$$D = \frac{1}{m} \sum_{j=1}^K d_j$$

then the second objective function is

$$\min_{x_{ij}} \frac{1}{M} \sum_{j=1}^K \left( \frac{\sum_{i=1}^M d_j x_{ij} - D}{D} \right)^r \quad (2)$$

where  $r$  is an integer, usually 1 or 2, that gives a computational effect on the difference between the total actual flying-time of crew- $i$  and the ideal flying time for each crew. For  $r = 2$ , the small differences become smaller and large difference are magnified, which are the same reasons for quadratic difference on least squares approximation [11].

The constrains of both problems are formulated based on BOM and local regulations, including maximum amount of 85 hours of flying time in one month, maximum number of 6 working days in seven day in a row, minimum number of 8 days for take a rest in one month, availability of the crew on the working days - which are depended on the planned annual leave, training times, medical examination, or others - and maximum number of 90 takeoffs in one month.

### III. MINIMUM NUMBER OF CREWS

The algorithm to find the minimum number of crew is quite simple in computational implementation, without using the computation of linear independence of vectors. Figure 1 below is the algorithm in searching the minimum number of crews needed. We will find value of  $m_k$  as small

as possible such that we can find a feasible solution  $x_{ij}$  for all CROPAs. For CROPA- $j$  starting from  $j=1$ , we choose randomly crew- $i$  to be a candidate. All constraints should be satisfied by crew- $i$  in order to be assigned to CROPA- $j$ . If it is successful, find another crew to be assigned to the next CROPA. If not, choose randomly another  $i$  and proceed the same way. If it is successful for all  $i$  and  $j$ , this means all crews have been assigned to all CROPA so a feasible solution  $x_{ij}$  has been made. Try to reduce the value of  $m_k$  and set up another feasible solution again using the same procedure. The trial to reduce value of  $m_k$  will stop when there is no feasible solution can be made from the latest value of crews  $m_k$ .

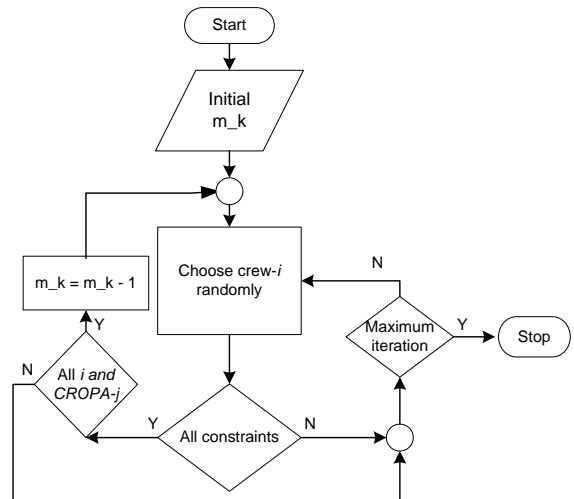


Fig. 1. Algorithm on searching minimum number of crews.

To speed up, an implementation of the simulated annealing method in order to have a balance of flying time in equation (2) had been done whenever a feasible solution has been found for a value of  $m_k$ . The implementation will be explained in the next section. There is significant impact on speeding the process when the initial number of crews  $m_0$  is large enough to begin with, for example 100 and 150.

### IV. SOLVING THE AIRCREW ASSIGNMENT PROBLEM

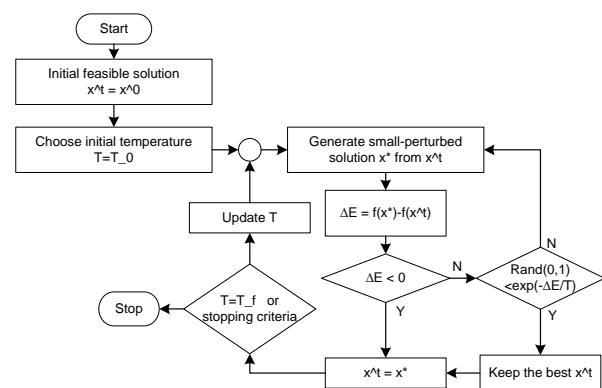


Fig. 2. Simulated annealing algorithm on fair assignment.

In the simulated annealing method, we use the objective function (2) which calculates the average of the relative difference between the actual total flying time and the ideal average daily flight time of each crew as in [5]. First we generate an initial feasible solution  $x_{ij}^0$ . Starting the

simulated annealing, choose an initial temperature  $T_0$  which will be decreased later by factor of  $\alpha$ . Generate another solution  $x_{ij}^*$ , called perturbed solution, which is slightly different from the existing solution. Then we calculate whether the perturbed solution is better than the existing solution or not by taking the difference between values of their objective function, or  $\Delta E = x_{ij}^* - x_{ij}^0$ . If  $\Delta E < 0$ , we accept the perturbed solution as the new solution. If not, we still have a chance to accept the perturbed solution in a particular condition, that is the value of the function  $f(x_{ij}^*, x_{ij}^0) = e^{-\Delta E/T}$  is greater than a randomly chosen value between 0 and 1. Here T is the current temperature. If this condition is satisfied, we accept the perturbed solution  $x_{ij}^*$  as the new solution, but keep the existing solution as the best, and continue to another perturbation. After all perturbations are done, the temperature will be updated by multiplication of decreasing factor  $\alpha$ . The searching of the new solution with the smallest value of  $f$  starts again with those perturbations and will stop when the temperature T has achieved the target  $T_f$  or the maximum iteration has been exceeded.

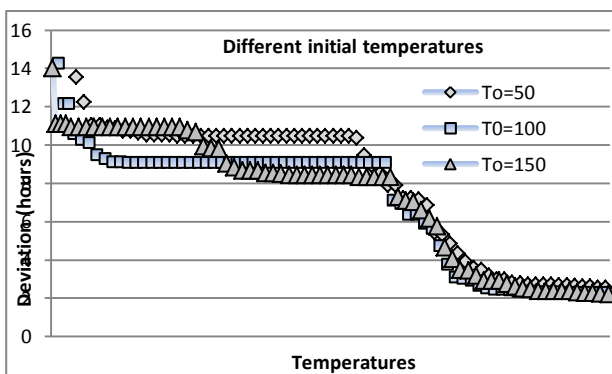


Fig.3. Different initial temperatures

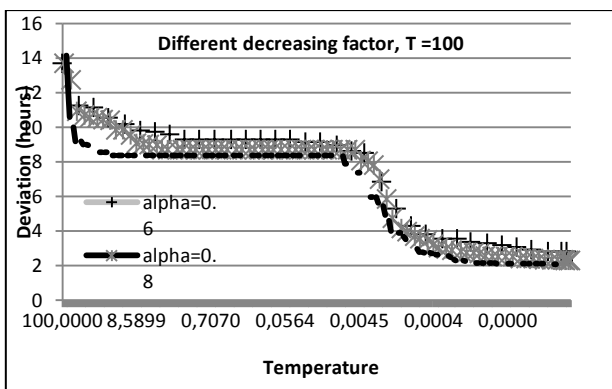


Fig.4. Different decreasing factors

Now we discuss some cases. Notes that graphics in the figures have deviations (hours) as the vertical axis, which are the average deviation between the personal flying time and there ideal one:

$$\text{deviation} = \sum_{j=1}^K d_j x_{ij} - D \quad (3)$$

First, the impacts on different choises of initial temperature and the decreasing factor  $\alpha$  are discussed. For example initial temperatures are 150, 100 and 50. Results shows values of (3) for  $T_0=100$  are much decreasing at the

beginning of iterations. However, these values for all  $T_0$  are much sloping down at the same temperatures. See Fig.3.

For different decreasing factors  $\alpha$ , there is no much difference because the values of (2) are sloping down at about the same temperature, which are about  $10^{-3} - 10^{-5}$ . Note that these values are still significantly decreasing when the temperature is very close to zero.

Now we discuss the procedure to generate a perturbed solution. This solution has slightly different crew assignment from the existing solution on a small number of pairings, which why it is called a perturbation. In one perturbation consisting of a small number of CROPAs, the assigned crews for these observed CROPAs will be replaced by other crews who are assigned on other CROPAs. We observe that different size of the perturbation, for instance 5, 10, and 15 CROPAs in one perturbation, show no difference ever all. The difference is only shown at the beginning iteration. See fig.5.

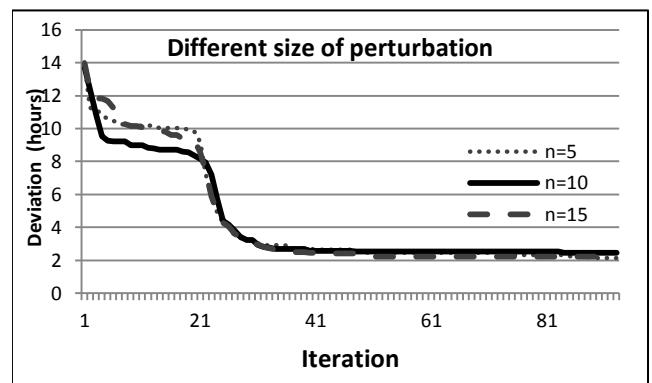


Fig.5. Different size of perturbation

### V. ADDING AND SUBTRACTING SCHEME

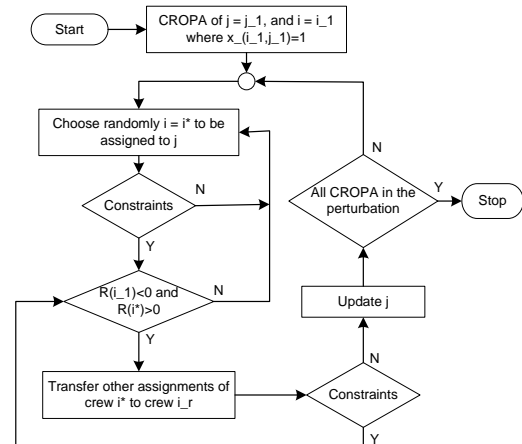


Fig. 6. A method for speeding up the process.

In this research, we use an additional method to speed up the process of decreasing of the objective function's value. For example one perturbation contains  $s$  CROPAs whose crews will be replaced. In the beginning we will replace crews assigned to CROPA-1 to CROPA- $s$ , those are  $i_1, i_2, \dots, i_s$ . For instance for CROPA-1, find a candidate by randomly choosing  $i^* \notin \{i_1, i_2, \dots, i_s\}$ . Notice that the candidate crew must be assigned to CROPAs other than the CROPAs in the observed perturbation. Check whether the candidate crew satisfies all constraints or not. If it is, we

transfer the assignment on CROPA-1 to crew- $i^*$ . If not, we find another candidate to replace  $i_1$  on assignment of CROPA-1. After the transfer, it is possible that crew  $i_1$  has very small number of assignments so that its discrepancy to the average flying time is large. It will show from the value of the function

$$R(i) = \frac{\sum_{j=1}^K d_j x_{ij} - D}{D}$$

If  $R(i_1) < 0$  and  $R(i^*) > 0$ , one or more crew- $i^*$ 's assignment needs to be transferred to crew- $i_1$  if the constraints are satisfied. This process will be done up to either  $R(i_1) > 0$  or  $R(i^*) < 0$ . The replacement of the crew is made for all CROPAs in the observed perturbation.

We compare the results from the algorithm with and without the scheme. The first algorithm performs better than the latter one. It begins to decrease quickly to reach 1.3378 which means in average the discrepancy between the real and ideal flying times is 1.3378 hours. The discrepancy from the algorithm without the scheme is 2.3849 hours. These results can be seen in Fig. 7.

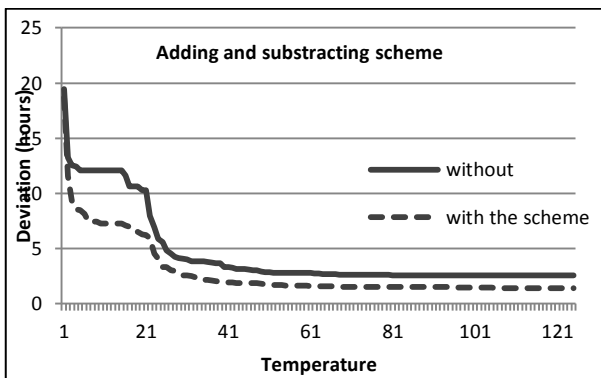


Fig.7. Results from algorithm with and without the scheme

## VI. RESULTS

Results of the crew assignments on the data from GI are shown in this section. From 702 flights paired in 225 CROPAs, the minimum number of crews needed is 84 crews. In practise, the number will exceed it regarding the crews' planned day-offs due to annual leaves, training times, medical examination, and others, that will reduce their availability of on the working days.

The algorithms on minimization of the crew number and the crew's schedule has been made into a user-friendly program that is shown in Fig. 8.

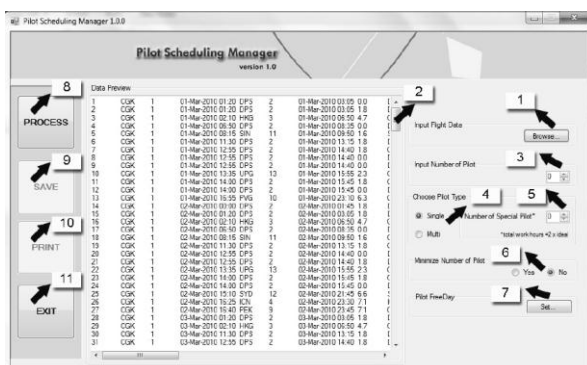


Fig. 8. A prototype of pilot scheduling manager.

First input data from MS excel file using (1) "Browse". In (2) "Preview window", the view of the input and output data will appear. The number of crews is typed in (3). If the minimization process is chosen in (6), this number is the initial one and the results will show the minimum number and the schedule. If not, the results will give the schedule and the back up crews that are reserved to be assigned to each CROPA, in case the assigned crews cannot work. The back up crews are sought through the same procedure after the best solution had been found.

The program allows a number of crews that have a special arrangement on their total hours. For example, they need 2 times larger than the total amount assigned to all crews. The number of these crews is inputted in (5). For example, if 2 crews have this privilege, the procedure will run with the number of the crews plus two additional dummy crews. At the end of the process, the workload of these dummy crews is added to the privileged crews, so they will have twice amount of flying time than others. On the other hand, crews with particular planned day-offs are inputted in (7).

In future research, flight rotation pairings of the input will be examined with simulated annealing method. Furthermore, the scheduling program is required to be improved with other functions that made the scheduling process as more adaptive as possible to the real implementation. For example, there is a possibility to re-run the process when half amount of the assignments has been occurred, and others.

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